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Loss-minimal Algorithmic Trading Based on Levy Processes

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***Abstract* – In this paper we optimize portfolios assuming that the value of the portfolio follows a Lévy process. First we identify the parameters of the underlying Lévy process and then portfolio optimization is performed by maximizing the probability of positive return. The method has been tested by extensive performance analysis on Forex and SP 500 historical time series. The proposed trading algorithm has achieved 4.9\% percent yearly return on average without leverage which proves its applicability to algorithmic trading.**

***Keywords* – Levy Process, Algorithmic trading, Portfolio optimization, FOREX.**

1. Introduction

Portfolio optimization was first investigated by Markowitzin the context of diversification to minimize the associated risk and maximize predictability [1]. Since the first results, many papers have been dealing with portfolio optimization[2, 3]. A usual approach is finding the portfolio which exhibits predictability and minimal risk [4, 5]. Another approach is to identify mean reverting portfolios where trading is launched when being out of the mean and a complementary action is taken after reverting to the mean [6, 7]. The traditional strategies are concerned with optimizing the mean reverting parameter (or predictability) which lends itself to analytical tractability by solving a generalized eigenvalue problem [8]. In this paper we generalize the previous results by assuming that the portfolio price follows a Lévy process which is the more general model than the traditional Ornstein-Uhlenbeck process [9]. Based on this assumption, after identifying the parameters of the underlying Lévy process, such portfolio is selected which maximizes the probability of positive return. The corresponding computational model is depicted by the figure bellow:

The structure of the paper is as follows.

- in Section~2, the model and notations are introduced
- together with the Lévy model;
- in Section~3 we identify the model parameters;
- in Section~4 we deal with portfolio optimization subject to minimizing the probability of loss;
- in Section~5, the trading strategy is discussed;
- in Section~6, a detailed performance analysis of the new method compared to traditional (e.g. mean reverting strategies) and the corresponding numerical results are given;
- in Section~7, conclusions are drawn.

2. System model

In this section we describe the system model, which we use in this paper.

2.1 Asset parameters and investors

The sell price of a given asset i at a given time instant t is denoted by $s_i(t)$. The same asset can be bought at a price denoted by $b_i(t)$, $b_i(t) > s_i(t)$, where $b_i(t) - s_i(t)$ is the so-called bid-ask spread. Assuming a given number of assets $i = 1, \ldots, N$ available for trading, we can describe the corresponding bid and ask prices by vectors

$$(\underline{b}(t),\ \underline{s}(t)).$$

Here we take $\underline{x}$ as a column vector, and $\underline{x}'$ denotes its transposed version. An investor is supposed to have a combination of assets in which the numbers of different assets at hand are denoted by $\underline{n}(t)$ (which is a sparse integer vector including many zero components for low transaction cost) and it represents the portfolio. It is noteworthy, that negative components in this vector are also permitted which refer to short positions. Additionally, the available cash the investor has is denoted by $c(t)$. As a result, the investor is fully described by the following quantities

$$(\underline{n}(t), c(t)).$$

The prompt value of assets depends on the sell price $\underline{s}(t)$ if the amount of assets are positive (long position), or on the buy price $\underline{b}(t)$ if the amount of assets are negative (short position). As a result, the prompt value of the portfolio held by the investor is given as

$$P(t) = \max(\underline{0}', \ \underline{n}'(t)) \ \underline{s}(t) + \min(\underline{0}', \ \underline{n}'(t)) \ \underline{b}(t), \qquad (1)$$

where vector $\underline{0}$ refers to the all zero vectors (each component is zero).

The wealth of the investor (the available cash plus the value the portfolio) at time instant t is

$$r(t) = P(t) + c(t).$$

This expression describes the amount of money the investor could obtain immediately, if he/she decides to quit.

At time instance t, the investor may make a decision to change the portfolio by $\underline{\nu}(t)$, where this vector can contain positive and negative integers, based on the type of transactions (i.e. positive value means that the investor buys the corresponding asset, negative value refers to selling). For technical reasons, we assume that at a given time t the investor either buys or sells an asset, but not permitted to buy and sell at the same time. That is, $\underline{\nu}(t)$ contains either only positive or only negative components. The number of assets owned by the investor in the next time instant $t + \Delta t$ is given as

$$\underline{n}(t + \Delta t) = \underline{n}(t) + \underline{\nu}(t),$$

and the cash of the investor is described as

$$c(t + \Delta t) = \begin{cases} c(t) - \underline{\nu}'(t) \ \underline{s}(t) & \text{if the investor sells,} \\ c(t) - \underline{\nu}'(t) \ \underline{b}(t) & \text{if the investor buys.} \end{cases}$$

Note that although in the last term there is a subtraction, the negative components of $\underline{\nu}(t)$ yield an increase. Here, we have neglected the transaction fee. This can, though, be represented in the sell and buy prices (modifying them with the corresponding transaction cost).

The investor follows an acceptable strategy, if his/her wealth increases over time, i.e. for most of the purchases $\nu_i(\tau)$, there exist a T_i, where $r(\tau + T_i) > r(\tau)$. Our objective is to find a portfolio which is not only acceptable, but maximizes the probability of positive return.

3. The Lévy model

As a generalization of the Samuelson model, one can describe financial markets' security prices as Lévy processes. Lévy process $X(t)$ is characterized by the following properties:

- The paths of $X(t)$ are right continuous, with left limits, i.e.,
- $\forall \gamma, \lim_{\varepsilon \to 0+} \Pr X(t+\varepsilon) - X(t) > \gamma = 0.$
- The increments have identical distribution, i.e., for $0 \le s \le t, X(t) - X(s)$ has the same distribution as $X(t-s)$.
- $X(t)$ has independent increments, that is, for $0 \le s \le t, X(t) - X(s)$ is independent of $\{X(u) : u \le s\}$.

There are some well-known properties of Lévy processes given as follows (a detailed treatment of these properties can be found in [9, 10, 11, 12].)

- the simplest Lévy process is the linear drift, which is a deterministic process, of course;
- the Brownian motion is the only (non-deterministic) Lévy process with continuous sample paths;
- other examples of Lévy processes are the Poisson and compound Poisson processes;
- the sum of two Lévy processes is again a Lévy process; thus a Brownian motion with linear drift is also a Lévy Process.

We will focus on the Lévy jump process, which is constructed from the linear drift and the Poisson point process and given as

$$P(t) = P(0) + \delta t + \sum_{k=1}^{N_t} J_k. \qquad (2)$$

Here δ is the drift parameter, $\{N_t\}$ is a Poisson point process, with mean $\mathsf{E}\{N_t\} = \Lambda t$, and J_k is a sequence of independent, identically distributed discrete random numbers with zero mean, i.e. $\mathsf{E}\{J_k\} = 0$. In financial markets, J_k represents the jumps between different ticks in subsequent transactions. As long as the ticks are from a discrete set then J_k must be discrete, too. Unbalanced portfolios (tending down or up) are described by the linear drift term. In balanced portfolios, $\delta = 0$.

3.1. Parameter identification of Lévy processes

Under the conditions described above, the mean of (2) can be calculated as

$$\mathsf{E}\left\{P(t)-P(0)\right\}=\mathsf{E}\left\{\delta t+\sum_{k=1}^{N_t}J_k\right\}$$

$$=\delta t+\mathsf{E}\left\{\sum_{k=1}^{N_t}J_k\right\}=\delta t+\Lambda t\mathsf{E}\left\{J_k\right\}=\delta t.$$

Thus, the statistical mean of $\mathsf{E}\left\{P(t+\tau)-P(t)\right\}/\tau$ yields parameter δ, for any T

$$\hat{\delta}=\frac{1}{K}\sum_{i=1}^{K}\left(\frac{P(iT)-P((i-1)T)}{T}\right)=\frac{P(KT)-P(0)}{KT} \qquad (3)$$

In most cases δ proved to be a very small number, hence we assume that its effect can be neglected in subsequent transactions i.e. $\delta\Delta t\approx 0$.

Since $\{N_t\}$ is a Poisson point process, its mean, Λ can be estimated by the average number of transactions,

$$\hat{\Lambda}=\frac{1}{KT}\sum_{i=1}^{K}\left(N_{(i+1)T}-N_{iT}\right)=\frac{N_{(K+1)T}-N_T}{KT} \qquad (4)$$

for any T.

Finally, the statistics of J_k must be simply counted by relative frequencies,

$$\hat{p}_i=\frac{1}{K}\sum_{k=1}^{K}I(J_k=i), \qquad (5)$$

where $I(.)$ is the indicator function and $\hat{p}_i$ estimates the probability $\mathsf{P}\left\{J_k=i\right\}$. Since the convolution of such random numbers will play a major role in the following, the 2-step probabilities can be calculated as:

$$\hat{p}_n^{(2)}=(\hat{p}*\hat{p})_n=\sum_i\hat{p}_i\hat{p}_{n-i}. \qquad (6)$$

Note that the limit of the sum depends on the number of non-zero elements in $\hat{p}$. That is, if $\hat{p}_i=0$ for all $|\mathrm{i}|>1$, the sum contains three elements only ($i\in\{-1,0,1\}$).

As a generalization, the n-step probabilities can be easily constructed; n such numbers generate a process with distribution $\hat{p}^{(n)}$, such that

$$\hat{p}^{(n)}=\underbrace{\hat{p}*\hat{p}*\hat{p}*\ldots*\hat{p}}_{n}\text{ ,or equivalently}$$

$$\hat{p}_i^{(n)}=\sum_j\hat{p}_j^{(n-1)}\hat{p}_{i-j}\,. \qquad (7)$$

4. Portfolio optimization by maximizing the probability of positive return

We start from (2) corresponding to the Wiener process $W(s)$ and we make a transformation to another random variable r, which satisfies to find the parameters thatyield the maximum probability for the minimum expected profit. That is, the loss (compared to the minimum expected profit) is minimized in probability.

We assume that parameters, i.e. $\underline{p},\Lambda$ could be derived for all possible portfolios and the investor defines a time frame t which limits the holding time of the portfolio and an expected return r. Without the loss of generality we assume that the investor opens a position (either short, or long) at $t=0$ at the price of $P(0)$ and observes the price movements, i.e. $\mathrm{P}(\mathrm{t})$. Parameter μ denotes the expected future value of the portfolio.

Note that for the sake of generality, parameters μ and $P(0)$ could be either positive, or negative and they could also have opposite signs. Negative $P(0)$ means: opening a long position yields cash, opening short position costs money. Similarly, negative $P(t)$ means: closing the long position needs cash, closing a short position results in cash.

If t is short $(t\approx 0)$, and $\mu>P(0)$, i.e., larger price is expected than the starting price, then the investor should take a long position, independently of the sign of $P(0)$: it is expected that the price of the portfolio will go up, and thus only long position yields profit. Otherwise, if $\mu<P(0)$, the investor should take a short position, expecting positive profit independently of the sign of $\mathrm{P}(0)$.If $\mu\approx P(0)$ then the investor should not take any position. If there is an open position, the investor can either win, or loose, depending on the noise (the random nature of the system).

If t is large, $P(t)$ must be compared against $P(0)(1+r)^t$. That is, holding the portfolio for a t period is expected to yield interest which could be obtained from alternative investments. The interest grows exponentially with time t. When a long position was opened at $t=0$, then $P(t)<P(0)(1+r)^t$ describes the situation of loss,

i.e. the prompt value of the portfolio is smaller than the price of the alternative investment. On the other hand, when a short position was opened at time $t = 0$, then $P(t) > P(0)(1+r)^t$ describes the event of loss. Figure 1 shows the region of loss in blue color.

Depending on the sign of the expected future price (we will denote the expected future price as μ in the following discussion) and the starting price $(\mathrm{P}(0))$ and their relation, six different cases could happen. These cases are depicted in Figure 1.In the odd cases (left hand side column), μ (the expected future price) is greater than $P(0)$, thus long position should be taken. On the contrary, even cases (right hand side column), μ is smaller than $P(0)$, thus short positions should be opened at $t = 0$. Please note that Case~1 can be considered as minus one times the portfolio of Case~6 (taking a long position in a positive portfolio is the same as taking a short position in the negative/opposite portfolio). Similarly, Case~2 can be transformed into Case~5. Case~3 and Case~4 could be also replaced by each other. As a result, one can say that one column (three cases) describes all possible events.

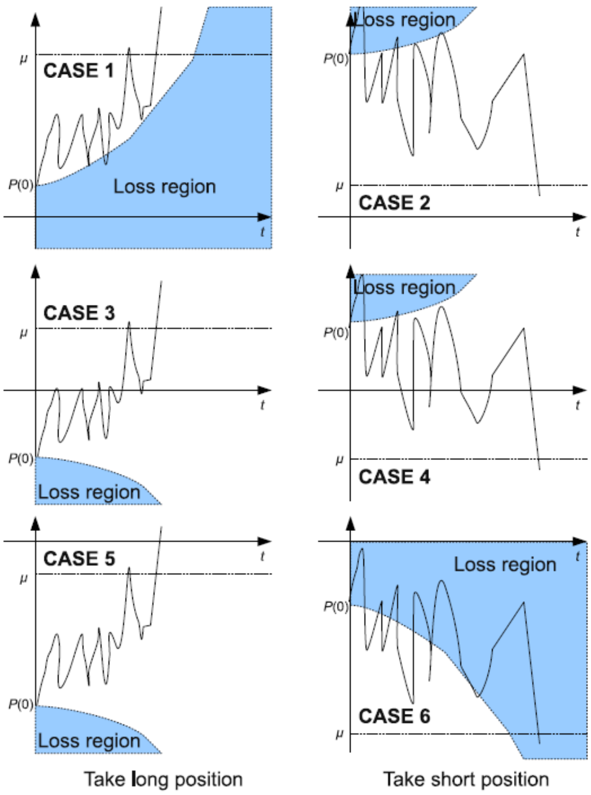

Figure 1 . Six different cases for $\mathrm{P}(0)$ *and* μ *values.*

The probability of loss (P_L) could be expressed as the probability that the worth of the purchased portfolio is less than a financial instrument with the expected return r for the given time t, i.e.

$$P_L = \mathbf{P}\left\{(\mu - P(0)\left(P(t) - (1+r)^t P(0)\right) < 0\right\}, \qquad (8)$$

which yields

$$P_L = \begin{cases} \mathbf{P}\left\{P(t) < (1+r)^t P(0)\right\}, & \text{if } \mu > \mathrm{P}(0), \text{ i.e., long position is taken,} \\ \mathbf{P}\left\{P(t) > (1+r)^t P(0)\right\}, & \text{if } \mu < \mathrm{P}(0), \text{ i.e., short position is taken.} \end{cases}$$

For the sake of simplicity, we continue with only one expression, with double inequality sign £ (or double binary operator, like $\pm$). The upper inequality (or binary operator) always refers to the long position, whilst the lower one refers to the short position related equations,

$$P_L = \mathbf{P}\left\{P(t) \text{ £ } (1+r)^t P(0)\right\}. \qquad (9)$$

In Lévy jump process portfolios, $P(t)$ must be substituted from (2) into (9).Thus one gets

$$P_L = \mathbf{P}\left\{P(0)(1+r)^t \text{ ¤ } P(0) + \delta t + \sum_{k=1}^{N_t} J_k\right\},$$

,which can be rewritten as

$$P_L = \mathbf{P}\left\{\sum_{k=1}^{N_t} J_k \text{ £ } P(0)\left((1+r)^t - 1\right) - \delta t\right\}. \qquad (10)$$

The left hand side of the inequality in the probability depends on several independent random variables. The number of points in N_t is a Poisson random variable. Since J_k follows the discrete distribution of p_i, their sum is another discrete random variable. The distribution of n independent J_k random variables (we will denote it as J_k^n in the following) has an important role here. Thus, we should first construct their distribution based on (7).

To calculate the probability of the portfolio value remaining under a certain limit, the discrete probability values must be summed up accordingly. Thus, (10) can be calculated as

$$P_L = \sum_{n=0}^{\infty} \mathbf{P}\left\{N_t = n\right\} \mathrm{P}\left\{J_k^n < P(0)\left((1+r)^t - 1\right) - \delta t\right.$$
$$= \sum_{n=0}^{\infty} \frac{e^{-\Lambda t}(\Lambda t)^n}{n!} \cdot \sum_{i=-\infty}^{\left\lfloor P(0)\left((1+r)^t - 1\right) - \delta t \right\rfloor} p_i^{(n)}. \qquad (11)$$

For $n = 0$, there is obviously no transaction in the time period t), $p_i^{(0)} = I(i = 0)$.

In (11) for technical reasons, we have considered only one case: we have put the inequality sign $<$ instead of the original double inequality £ . We will follow only this path of (11) in the following, due to space limitations. The other case can be similarly constructed; however the limits of the sum must be changed accordingly.

Note that although the second sum starts from $-\infty$, its real starting value depends on the measured properties of subsequent transactions. If $J_k = -l$ is the lowest value which happens in the stock market, $J_k^n < -ln$ has zero probability ($p_i^{(n)} = 0, \forall i < -ln$), thus the sum can be started from $-ln$. For example, in FOREX, subsequent EURUSD jumps are limited by approximately 50 ticks (see Figure 2).

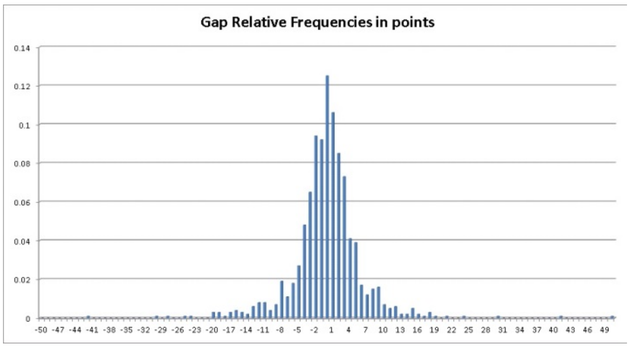

Figure 1. Relative frequencies of different hop sizes for the FOREX EURUSD charts

With these probabilities at hand one can launch the appropriate trading actions.

5. Numerical results

We analyzed the performance of trading based on the Lévy model in two different cases:

In the first case, we did the back testing for a long period (almost 3 years) on a daily timeframe to reveal the capability of our model in selecting the best portfolio out of the pool.

In the second case, we did forward testing on shorter periods, such as 5 minute timeframe, for one month and investigated the performance of the results.

In both cases we have a sparse portfolio, i.e. a combination of maximum 3 assets out of four (EURUSD, GBPUSD, AUDUSD, NZDUSD) in the FOREX market was chosen by the Lévy based trading algorithm. Our pool, which we call it fully-hedged pool, contains those kinds of portfolios which has equal volume of short and long positions.

The trading strategy applied the following decisions:

- Entry point -- Due to different time scales, the two cases differ in the position-opening schedule. The following two subsections give details about the entry point decisions.
- No Stop Loss point -- we do not have any stop loss, instead we have maximum waiting time which is one of our predefined critical parameters. If this period has elapsed, the positions are closed independently of the actual prices. The two cases have different waiting times, which are given in the following two subsections.
- Take Profit point -- Due to different time scales, the two cases differ in the portfolio closing decision. The following two subsections give details about the taken profit points.
- Risk/Money management -- as we do not have an explicit stop loss point, we trade with full margin and thus we are not able to really manage the risk. Risk management is limited to setting the leverage ratio. The two cases applied different leverages which are given in the following two subsections.

The initial deposit was 1000$ in both cases.

5.1. Back-testing

Back-testing was performed on daily time frame. That is, all positions were opened at the beginning of the day and were closed at the end of the day.

The trade was immediately started right after the portfolio was identified.

In our test we used 10 days of waiting time. We closed the trades at the end of each day if our profit was positive. If not, at the end of the next day the same comparison was done ....etc. After 10 days, the portfolio is closed if still remained in the negative profit region.

We did not use any leverage for back-testing. The results are indicated in Figure 3.

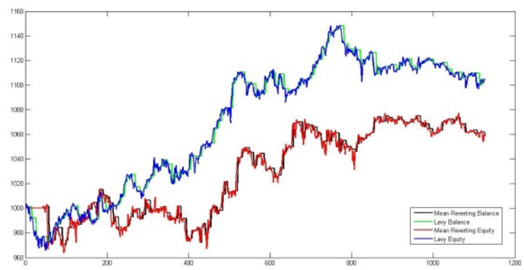

Figure 2: The performance of the proposed method in back-testing mode: balanceand equity of the proposed method (Lévy) and the mean reverting algorithm [8].

As one can see from the figure, the profit was approximately 11%; the maximum drawdown was less than 5% in this three year period.

5.2. Forward-testing

In forward-testing, we switched from the daily timeframe to five minute timeframe.

The process of searching the best portfolio proved to be a time consuming task. During the identification of the best portfolio, the price usually changed. Therefore -- despite of the strategy in back-testing -- we waited till we get the same price, or lower price, and the position was opened after the criteria was met.

Here, we used 50 minutes of waiting time. We had implicit take profit point which is equal to our expected interest rate (r), in our case 10$ per 1 standard lot was the profit point.

We used leverage of 100 for forward-testing. The results are indicated in Figure 4. On the horizontal axis, the transaction steps are given

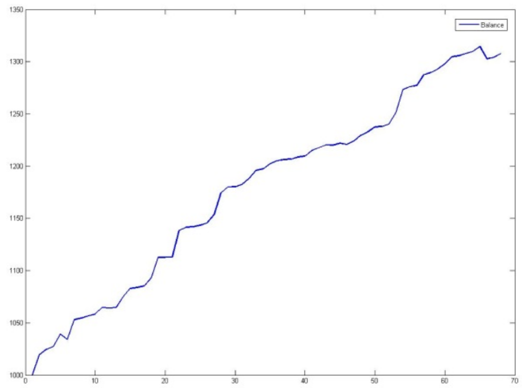

Figure 3. The performance of the proposed method in forward-testing mode: balance

As visible from the figure, the profit was 31%; the maximum drawdown was approximately 1% in the three week period.

6. Conclusions

In this paper, we have introduced a novel portfolio optimization method and trading algorithm based on the Lévy model. By using the Lévy model, one can capture a wild class of random portfolio value sequence. Estimates for the identification of model parameters were also given. Based on the Lévy process one can analytically calculate the probability of positive returns and the optimal portfolio holding time. As a result, the optimal portfolio could be selected which maximizes the probability of positive return. The method was tested by back- and forward testing and its performance outdid the traditional mean reverting trading.

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