Self-Tuning Control of Linear Systems Followed by Deadzones

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***Abstract* — The aim of the present paper is to increase the efficiency of self-tuning generalized minimum variance (GMV) control of linear time-invariant (LTI) systems followed by deadzone nonlinearities. An approach, based on reordering of observations to be processed for the reconstruction of an unknown internal signal that acts between LTI system and a static nonlinear block of the closed-loop Wiener system, has been developed. The results of GMV self-tuning control of the second order LTI system with an ordinary deadzone are given.**

***Keywords* — Self-tuning, GMV control, Wiener system, deadzone, observations.**

1. Introduction

Static nonlinearity, such as a deadzone, characterizes the response of the system to small signal values and is common in most control systems [1], especially, in hydraulic and electro-mechanical ones. Frequently, nonlinearities of actuator devices occur on the output of the system to be controlled that limit the system performance considerably [2, 3]. Therefore, Wiener systems, consisting of a linear dynamic block followed by a static nonlinear one, are considered to be suitable for a broad spectrum of nonlinearities [4].

Diverse compensators have been made up, and various techniques have been proposed to compensate deadzone nonlinearities: the general actuator deadzone compensator adaptive control schemes have been developed in [5,6]; an intelligent compensator of deadzone nonlinearity, based on tuning neural network algorithms, is given in [7], while a fuzzy logic is used in [8], etc. Thus, various compensators have been tried to adjust the performance of control systems by reducing parasitic effects of a deadzone. On the other hand, we describe here the approach based on reconstruction of the unmeasurable internal signal, acting between both blocks of the Wiener system, without designing special and complex enough compensators. Afterwards, instead of measurable output of the Wiener system, affected by the deadzone nonlinearity, the reconstructed signal, free of the parasitic effects, could be used for self-tuning control of the LTI system.

In Section 2, a statement of the problem is presented. In Section 3, the general method is given for determining an auxiliary signal that corresponds to the extracted version of the internal one. Section 4 presents the simulation results of the Wiener system to be controlled. Section 5 contains conclusions.

2. Statement of the problem

Assume that the output $y(k), \forall k = \overline{1, N}$ of the self-tuning LTI system, controlled by the GMV control law, has failed to correctly track the given set-point signal $\{r(k)\}$ in an noisy environment (Fig. 1). Here signals: $\{r(k)\}$ is a set-point signal; $\{y(k)\}$ is an output signal; $\{n(k)\}$ is a measurement noise. The reason is the deadzone nonlinearity in the output of the given

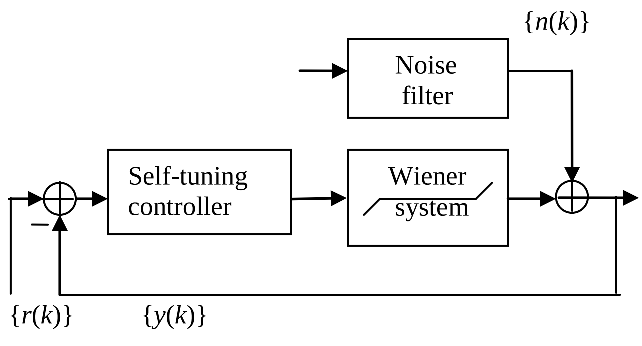

Figure 1. A closed-loop Wiener system, consisting of LTI system and a deadzone (Fig. 2) with a self-tuning GMV controller. The process noise filter is not shown here.

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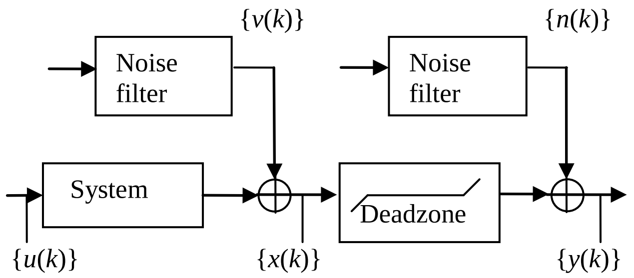

Figure 2. An open-loop Wiener system, consisting of LTI system and a deadzone. The internal unmeasurable noisy signal {x(k)} is acting between the LTI system and the deadzone. The output of the LTI system is corrupted by process noise {v(k)}, while the output of the deadzone by the measurement noise {n(k)}. Other signals: {u(k)} is an input signal; {y(k)} is an output signal.

LTI system. Thus, the system to be controlled indeed consists of a linear block followed by a static nonlinearity (Fig. 2), and it is called a Wiener system. The linear block of the Wiener system can be represented by an LTI system with the transfer function of the form

$$G(q^{-1},\mathbf{\Theta}) = \frac{b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}} = \frac{B(q^{-1},\mathbf{b})}{1 + A(q^{-1},\mathbf{a})},$$
$$\mathbf{\Theta}^T = (b_1,\ldots,b_m,a_1,\ldots,a_m)$$

with a finite number of parameters

$$\mathbf{b}^T = (b_1,\ldots,b_m), \quad \mathbf{a}^T = (a_1,\ldots,a_m) \tag{1}$$

that are determined from the set $\mathbf{\Omega}$ of permissible parameter values $\mathbf{\Theta}$. Here q^{-1} is a time-shift operator. The unmeasurable internal intermediate signal

$$x(k) = \frac{B(q^{-1},\mathbf{b})}{1 + A(q^{-1},\mathbf{a})} u(k) + v(k), \tag{2}$$

$\forall k = \overline{1,N}$ generated by the linear part of the Wiener system as a response to the input $u(k)\,\forall k = \overline{1,N}$ and corrupted by the additive noise $v(k)\,\forall k = \overline{1,N}$, is acting on the static nonlinear part $f(\cdot,d)$ with $d > 0$, i.e.,

$$y(k) = f(\cdot,d) + n(k). \tag{3}$$

Here the nonlinear part of the Wiener system is a deadzone of the form

$$f(\cdot,d) = \begin{cases} x(k) - d & if \;\; x(k) \geq d \\ 0 & if \;\; -d < x(k) < d. \\ x(k) + d & if \;\; x(k) \leq -d \end{cases} \tag{4}$$

The process noise $v(k), \forall k = \overline{1,N}$ and the measurement noise $n(k), \forall k = \overline{1,N}$ are added to the intermediate signal $x(k), \forall k = \overline{1,N}$ and to the output signal $y(k), \forall k = \overline{1,N}$, respectively.

The aim of the given paper is to avoid the parasitic effects of nonlinear distortions, induced by the deadzone nonlinearity (4), that appear in the output $\{y(k)\}$ of the self-tuned GMV control LTI system (Fig. 1) without designing special enough compensators.

3. Internal signal reconstruction

Now, to calculate the auxiliary signal $\hat{x}(k), \forall k = \overline{1,N}$, i.e., the estimate of unmeasurable $x(k), \forall k = \overline{1,N}$, of the closed-loop Wiener system (Fig. 1) by processing N pairs of observations of the set-point signal $\{r(k)\}$ and noisy cut-off output $\{y(k)\}$, one can use the finite impulse response (FIR) model

$$y(k) = \gamma_0 r(k) + \gamma_1 r(k-1) + \ldots + \gamma_\mu r(k-\mu) + n(k), \tag{5}$$

$\forall k = \overline{\mu+1,N}$, or using the expression in a matrix form

$$\mathbf{V} = \mathbf{\Pi}\boldsymbol{\gamma} + \mathbf{n}. \tag{6}$$

Here

$$\mathbf{V} = (y(\mu+1),\, y(\mu+2),\ldots, y(N-1),\, y(N))^T \tag{7}$$

is the vector $(N-\mu)\times 1$;

$$\mathbf{\Pi} = \begin{bmatrix} r(\mu+1) & r(\mu)\ldots & r(2) & r(1) \\ r(\mu+2) & r(\mu+1)\ldots & r(3) & r(2) \\ \vdots & \vdots\ldots & \vdots & \vdots \\ r(N) & r(N-1)\ldots & r(N-\mu+1) & r(N-\mu) \end{bmatrix} \tag{8}$$

is the full rank regression matrix $(N-\mu)\times(\mu+1)$, consisting only of observations of the non-noisy set-point signal $\{r(k)\}$; $\boldsymbol{\gamma}^T=(\gamma_0,\gamma_1,\ldots,\gamma_\mu)$ is the vector $(\mu+1)\times 1$ of unknown parameters of FIR model (5);

$$\mathbf{n}=(n(\mu+1),n(\mu+2),\ldots,n(N-1),n(N))^T \qquad (9)$$

is the vector $(N-\mu)\times 1$, consisting of the values $\{n(k)\}$; μ is the order of FIR filter that can be arbitrarily large, but fixed.

The reason for the use of the FIR model is as follows: the dependence of some regressors on the process output will be facilitated, and the assumption of ordinary LS that the regressors depend only on the non-noisy input signal, will be satisfied. Next, let us rearrange the data in the vector $\mathbf{V}$ in an ascending order of their values, reordering the associated rows of the matrix $\boldsymbol{\Pi}$ as well [9]. We could do that by interchanging equations in the initial system (6). Note that the interchange of equations does not influence the accuracy of LS parameter estimates $\hat{\boldsymbol{\gamma}}^T=(\hat{\gamma}_0,\hat{\gamma}_1,\ldots,\hat{\gamma}_\mu)$ to be calculated. Then, assuming that a deadzone is present in the given closed-loop system, the vector $\mathbf{V}$ and the matrix $\boldsymbol{\Pi}$ should be partitioned into three data sets: the left-hand data set $\mathbf{V}_1=\boldsymbol{\Pi}_1\boldsymbol{\gamma}$, the middle data set $\mathbf{V}_2=\boldsymbol{\Pi}_2\boldsymbol{\gamma}$, and the right-hand data set $\mathbf{V}_3=\boldsymbol{\Pi}_3\boldsymbol{\gamma}$, according to the three regimes of the deadzone nonlinearity. Here $\mathbf{V}_1,\mathbf{V}_2,\mathbf{V}_3$ are $N_1\times 1$, $N_2\times 1$ and $N_3\times 1$ vectors, respectively, $\boldsymbol{\Pi}_1,\boldsymbol{\Pi}_2,\boldsymbol{\Pi}_3$ are $N_1\times(\mu+1)$, $N_2\times(\mu+1)$ and $N_3\times(\mu+1)$ matrices, correspondingly, where $N=N_1+N_2+N_3$. Thus, initial system (6) is reordered into a system

$$\overline{\mathbf{V}}=\overline{\boldsymbol{\Pi}}\boldsymbol{\gamma}+\overline{\mathbf{n}}. \qquad (10)$$

with $\overline{\mathbf{V}}=[\mathbf{V}_1,\mathbf{V}_2,\mathbf{V}_3]^T$, and $\overline{\boldsymbol{\Pi}}=[\boldsymbol{\Pi}_1,\boldsymbol{\Pi}_2,\boldsymbol{\Pi}_3]^T$ by simply interchanging equations in the initial system (6). The left-hand side data set $\mathbf{V}_1$ (N_1 samples) consists of the reordered $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N_1}$, equal or less than negative d, the middle data set $\mathbf{V}_2$ (N_2 samples) consists of the reordered cut-off values $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N_2}$ higher than negative d, but lower than d, and the right-hand side data set $\mathbf{V}_3$ (N_3 samples) consists of the reordered values $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N_3}$ equal to or more than d. Here $\tilde{k}$ is any integer k rearranged in an ascending order, dependent on the reordered values of observations $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N}$.

Hence, the reordered observations with the positive values, equal or higher than d, will be concentrated on the right-hand side set, while the reordered observations with the negative values, equal or smaller than $-d$, on the left-hand side set. The observations of the middle data set of $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N_2}$ are cut-off. It follows that, for a deadzone there exist three regimes in total, but only the systems of linear equations $\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_3$ are valid, because they both are based on the true reordered values of $\{y(k)\}$. The system $\boldsymbol{\Pi}_2$ is not usable because of the presence of cut-off observations. It is easy to separate the particles of $\{\tilde{y}(k)\}$ by deleting the middle data set $\mathbf{V}_2$ (N_2 samples) and the system $\boldsymbol{\Pi}_2$, if the values of the measurement noise $\{n(k)\}$ are small enough. In this case, only $\mathbf{V}_1$, $\mathbf{V}_3$ and $\boldsymbol{\Pi}_1$, $\boldsymbol{\Pi}_3$ are needed for a further extraction of the unmeasurable internal signal $x(k),\forall k=\overline{1,N}$. Note that the estimate $\hat{d}$ of the size d of the deadzone nonlinearity is determined after separating the particles. Then, the reordered values $\tilde{y}(\tilde{k}),\forall\tilde{k}=\overline{1,N_1+N_3}$ in $\mathbf{V}_1$, $\mathbf{V}_3$ ought to be processed by the deadzone inverse according to [5]. This can be done simply by adding a constant $\hat{d}$ equal to the estimated size of the deadzone, too.

To estimate the parameters $\boldsymbol{\gamma}^T=(\gamma_0,\gamma_1,\ldots,\gamma_\mu)$ of FIR model (5), we can use the expression of the form

$$\hat{\boldsymbol{\gamma}}=(\hat{\boldsymbol{\Pi}}^T\hat{\boldsymbol{\Pi}})^{-1}\hat{\boldsymbol{\Pi}}^T\hat{\mathbf{V}}. \qquad (11)$$

Here $\hat{\boldsymbol{\gamma}}^T=(\hat{\gamma}_0,\hat{\gamma}_1,\ldots,\hat{\gamma}_\mu)$ is a vector $(\mu+1)\times 1$ of the estimates of parameters $\boldsymbol{\gamma}^T=(\gamma_0,\gamma_1,\ldots,\gamma_\mu)$, respectively; $\hat{\boldsymbol{\Pi}}$ and $\hat{\mathbf{V}}$ are the same as $\tilde{\boldsymbol{\Pi}}$ and $\tilde{\mathbf{V}}$, correspondingly, where $\boldsymbol{\Pi}_2$ and $\mathbf{V}_2$ are deleted. Then, the estimate $\hat{x}(k),\forall k=\overline{1,N}$ of the intermediate unmeasurable signal $x(k),\forall k=\overline{1,N}$ could be extracted by

$$\hat{x}(k)=\hat{\gamma}_0 r(k)+\hat{\gamma}_1 r(k-1)+\ldots+\hat{\gamma}_\mu r(k-\mu), \qquad (12)$$

where the true values $\gamma_0,\gamma_1,\ldots,\gamma_\mu$ are replaced by their estimates $\hat{\boldsymbol{\gamma}}^T=(\hat{\gamma}_0,\hat{\gamma}_1,\ldots,\hat{\gamma}_\mu)$, respectively, calculated in (11) [9]. Note that the result of this step is the auxiliary signal (12) that is a reconstructed version of the intermediate signal $x(k), \forall k = \overline{1, N}$ of the closed-loop system. To estimate the parameters of FIR model (5) recursively, one can use the well-known ordinary recursive least squares (RLS). Now, instead of the cut-off signal $y(k), \forall k = \overline{1, N},$ that has parasitic effects induced by the deadzone, the current values of $\hat{x}(k), \forall k = \overline{1, N}$ calculated by (12), will be used for the self-tuning GMV control of the basic LTI system.

4. Simulation results

The linear block of the control system (Fig. 2) is given by transfer function $G(q^{-1}, \mathbf{\Theta})$. Here the true parameters are: $b_1 = 1, b_2 = 0.5, a_1 = -1.5, a_2 = 0.7$. The output of the Wiener system $\{y(k)\}$ is generated by (2), (3), where the deadzone size $d = 0.5$. The GMV controller for an LTI system is given by

$$u(k) = \frac{a_1 y(k) + a_2 y(k-1) - b_2 u(k-1) + r(k)}{b_1 + q} \quad (13)$$

with $q = 2.5$. Then, $N = 2000$ data points have been generated with additive process and measurement noises according to

$$x(k) = x(k) + \lambda_1 v(k), y(k) = y(k) + \lambda_2 v(k), \ \forall k = \overline{1, N},$$

respectively. The values of λ_1, λ_2 were chosen so that SNRs (the square root of the ratio of signal and noise variances) were equal to 100 for both noises.

To implement the self-tuning GMV controller correctly, it is necessary, firstly, to estimate the LTI system parameters recursively by the next RLS, and, secondly, in each current operation, to substitute their estimated values in expression (13). Finally, the current estimate $\hat{u}(k), \forall k = \overline{1{,}2000}$ of the value $u(k), \forall k = \overline{1{,}2000}$ is determined by (13), if the cut-off and noisy $y(k)$, and $y(k-1)$ are changed by the current values $\hat{x}(k)$ and $\hat{x}(k-1)$, respectively.

The simulation results, obtained in the noisy frame, are shown in (Figs 3, 4). They imply that the iterative technique, based on the set-point signal $\{r(k)\}$ and output data rearrangement $\{y(k)\}$, and on FIR model (5), rejecting cut-off observations $\{y(k)\}$, could be used for the recursive parametric identification of the Wiener system, if the deadzone inverse were employed (Fig.3) with estimated $\hat{d} = 0.5$. In such a case, the linear block parametric identification results of the Wiener system (Fig. 3d) are better than that shown in (Fig. 4d), where the value of $\hat{b}_2$ is unexpected negative (see Table 1). The true value of b_2 is 0.5.

The accuracy of self-tuning GMV control of an LTI system, followed by the deadzone nonlinearity, with the unmeasurable internal signal extraction and deadzone inverse is increased (Fig. 3b) as compared with the same parametric estimation scheme but without the deadzone inverse (Fig. 4b).

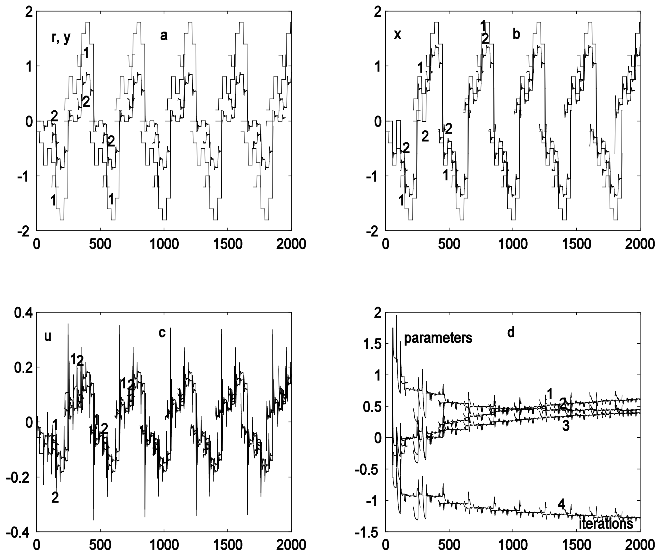

Figure 3. GMV self-tuning control of the Wiener system (Figs 1, 2) with the internal signal $x(k), \forall k = \overline{1{,}2000}$ extraction using the FIR model (5) and employing the inverse deadzone with $\hat{d} = 0.5$, dependent on the processed observations.

Signals in Fig. 3: set-point signal $x(k), \forall k = \overline{1{,}2000}$ (1a, 1b), cut-off and noisy output $y(k), \forall k = \overline{1{,}2000}$ (2a), estimate of intermediate signal $\hat{x}(k), \forall k = \overline{1{,}2000}$ (2b), true input $u(k), \forall k = \overline{1{,}2000}$ (1c), its estimate $\hat{u}(k), \forall k = \overline{1{,}2000}$ (2c), current vector of estimates of parameter $\hat{\mathbf{\Theta}}^T = (\hat{b}_1, \hat{b}_2, \hat{a}_1, \hat{a}_2)$ (d). Curves in (d): $\hat{b}_1$ (1), $\hat{b}_2$ (2), $\hat{a}_1$ (4), $\hat{a}_2$ (3).

Table 1. *Parameter estimates after 2000 iterations. True values are:* $b_1 = 1, b_2 = 0.5, a_1 = -1.5, a_2 = 0.7$.

Fig.	$\hat{b}_1$	$\hat{b}_2$	$\hat{a}_1$	$\hat{a}_2$
3d	0.6127	0.4396	-1.2709	0.3924
4d	1.0029	-0.6672	-1.3726	0.5930

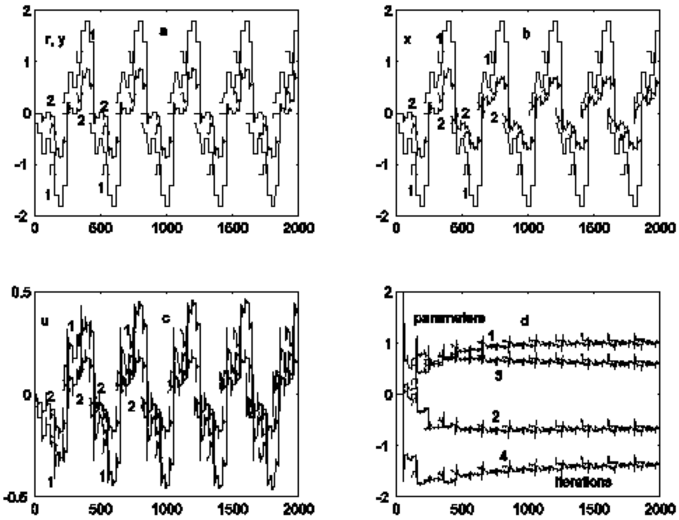

Figure 4. The values and markings are the same as in (Fig.3). The inverse deadzone is not applied here.

5. Conclusions

Neural network and fuzzy logic models are used broadly now for self-tuning control of nonlinear processes. However, it is known that such model representations frequently are too complex and difficult to implement. Besides, their use to adopt the changes in real processes is problematic, too.

In practice, one has an interest in approaches that are computationally efficient, speedy, and easily implemented because "the calculation time for digital controllers is usually between a hundredth of a second and one second". That is why, the original approach is presented here, based on the extraction of an unknown internal intermediate signal, acting between linear and nonlinear blocks of the Wiener system with an ordinary deadzone nonlinearity, avoiding complex compensators. It shows the efficient performance of self-tuning GMV control of the initial LTI system, if the deadzone inverse is also used (Figs 3, 4).

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