Spur Gear Tooth Topology Optimization: Finding Optimal Shell Thickness for Spur Gear Tooth produced using Additive Manufacturing

Adis J. Muminovic, Adil Muminovic, Elmedin Mesic, Isad Saric, Nedim Pervan

University of Sarajevo, Faculty of Mechanical Engineering, Department of Mechanical Design, Vilsonovo setaliste no. 9, 71000 Sarajevo, Bosnia and Herzegovina

Abstract **– There is a lot of papers concerning gears topology optimization, but all of them are limited to the topology optimization of gear body. Gear tooth is not taken into consideration so far. Gear tooth was not optimized before because optimised structure could not be produced. By advancing of additive manufacturing technology, now, gear tooth can be produced in a form of shell body with empty space inside. In this paper, mass of spur gear tooth is optimized in relation to Von-Misses stress on critical places on the gear tooth. Optimization goal is to find optimal shell thickness of shell body spur gear tooth.**

Keywords **– Spur gear tooth, topology, optimization, shell thickness, additive manufacturing.**

1. Introduction

Today, gears are crucial parts of most mechanical power transmissions. History concerning the gears starts, comprising some early examples of gears, in the 4th century BC in China [1].

DOI: 10.18421/TEM83-13
https://dx.doi.org/10.18421/TEM83-13

Corresponding author: Adis J. Muminovic,
University of Sarajevo, Faculty of Mechanical Engineering, Department of Mechanical Design, Sarajevo, Bosnia and Herzegovina
Email: *adis.muminovic@mef.unsa.ba*

Received: 15 May 2019.
Revised: 24 July 2019.
Accepted: 02 August 2019.
Published: 28 August 2019.

In Europe, in contrast, a gear driven device for calculating the motions of stars and planets was discovered in 1901, and it is concluded that it dates form Greek period.

One can claim that gears were invented by the Greek mechanics in Alexandria in the third century B.C., and they were considerably developed by Archimedes, having a wide use in the Roman world, as well [2]. They were used in two main applications: in heavy-duty machines, such as mills and irrigation wheels, by which they transmitted considerable power, and in small-scale water-clocks, calendrical instruments and automata which could be of extraordinary sophistication, incorporating the differential and perhaps the hypoid gear. They became the ancestors of all modern gearing [2]. From Greek period up to industrial revolution there was no major development concerning gears. Modern gears with involute curves profile were invented in the late 19th and at the beginning of 20th century. Development of gears was generally delayed by the production possibilities. Before 19th and 20th century it was impossible to produce gears with involute curves profile. Involute gears are mostly used today. After the inventions of involute gears there was no major development concerning gears up today. A lot of research can be found about optimization of gear profile [3], [4], [5]. Paper [3] presents modification analysis with regard to tooting of spur gears. The aim of the modification the profile on the tooth tip is to compensate for the tooth deflection under loading and to improve the conditions of gear operation. In paper [4] the maximum radial stress via the optimization design of the gear decreases by about 25% in contrast to the maximum radial stress of the gear with the even thickness profile. In paper [5] authors developed method for determining the wear and durability of gears. There is a lot of ongoing research comprising vibration and the noise of gears, like paper [6] in which author investigated relation between design of gears and noise which they produce. There is a lot of similar available researches as in [7], [8]. One more area of research, regarding gears, is in the focus right now. That area is topology optimization of gears. Topology optimization of machine parts become more popular in the last few years because additive manufacturing enables productions of optimized parts [9]. Completely new field of mechanical design is born, popularly called Design for additive manufacturing. Goal of topology optimization is to find minimum mass of gear in correlation to allowed stresses [10]. Paper [11] presents a new approach which aim is to reduce gear vibration and weight by modifying its body structure. As it can be seen from papers [10] and [11], and others similar papers available, all researchers try to optimize gear body, none of them tries to optimize gear tooth (Figure 1. [10]).

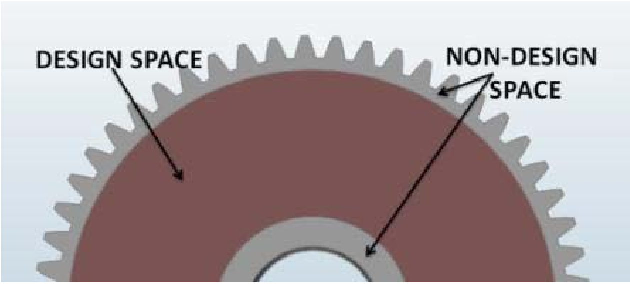

Figure 1. Design space and non-design space. [10]

Goal of this research is to optimize gear tooth. Main hypothesis is that it is possible to reduce mass of gear tooth in correlation to the minimum stresses allowed.

2. Research methodology

Optimization in this research is not done conventionally by using topology optimization software. It is carried out using numerical finite element method (FEM) simulation for already chosen different designs solutions of gear tooth. Chosen design solutions of gear tooth are presented later in this paper. There are a lot of papers about numerical structural analysis of gears and gears tooth's. Bending stress analysis of bevel gears is investigated in paper [12]. Research about stress distribution of gear tooth due to axial misalignment condition is carried out in paper [13]. Research of error analysis on finite element modeling of involute spur gears is done in paper [14]. Finite element analysis of a spur gear tooth using ANSYS and stress reduction by stress relief hole is done in paper [15]. Static analysis bending stress on gear tooth profile by variation of gear parameters with the help of FEA is carried out in paper [16].

2.1. Problem statement

Looking to the results of Von-Misses and principal stresses from papers [13], [15] and [16], we can notice that there are some areas on gear tooth where stresses have lover values (black rectangles at figures 2, 3 and 4). Same results can be seen in many other researches regarding stress distribution in gear tooth. The idea of this research is the claim that it is possible to remove some materials from this areas in correlation to maintain values of stresses within the permitted limits. From figures 2, 3 and 4 it can be noticed that there are two main areas in which stress increases, in contact area and in tooth root area. Contact stress, also known as Hertz pressure [17] always goes deeper inside the gear tooth in relation to tooth root stress (also known as bending stress [16]).

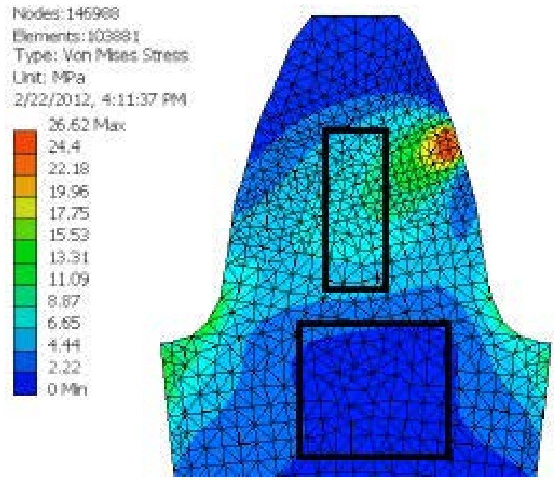

Figure 2. Von-Misses stress in contact region and the tooth root. [13]

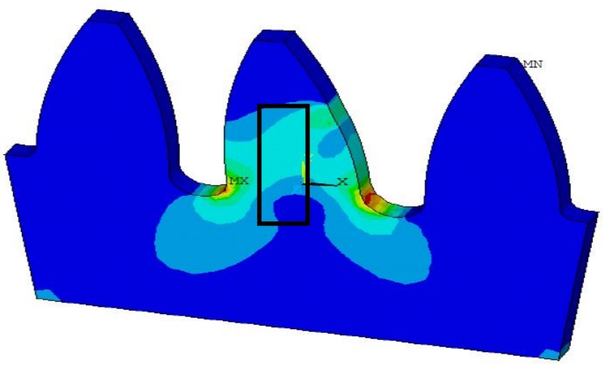

Figure 3. Principal stress in contact region and the tooth root. [15]

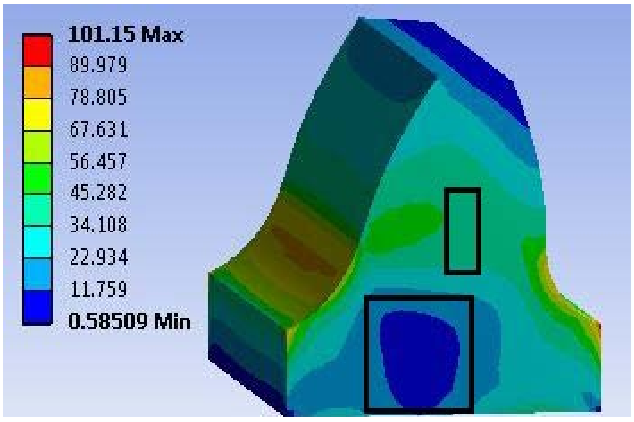

Figure 4. Von-Misses stress in contact region and the tooth root. [16]

Heretofore all gear teeth's are fully body elements. The idea of this research is to model gear tooth as shell body with shell thickness all around the tooth and with free empty space inside the tooth (Figure 5). In this way, mass of the tooth (and the whole gear in total) will be decreased, which is usually main goal of topology optimization. This type of shell body gear tooth can be manufactured only using additive manufacturing. This research is trying to answer one more question which is often asked these days, from gears manufactures who want to produce the gear using additive manufacturing. That question is how thick shell of gear tooth should be in order to maintain values of stresses within the permitted limits.

This research proposes that initial values of shell thickness should be the value of Hertz pressure depth obtained for full body tooth. This value can be increased or decreased regarding values of stresses for shell body gear tooth. To obtain shell thickness following steps needs to be carried out:

- carry out FEM analysis of full body tooth,
- measure depth of Hertz pressure,
- carry out FEM analysis of shell body gear tooth, in which shell thickness is equal to the depth of Hertz pressure,
- according to the values of stresses for shell body gear, increase or decrease the shell thickness.

To carry out above mentioned steps CAD models and FEM models of full body and shell body gear teeth's have to be developed.

2.2. CAD models of gear teeth's

To be able to compare results, and to be able to validate created FEM model, geometrical characteristics and material properties of gear (and gear tooth in the same time) are chosen the same as geometrical characteristics and material properties of gear analyzed in paper [13].

Table 1. Geometrical characteristics of chosen gear

Geometrical characteristics	Units	Value
Number of teeth	-	24
Module	mm	4
Pitch Circle Diameter	mm	96
Base circle diameter	mm	90
Pressure angle	mm	20
Addendum circle diameter	mm	104
Circular pitch	mm	12.56
Face width	mm	10

Two gears tooth are modeled, full body tooth (Figure 5a) and shell body gear tooth (Figure 5b). Shell body tooth is shown with a section, because inside the tooth needs to be shown. Root radius on both tooth's are 1 mm. All radiuses inside shell body tooth are 0.5 mm. Radius on the outer edges on both tooth's are 0.1 mm.

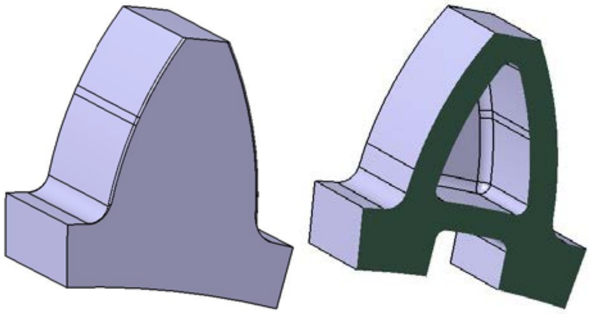

Figure 5. Full body tooth (a), shell body tooth (with a section view) (b).

Material of gears is standard steel with Modulus of elasticity 210 GPa and Yield Strength 350 MPa.

2.3. FEM models of gear teeth's

For numerical structural analysis of both teeth's, finite element method is used. FEM model of shell body tooth is shown on Figure 6.

Figure 6. FEM model of gear tooth

To develop FEM model first step is to choose finite element type. Parabolic tetrahedron volume element is chosen with size 0.3 mm and absolute sag 0.03 mm. Second step is to define boundary conditions. As it is mentioned before, for this study only gear tooth is taken into consideration, rest of the gear is considered as rigid body. Because of this assumption lover surface of gear tooth is constrained with fixed constrains (Figure 6). Final step is to apply loads. Loads are applied according to the Lewis equations [16] in a form of Force density load (F), with different values for different stage of analysis, on the small surface with the width of 0.4 mm (Figure 7). Gear tooth is maximally loaded at the moment when only one tooth pair is in contact. This moment happens when contact line of two gears is on the pitch circle diameter of gears [17]. Force F is applied at the angle of 20° in regard to horizontal axis. FEM model is the same for both teeth's, only difference is the shape of the tooth.

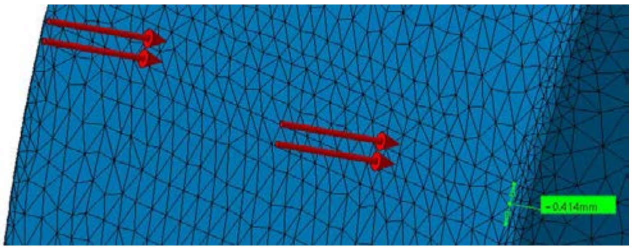

Figure 7. Contact surface of applied loads

3. Results

3.1. Validation of FEM model

After CAD and FEM model formulation. FEM model needs to be validated. Validation of FEM model is carried out comparing results for full body tooth with the results from [16]. Vonn-Misses stress in gear root area is compared for gear root radius 0 mm and force load 2500 N. Figure 8 shows results from [16] and Figure 9 shows results from this research.

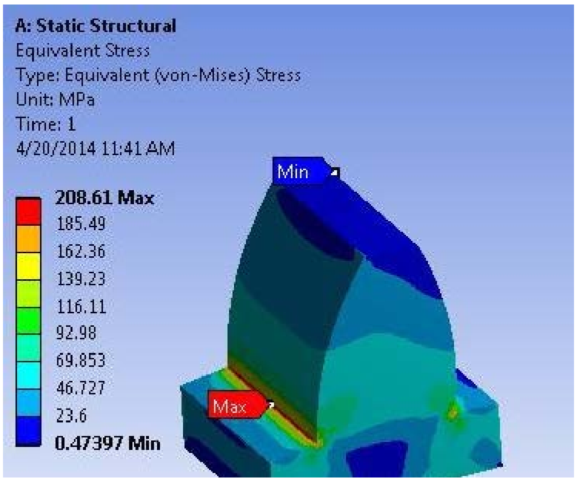

Figure 8. Vonn-Misses stress results in gear root area from [16]

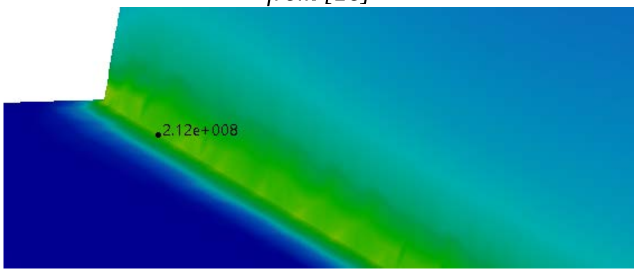

Figure 9. Vonn-Misses stress results in gear root area from this research

Vonn-Misses stress in [16] is 208,61 MPa, in this research it is 212 MPa. In this way FEM model used for this research is validated. It is important to notice that results from [16] can be taken for validation because they are also validated by analytical results in [16].

3.2. Depth of Hertz pressure

As mentioned earlier, this research propose that initial values of shell thickness should be the value of Hertz pressure depth obtained from full body tooth FEM analysis. Numerical FEM analysis is carried out for force F from 2000 N up to 2500 N in increment of 100N. These values are chosen because this gear is initially designed as full body gear for force F of 2500N. Lower values of force F are chosen to investigate its depth of Hertz pressure, which is enough to be taken as shell thickens for different value of force F. Values bigger than 2500 N are not chosen because Hertz pressure for that values is too deep. In that case, there is not any place for material remove and optimization, which will be shown later in this paper. Depth of Hertz pressure is measured manually from FEM results for full body tooth (Figure 10). Point where influences of Hertz pressure become less intense is chosen as depth of the Hertz pressure.

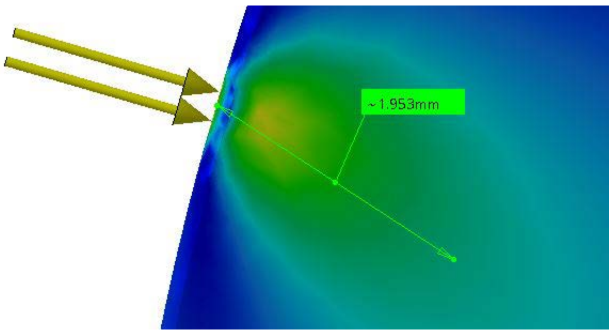

Figure 10. Depth of Hertz pressure (F = 2500 N, full body gear tooth)

Table 2 shows results for depth of Hertz pressure for different values of force load.

Table 2. Depth of Hertz pressure

Force [N]	Depth of Hertz pressure [mm]
2000	1.335
2100	1.369
2200	1.558
2300	1.620
2400	1.766
2500	1.942

3.3. FEM analysis of shell body gear tooth

To find out is it the value of Hertz pressure enough to be taken as shell thickness of shell body gear tooth, numerical structural FEM analysis of shell body gear tooth is carried out. At the same time analysis is carried out for different values of the shell thickness, in order to find out if it is possible to decrease or necessary to increase shell thickness in relation to depth of Hertz pressure. Analysis shows that for shell body tooth it is not enough to look only to Von-Misses stresses at the contact of two teeth's and at the root of the tooth. Other places on the tooth needs to be analyzed (Figures 11 and 12).

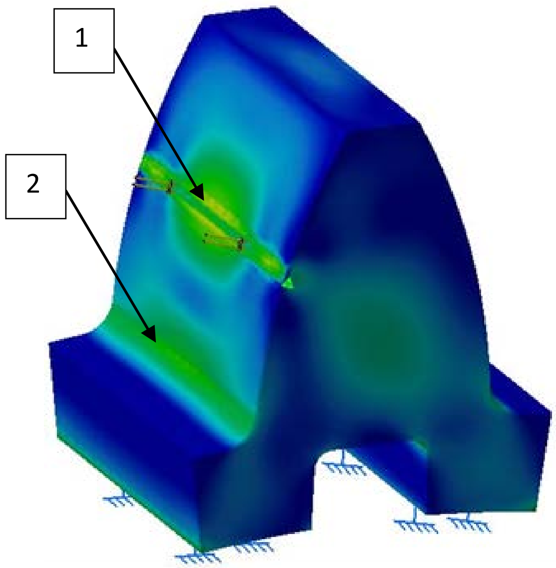

Figure 11. Von-Misses stress of shell body gear tooth. Places of analysis (increased stress).

Table 3. Result values of Von-Misses stresses in MPa for different shell thickness. F = 2500 N.

Place of analysis			1	2	3	4	5	6	7	8
Full body tooth			392	176	-	188	-	-	-	-
Optimized tooth	Shell thickness [mm]	1	960	686	994	390	380	1240	1090	628
		1.5	450	320	447	280	280	606	456	298
		2	422	208	244	230	209	380	330	253
		2.5	409	177	180	202	190	210	235	233

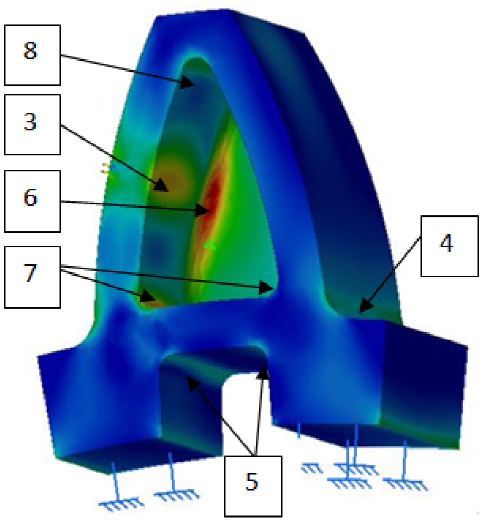

Figure 12. Von-Misses stress of shell body gear tooth. Places of analysis (increased stress).

Table 3 shows results of Von-Mises stress for analyzed places shown on Figures 11 and 12, for different shell thickness and for force F = 2500N. Depth of Hertz pressure for this value of force is 1.942 mm. From Table 3, it can be noticed that values for Von-Misses stress are not significantly increased (all values can be seen in Table 4) in comparison to the values of full body tooth for shell thickness of 2 mm and 2.5 mm for places 1, 2 and 4. These values of shell thickness are close to the values of Hertz pressure. The values of Von-Misses stress in other places are in the same range (Table 3 and Table 4). This can be taken as fact that this gear tooth can be produced using additive manufacturing, such as shell body tooth gear with the almost the same stress conditions. Mass of this shell body gear tooth is reduced by 16.31% for shell thickness of 2 mm and 10.27% for shell thickness of 2.5 mm in comparison to the same full body gear tooth (Table 4). Values of Von-Misses stress for lower values of shell thickness are significantly increased, especially inside the gear tooth, at place number 6. This mean that values lower than depth of Hertz pressure cannot be taken as values for shell thickness. Next step in analysis

Table 4. Increases of Von-Misses stress in % for optimized shell body tooth in relation to full body tooth. Analyzing stress increase in relation to the reduction of mass. Von-Misses stress is given in MPa.

Place of analysis			1	% for place 1	3	% for place 3	4	% for place 4	Mass of tooth	Reduced mass [%]
Full body tooth			392	-	176	-	188	-	0.00662	-
Optimized tooth	Shell thickness [mm]	1	960	144.89	686	289.77	390	107.44	0.00412	37.76
		1.5	450	14.79	320	81.81	280	48.93	0.00494	25.37
		2	422	7.65	208	18.18	230	22.34	0.00554	16.31
		2.5	409	4.33	177	0.568	202	7.44	0.00594	10.27

Table 5. Result values of Von-Misses stresses in MPa for different shell thickness and appropriate forces

Force [N]	Type of tooth	Shell thickness [mm]	Place of analysis							
			1	2	3	4	5	6	7	8
2000	Full body tooth	-	313	142	-	151	-	-	-	-
	Optimized tooth	1.335	420	289	458	249	230	616	456	270
2100	Full body tooth	-	320	147	-	161	-	-	-	-
	Optimized tooth	1.369	407	309	457	256	227	629	480	275
2200	Full body tooth	-	340	157	-	168	-	-	-	-
	Optimized tooth	1.558	382	261	368	242	220	513	384	250
2300	Full body tooth	-	360	163	-	173	-	-	-	-
	Optimized tooth	1.620	386	240	355	240	240	478	411	226
2400	Full body tooth	-	367	170	-	179	-	-	-	-
	Optimized tooth	1.766	410	225	324	240	227	460	360	255
2500	Full body tooth	-	392	176	-	188	-	-	-	-
	Optimized tooth	1.942	427	208	264	235	224	398	336	244

process is to find out is it possible to reduce depth of Hertz pressure, and in that regard reduce shell thickness of shell body gear tooth by reducing value of force F. Results of numerical analysis for different values of force F and corresponding values of shell thickness are showed in Table 5.

From Table 5 it can be seen that by reducing value of force F, depth of Hertz pressure, and the shell thickness also reduce.

Values of Von-Misses stress increase in the same time, especially on the places inside shell body gear. Increase is especially visible on the place number 6. Place number 6 can be noticed as a critical place for shell body gear tooth. This can be explained by the fact that in the case of shell body gear tooth, shell of the gear is loaded with bending stress, which is bigger for bigger values of gear face width.

Future researches on the subject of how to choose shell thickness for gears produced using additive manufacturing should include face width. Stress at the place number 6 can be reduced by putting infill grid inside shell body gear. Infill grid will increase the stiffness of tooth.

4. Conclusion

Additive manufacturing opens up completely new area for design and production of machine elements. Because of the possibilities, which additive manufacturing offers, a lot of machine elements can be redesigned, especially from the aspect of weight reduction thought topology optimization. Additive manufacturing enables production of elements with complex shapes and with holes or empty spaces inside the elements, popularly called shell body elements.

Gears can be considered as one of the most commonly used machine elements, with long history of research. Gear tooth as a part of gear that is usually considered as optimized as possible, a lot of authors will say that there is nothing which can be done regarding gear tooth shape in order to improve it. This is true for outside shape of the gear tooth, but inside of gear tooth can be optimized, especially for gear tooth's with bigger dimensions.

This paper proves that gear tooth can be produced as a shell body gear tooth with the reduction of mass, and at the same time with the stresses in allowed limits. This research and this paper is just a the beginning of research about shell body gear teeth's.

There is a lot of research which needs to be done to fully understand stress conditions of shell body gear tooth and to formulate new ways how to choose or calculate best shell thickness for shell body gear tooth. This paper proposed and proved that initial values of shell thickness should be the depth of Hertz pressure for full body gear tooth. In future research, researchers will try to formulate analytical equation which can be used to calculate shell thickness.

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