Mathematical Model of Induction Heating Processes in Axial Symmetric Inductor-Detail Systems

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***Abstract* - The wide variety of models for analysis of processes in the inductor-detail systems makes it necessary to summarize them. This is a difficult task because of the variety of inductor-detail system configurations. This paper aims to present a multi physics mathematical model for complex analysis of electromagnetic and thermal fields in axial symmetric systems inductor-detail.**

Keywords – Mathematical model, Induction heating, Axial symmetric model

1. Introduction

A huge part of the studied literature sources presents theoretical studies based on models of inductor-detail systems analyzing the electromagnetic and thermal fields. The presented mathematical models are for a particular type of inductor-detail system, processing a ferromagnetic [1], [2], [3], [6], [7], [9], [10], [12], [13], [14], [15], [17], or a non-magnetic detail [4], [7], [11] mainly supplied from a current source and less than a voltage source [4], [15].

The analysis of the temperature field is realized with the boundary conditions of the third and fourth ranges. This requires the determination of the coefficients of convection and radiation, which are rarely given as a function of the temperature [1], [9], [10]. Often it is worked with constant values of these coefficients [2], [3], [4], [7], [11], [12], [13], [14], [15].

The purpose of this paper is to present a multi physics mathematical model and algorithm for complex analysis of electromagnetic and temperature fields in axial symmetric inductor-detail systems. The kind of power source and the magnetic properties of the detail, according to the intensity of the magnetic field and the temperature, are estimated.

2. Mathematical model of the electromagnetic field

Figure 1, shows geometry of the model of axial symmetric inductor-detail system. The inductor is single winding type and the detail is solid cylinder of ferromagnetic or non-magnetic material.

Three distinct areas are determined: Ω_1 - air, Ω_2 - inducer; Ω_3 - detail. With $d\Omega$ are mentioned the external and internal borders in the model.

In determining the size of the overhead area it is assumed that the distance from the center of the pattern to the lowest limit must be five times larger than the distance from the center to the outer limits of the investigated object [5],[8].

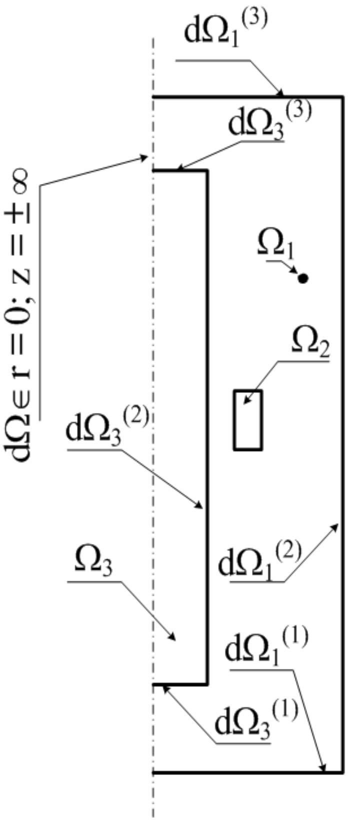

Figure 1. The geometry of the model.

The mathematical model describing the electromagnetic processes that take place during the induction heating is based on Maxwell's equations [21] of the quasi steady fields in conductive medium:

$$\nabla \times \mathbf{H} = \gamma \cdot \mathbf{E} = \mathbf{J}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (1)$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \cdot \mathbf{D} = 0$$

Wherein: $\mathbf{H}, A/m$ - the intensity vector of the magnetic field; $\mathbf{J}, A/m^2$ - a vector of the current density; $\gamma, S/m$ - a specific electric conductivity; $\mathbf{E}, V/m$ - a vector of the electric field strength; $\mathbf{B}, T$ - a magnetic induction vector; t, s - time; $\mathbf{D}, C/m^2$ - a vector of the electric induction.

In the analysis of an electromagnetic field a magnetic vector potential is used [2]:

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (2)$$

$$\nabla \cdot \mathbf{A} = 0 \qquad (3)$$

Wherein: $\mathbf{A}, Wb/m$ – a magnetic vector potential.

It is assumed that the power source in the system could determine the sinusoidal varying electromagnetic quantities. It is taken into account their phase shift too.

On the basis of equations (1), (2) and (3) an electromagnetic field in complex form is described by equations (4) and (5).

This corresponds to the case when the inductor-detail system is supplied with a current or voltage source both for ferromagnetic and nonmagnetic details:

$$\nabla \times \left(\frac{1}{\mu(H,T)} \cdot \nabla \times \dot{\mathbf{A}} \right) + j \cdot \omega \cdot \gamma(T) \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \qquad (4)$$

$$-\frac{1}{\mu} \nabla^2 \dot{\mathbf{A}} + j \cdot \omega \cdot \gamma(T) \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \qquad (5)$$

Wherein: $\dot{\mathbf{J}}_c, A/m^2$ - a vector of the current density in the inductor, $\dot{U}_c, V$ - full voltage on the inductor, T, K - absolute temperature, $\mu, H/m$ - an absolute magnetic permeability (it depends on the intensity of the magnetic field and temperature), L, m - length of the conductor of the inductor.

In cases where the variables are non-sinusoidal, the equivalent sinusoidal functions are estimated [20]. In the stated dependencies the specific electrical conductivity of the detail is set as a function of the temperature.

It is known from source [19], that during the heating of ferromagnetic details, the magnetic permeability is affected by the temperature at values lower than that corresponding to the Curie point. At higher temperatures the detail is losing its magnetic properties.

This fact allows the mathematical model of the electromagnetic field to be examined in two separate steps: step 1 - at temperatures below the Curie point, and step 2 - at a temperature equal to or higher than the Curie point temperature.

Taking into account the property from [21]:

$$\nabla \times \nabla \times \dot{\mathbf{A}} = -\nabla^2 \dot{\mathbf{A}} \qquad (6)$$

The model of the electromagnetic field for each area is presented as follows:

<u>Step 1:</u>

$$\nabla^2 \dot{\mathbf{A}} = 0 \quad \in \Omega_1$$
$$-\frac{1}{\mu} \nabla^2 \dot{\mathbf{A}} + j \cdot \omega \cdot \gamma \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \quad \in \Omega_2 \qquad (7)$$
$$\nabla \times \left(\frac{1}{\mu(H,T)} \nabla \times \dot{\mathbf{A}} \right) + j \cdot \omega \cdot \gamma \cdot (T) \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \quad \in \Omega_3$$

<u>Step 2:</u>

$$\nabla^2 \dot{\mathbf{A}} = 0 \quad \in \Omega_1$$
$$-\frac{1}{\mu} \nabla^2 \dot{\mathbf{A}} + j \cdot \omega \cdot \gamma \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \quad \in \Omega_2 \qquad (8)$$
$$-\frac{1}{\mu} \nabla^2 \dot{\mathbf{A}} + j \cdot \omega \cdot \gamma(T) \cdot \dot{\mathbf{A}} = \dot{\mathbf{J}}_c + \frac{\gamma \cdot \dot{U}_c}{L} \cdot \mathbf{e}_\varphi \quad \in \Omega_3$$

Upon heating of non-magnetic details the model of the system is determined by the system of equations (8) corresponding to the step 2.

3. Mathematical model of the temperature field.

The temperature field, which is generated and distributed in detail is described by the following differential equation [20]:

$$\rho(T) \cdot c(T) \cdot \frac{\partial T}{\partial t} = \nabla \cdot (\lambda(T) \cdot \nabla T) + q_v \in \Omega_3 \qquad (9)$$

Wherein : $\rho, kg/m^3$ - density of the material as a function of the temperature of the detail; $c, J/kg.K$ - a specific heat capacity as a function of the temperature; $\lambda, W/m.K$ - a thermal conductivity as a function of the temperature; $q_V, W/m^3$ - a bulk density of the power determined by the magnitude of the induced eddy currents in the detail:

$$q_v = \frac{1}{2}\cdot\omega^2\cdot\gamma(T)\cdot\dot{\mathbf{A}}^2 \tag{10}$$

4. Multi physics mathematical model for complex analysis.

In designing the multi physics model for analysis of the electromagnetic and temperature fields the following assumptions are made:

- The electromagnetic field is considered as continuous at the boundary between two mediums with different values of the magnetic permeability;
- The electromagnetic field decays off at five times larger distance than the largest radial dimension of the system-inductor detail;
- The temperature field is only considered in the detail and it is assumed that the heating process is adiabatic;
- It is assumed that the temperature of the inductor is constant.

The general form of the multi physics mathematical model of the inductor-detail system in cylindrical coordinates is as follows:

Step 1:

$$\begin{aligned}
&\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial r\cdot\dot{\mathbf{A}}}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)=0 && \in\Omega_1\\
&-\frac{1}{\mu}\cdot\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial r\cdot\dot{\mathbf{A}}}{\partial r}\right)-\frac{1}{\mu}\cdot\frac{\partial}{\partial z}\left(\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)+j\cdot\omega\cdot\gamma\cdot\dot{\mathbf{A}}=\\
&=\dot{\mathbf{J}}_c+\frac{\gamma\cdot\dot{U}_c}{2\cdot\pi\cdot r}\cdot\mathbf{e}_\varphi && \in\Omega_2\\
&\frac{\partial}{\partial r}\left(\frac{1}{\mu(H,T)}\frac{1}{r}\frac{\partial r\dot{\mathbf{A}}}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\mu(H,T)}\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)+j\cdot\omega\cdot\gamma(T)\cdot\dot{\mathbf{A}}=\\
&=\dot{\mathbf{J}}_c+\frac{\gamma\cdot\dot{U}_c}{2\cdot\pi\cdot r}\cdot\mathbf{e}_\varphi && \in\Omega_3\\
&\rho(T)\cdot c(T)\cdot\frac{\partial T}{\partial t}=\frac{1}{r}\cdot\frac{\partial}{\partial r}\cdot\left(\lambda(T)\cdot r\cdot\frac{\partial T}{\partial r}\right)+\lambda(T)\cdot\frac{\partial^2 T}{\partial z^2}+q_v && \in\Omega_3
\end{aligned} \tag{11}$$

Step 2:

$$\begin{aligned}
&\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial r\cdot\dot{\mathbf{A}}}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)=0 && \in\Omega_1\\
&-\frac{1}{\mu}\cdot\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial r\cdot\dot{\mathbf{A}}}{\partial r}\right)-\frac{1}{\mu}\cdot\frac{\partial}{\partial z}\left(\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)+j\cdot\omega\cdot\gamma\cdot\dot{\mathbf{A}}=\\
&=\dot{\mathbf{J}}_c+\frac{\gamma\cdot\dot{U}_c}{2\cdot\pi\cdot r}\cdot\mathbf{e}_\varphi && \in\Omega_2\\
&-\frac{1}{\mu}\cdot\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial r\cdot\dot{\mathbf{A}}}{\partial r}\right)-\frac{1}{\mu}\cdot\frac{\partial}{\partial z}\left(\frac{\partial\dot{\mathbf{A}}}{\partial z}\right)+j\cdot\omega\cdot\gamma(T)\cdot\dot{\mathbf{A}}=\\
&=\dot{\mathbf{J}}_c+\frac{\gamma\cdot\dot{U}_c}{2\cdot\pi\cdot r}\cdot\mathbf{e}_\varphi && \in\Omega_3\\
&\rho(T)\cdot c(T)\cdot\frac{\partial T}{\partial t}=\frac{1}{r}\cdot\frac{\partial}{\partial r}\cdot\left(\lambda(T)\cdot r\cdot\frac{\partial T}{\partial r}\right)+\lambda(T)\cdot\frac{\partial^2 T}{\partial z^2}+q_v && \in\Omega_3
\end{aligned} \tag{12}$$

The boundary conditions assigned at the boundaries of the model shown on Figure 1 are as follows:

- **Electromagnetic field:**

- Boundaries: $d\Omega\in(r=0, z=\mp\infty);\ d\Omega_1^{(1)};\ d\Omega_1^{(2)};\ d\Omega_1^{(3)}$

$$\dot{\mathbf{A}}=0 \tag{13}$$

- Boundary: $d\Omega_2$

$$\mathbf{n}\times\left(\frac{1}{\mu_2}\nabla\times\dot{\mathbf{A}}_2-\frac{1}{\mu_0}\nabla\times\dot{\mathbf{A}}_1\right)=0 \tag{14}$$

- Boundaries: $d\Omega_3^{(1)};\ d\Omega_3^{(2)};\ d\Omega_3^{(3)}$

$$\mathbf{n}\times\left(\frac{1}{\mu_3}\nabla\times\dot{\mathbf{A}}_3-\frac{1}{\mu_0}\nabla\times\dot{\mathbf{A}}_1\right)=0 \tag{15}$$

- **Temperature field:**

- Boundaries: $d\Omega\in(r=0, z=\mp\infty);\ d\Omega_3^{(1)};\ d\Omega_3^{(2)};\ d\Omega_3^{(3)}$

$$\frac{\partial T}{\partial n}=0 \tag{16}$$

The analysis presented by a mathematical model composed by the systems of equations (11) and (12) is held by the specially developed algorithm presented in Figure 2.

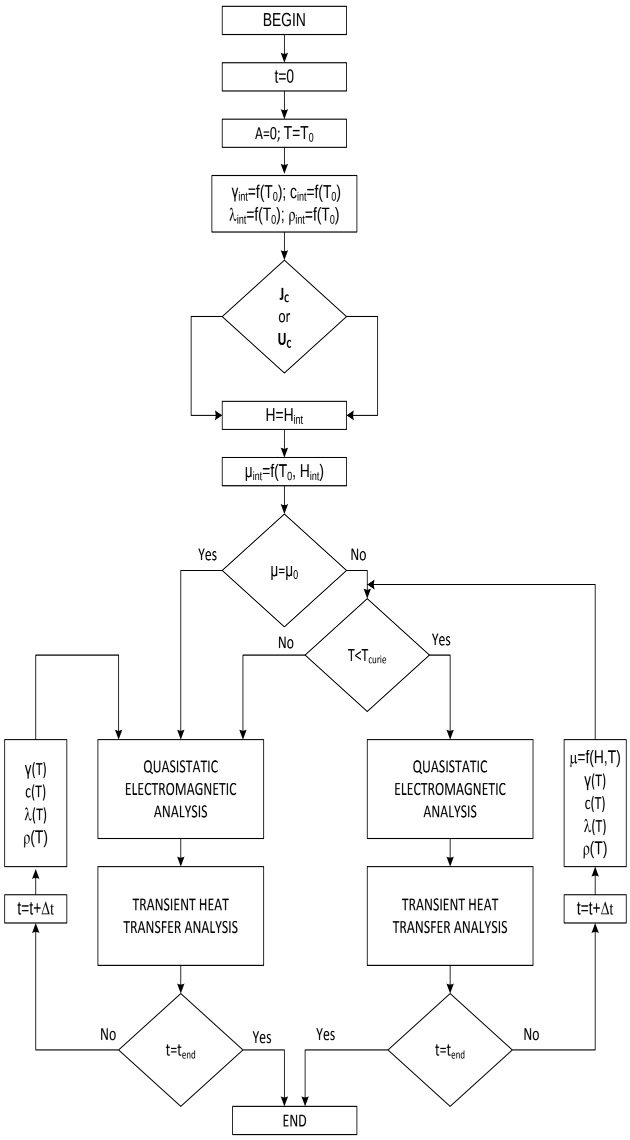

Figure 2. Algorithm for complex analysis of electromagnetic and thermal fields in the inductor-detail systems based on the presented multi physics mathematical model.

5. Conclusions

With the presented mathematical model and the proposed algorithm for analysis of electromagnetic and thermal fields it is given an approach which is aimed to summarize the methodology for implementation of models of axial symmetric inductor-detail systems. It is applicable to both regimes of simultaneously and stepwise heating.

In the case of continuous-sequential regime of heating (hardening), it is imperative to set a boundary condition of a third kind.

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