Numerical and Analytical Analysis of Elastic Rotor Natural Frequency

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***Abstract* –In this paper simulation model which enables quick analysis of elastic rotor natural frequency modes is developed using Matlab. This simulation model enables users to get dependency diagram of natural frequency in relation to diameter and length of the rotor,density of the material or modulus of elasticity. Testing of the model is done using numerical analysis in SolidWorks software.**

***Keywords* – Elastic rotor, natural frequency, critical speed, numerical analysis.**

1. Introduction

Industrial world today can't be imagined without rotary machinery. Regular and reliable operation of mechanical assemblies at high speeds is one of the biggest problems that machine industry faces since the Industrial Revolution.Any form of high speed rotation in mechanical assemblies carries out a number of problems such as vibration and deformation of rotating elements, friction, the problem of choosing proper bearings, the problem of active control, etc [1]. Also, if there is elastic rotor in that kind ofassemblies a couple of very important problems occur[2], [3], [4].One of these problems regarding elastic rotor is the problem of passing through the critical speeds of rotation.A rotor is generally considered to be flexible in nature when it operates close to or above its critical speed. The rule of a thumb is to consider a rotor flexible if it operates at 70% of 1^{st}critical speed or faster. For elastic rotor it can be said that it reaches critical speed when the speed of its rotation corresponds to one of its natural frequencies. This type of speed is studied within a branch of physics known as rotordynamics, which deals with rotational, or angular, motion. A rotating object, such as a propeller or a centrifugal pump, must often pass through one or more of its critical speeds as it accelerates or decelerates. While operating at critical speed, these objects vibrate at high amplitude, which can cause damage.

All elastic bodies have natural frequencies depending on how many degrees of freedom they are described. The object will be in resonance when it is excited with frequency which is equal to one of its natural frequencies. In that case object will suffer from large vibrational amplitudes. In a musical instrument, for example, this resonance is desirable because it causes a natural amplification of the instrument's sound. In rotordynamics, however, this resonance is undesirable because it makes the rotor vibrate strongly, what can lead to its damage.

There are a number of excitations that can cause resonance,one of them is rotational motionwith present unbalance. When an unbalanced rotor rotates at natural frequency, causes resonance, this is called critical speed. Rotating mechanical objects must be designed to pass quickly through these speeds so the amplified vibration that occurs at this velocity does not result in damage [5]. According to the above mentioned physical properties of elastic rotor, it is important, for engineers, to be able to quickly and easily find values of natural frequencies and critical speeds of flexible rotors so they can design it in a way that it does not operate too long into areas of critical speeds.Like others machine elements, elastic rotors also have more than one natural frequency, for practical purpose it is important to be familiar with at least first three.

Experiments performed so far have shown that every elastic rotor have almost the same deformation form when it passes through first three natural frequencies, as it can be seen from Figure 1. [2].

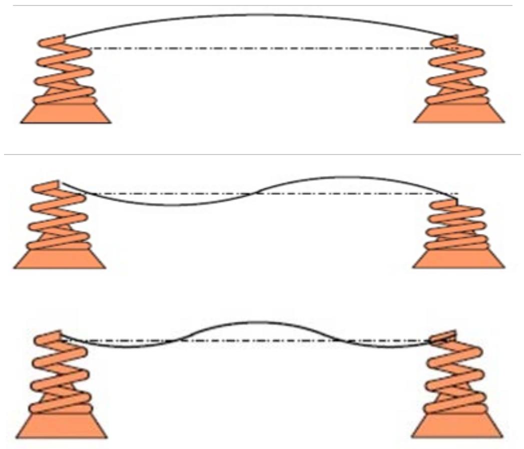

Figure 1.First threenormal modes of elastic rotors

2. Simulation model

2.1. Mathematical background

When elastic rotor (shaft) rotates it may well go into transverse oscillations. If the rotor is out of balance, the resulting centrifugal force will induce rotor to vibrate. When rotor rotates at the speed equal to the natural frequency of the transverse oscillations, this vibration becomes large and shows up as a whirling of the shaft. It also occurs at multiples of the resonant speed.This can be very damaging to heavy rotary machines such as turbine generator sets and the system must be carefully balanced to reduce this effect and designed to have a natural frequency different to the speed of rotation.When starting or stopping such machinery, the critical speeds must be avoided to prevent damage to the bearings and turbine blades [6].Consider a weightless rotor with a mass M at the middle (Figure 2.). Suppose the centre of the mass is not on the centre line. When the rotor rotates, centrifugal force will cause it to bend out. Let the deflection of the rotor be r $[m]$. The distance to the centre of gravity is then $r + e$. The shaft rotates at ω $[rad/s]$. The transverse stiffness is k $[N/m]$.

The deflection force is:

$$F = k * r \tag{1}$$

The centrifugal force is:

$$F_c = M * \omega^2 * (r + e) \tag{2}$$

Forces can be equated:

$$k * r = M * \omega^2 * (r + e) \tag{3}$$

from which:

$$r = \frac{M * \omega^2 * e}{k * \left(1 - \frac{M * \omega^2}{k}\right)} \tag{4}$$

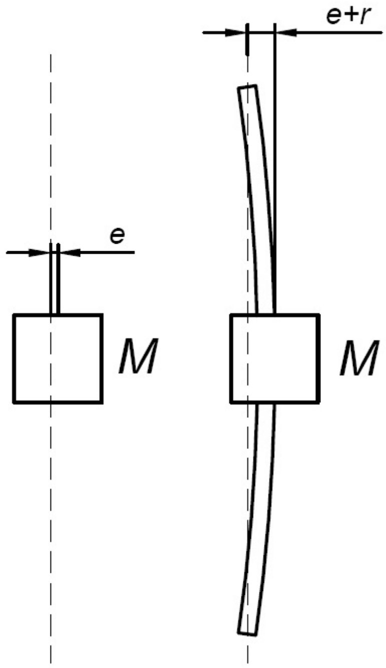

Figure 2.Rotor with a mass M at the middle

It is already known that:

$$\frac{k}{M} = {\omega_n}^2 \tag{5}$$

Where ω_n is critical rotationalspeed. Now r is:

$$r = \frac{e}{\left(\frac{\omega_n}{\omega}\right) - 1} \tag{6}$$

From last equation it can be seen that when$\omega_n = \omega$, r will give infinity, this means that no matter how small the imbalance distance e is, the shaft amplitude will grow to infinity. The frequencies at which whirling occurs are calculated by the same methods as for transverse vibrations of beams and the derivation is not given here. How to calculate natural frequencies depends on the type of the supports, modulus of elasticity E $[GPa]$, moment of inertia I $[m^4]$, weight per square meter w $[N/m]$ and length of the shaft$L$ $[m]$.

$$I = \frac{\pi * d^4}{2} \; [m^4] \tag{7}$$

$$w = \rho * A * g \; [N/m] \tag{8}$$

Where $\rho\ [kg/m^3]$ is density of the material,$g\ [m/s^2]$ is gravity acceleration,$d\ [m]$ is diameter of the rotor and A is cross section of the rotor:

$$A = \frac{d^2 * \pi}{4}\ [m^2] \qquad (9)$$

- Simply supported, the ends are free to rotate normal to the axis (e.g. self-aligning bearings):

$$f = \frac{\pi}{2} * n^2 * \sqrt{\frac{g * E * I}{w * L^4}}\ [Hz] \qquad (10)$$

n – number of natural frequency (mode), it must me integer number, for example, to calculate first natural frequency n must be 1, for second n must be 2 and so on.

- Fixed ends, fixed bearings:

$$f = \frac{\pi}{2} * \left(n + \frac{1}{2}\right)^2 * \sqrt{\frac{g * E * I}{w * L^4}}\ [Hz] \qquad (11)$$

- Cantilever, fixed bearings at one end:

$$f = \frac{\pi}{2} * \left(n - \frac{1}{2}\right)^2 * \sqrt{\frac{g * E * I}{w * L^4}}\ [Hz] \qquad (12)$$

2.2. Model development

Simulation model is developed using Matlab software and its programming capability. The goal of this simulation model is to enable engineers and users who are not deeply familiar with the theory of natural frequency and critical speeds to make analysis of natural frequency for simple rotor and to avoid need for buying expensive software for numerical analysis.Also, this simulation model enables user to investigate how different variables effect on the natural frequencies of the rotor. Results are given in form of values for natural frequencies and dependency diagrams of all natural frequencies in relation to the change of diameter, length, density or modulus of elasticity. Figure 3.shows flow chart of developed simulation model.

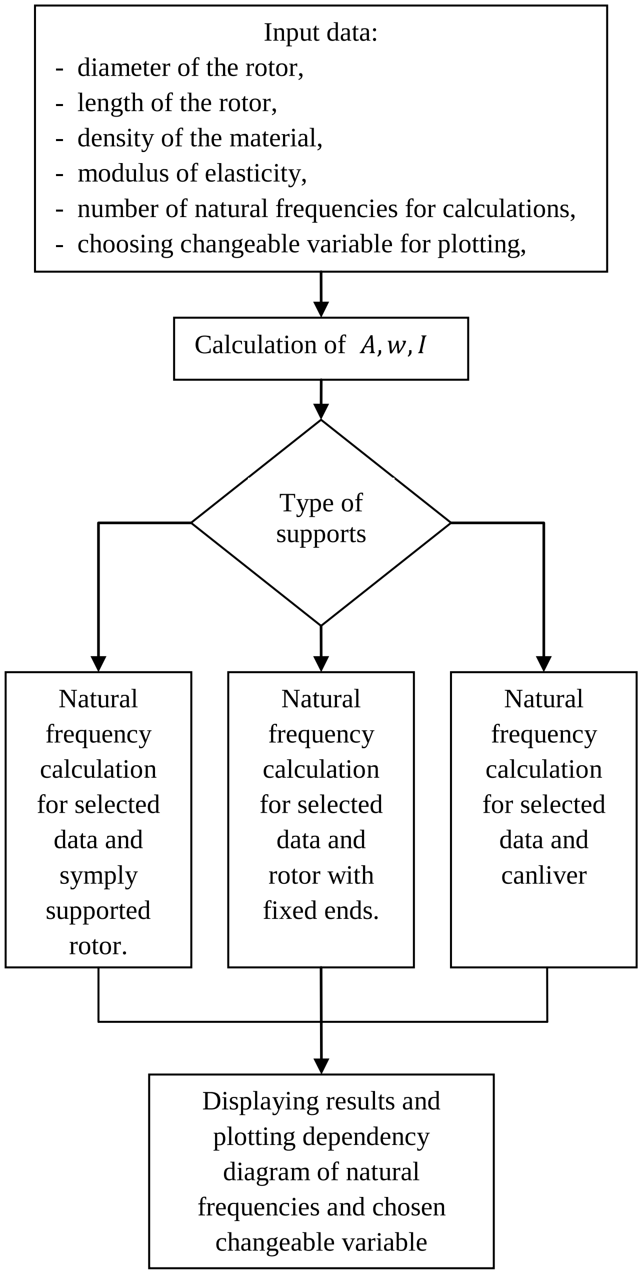

Figure 3.Flow chart of developed simulation model

To test developed simulation model, elastic rotor with following characteristic is selected: diameter of the rotor:$d = 0.04\ m$, length of the rotor: $L =\ 6\ m$, density of the material:$\rho = 7830\ kg/m^3$, modulus of elasticity:$E = 205e9\ GPa$.First three natural frequencies are investigated for three types of supports.

Results are given in Table 1.

Table 1. Values of natural frequencies for above mentioned example and for three types of supports

Type of supports	First natural frequency [Hz]	Second natural frequency [Hz]	Third natural frequency [Hz]
Simply supported	2.23	8.93	20.09
Fixed ends	5.02	13.95	27.34
Cantilever	0.55	5.02	13.95

As it is mentioned before, this simulation model enables quick analysis of natural frequency behavior in relation to the change of some variables.This can be very useful to the designers of rotary machinery to find out which variable they can change to make influence to the value of natural frequency and to calculate critical speed of elastic rotor.

Figures 4-7shows how first natural frequency behave for change of diameter, length, density and modulus of elasticity.

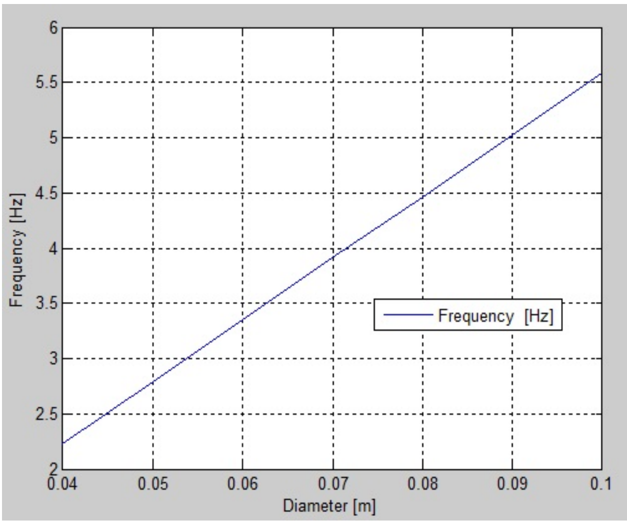

Figure 4.Dependency diagram of first natural frequency in relation to change of diameter for simply supported elastic rotor

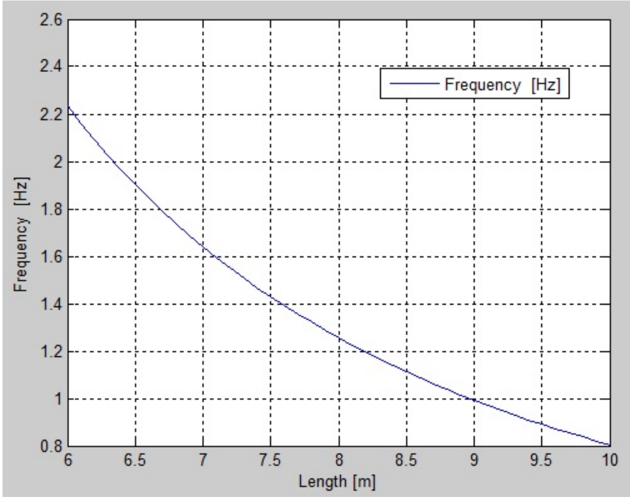

Figure 5.Dependency diagram of first natural frequency in relation to change of length for simply supported elastic rotor

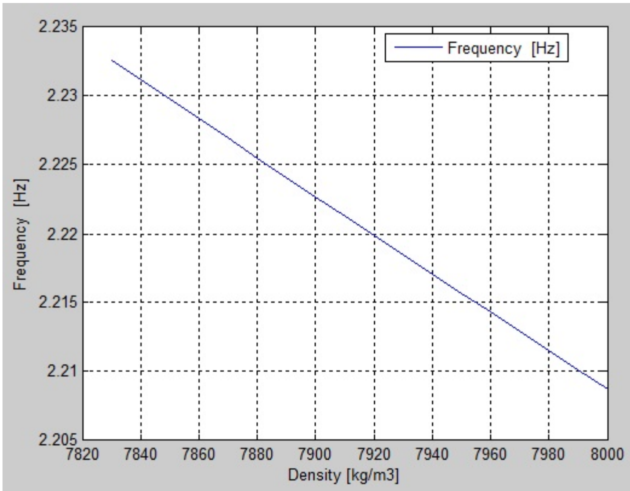

Figure 6.Dependency diagram of first natural frequency in relation to change of density for simply supported elastic rotor

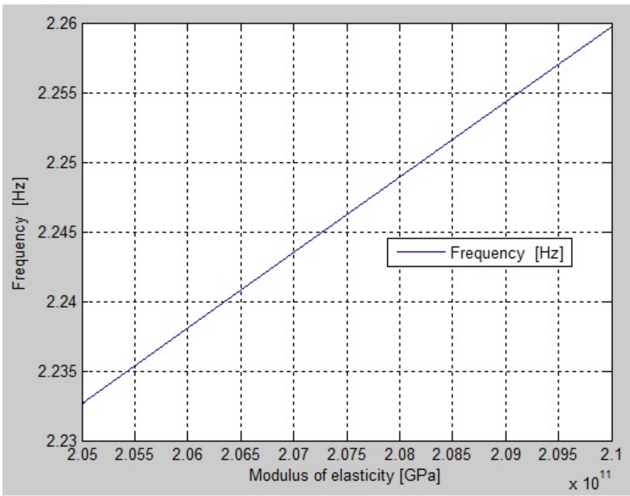

Figure 7. Dependency diagram of first natural frequency in relation to change of modulus of elasticity for simply supported elastic rotor

3. Numerical analysis

Numerical analysis is performed to test the above mentioned developed simulation model. Analysis is done using SolidWorkssoftware and its simulation module.Same input data from chapter 2 are selected. Results can be compared to the result from developed simulation model.

Imported properties of numerical simulation are:
Software: Solid Works,
Solver: FFEPlus,

Mesh properties:

- Element size 19.6121 mm
- Tolerance 0.980606 mm
- Mesh quality High
- Total nodes 11587
- Total elements 5458

Material of the rotor: Carbon Steel

Results are shown in Table 2, also, how first three natural frequency look like can be seen from Figures 8,9 and 10.

Table 2. Values of natural frequencies from numerical analysis

Type of supports	First natural frequency [Hz]	Second natural frequency [Hz]	Third natural frequency [Hz]
Simply supported,	2.24	8.87	19.75
Fixed ends	5.06	13.95	27.35
Cantilever	0.79	4.95	13.95

It is important to notice that Solid Works software gives all natural frequency in two perpendicular planes. That is reason why second natural frequency is basically third mode shape in Solid Works, as is shown in Figure 9.

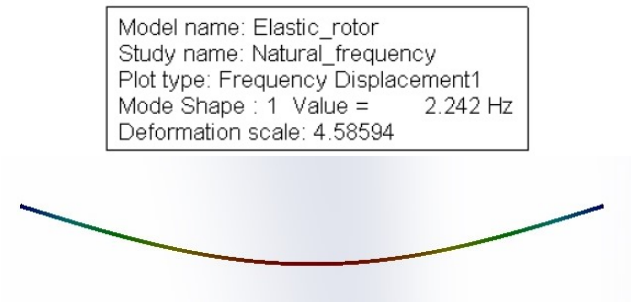

Figure 8. Elastic rotor first normal mode for selected input data (simply supported)

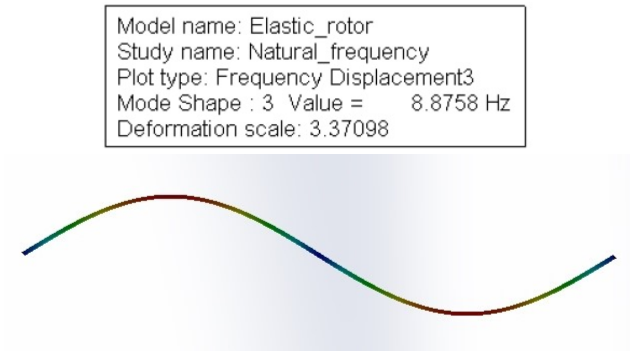

Figure 9.Elastic rotor second normal mode for selected input data(simply supported)

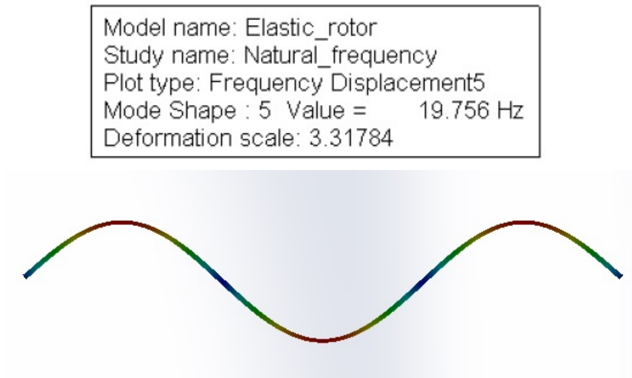

Figure 10. Elastic rotor third normal mode for selected input data (simply supported)

4. Results

Table 3. Values of natural freqencies from analytical and numerical analysis

Type of supports	First nat. fre. [Hz]		Second nat. fre. [Hz]		Third nat. fre.[Hz]	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
Simply sup.	2.23	2.24	8.93	8.87	20.09	19.75
Fixed ends	5.02	5.06	13.95	13.95	27.34	27.35
Canti.	0.55	0.79	5.02	4.95	13.95	13.95

Table 3.shows comparing results of analytical and numerical analysis. Good mach of results can be seen from the table. In this way, the developed simulation model and numerical analysis are confirmed.

5. Conclusion

This paper successfully presented developed simulation model for analysis of elastic rotor natural frequencies.Using this model it is possible to find natural frequencies for elastic rotor without deeply understanding theory of natural frequency and without buying expensive software for numerical analysis.Also, using this model it is possible to analyze how the change of diameter, length, density and modulus of elasticity effect natural frequencies of the elastic rotor.Developed model is confirmed using numerical analysis which is done by the commercial software Solid Works.

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