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FEM Aided Design of Distribution Transformer

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***Abstract* – Computer aided procedure for design of distribution transformer is implemented on transformer type ETN 50-1-/0.4 a product of “EMO” company. All transformer parameters are calculated in Matlab as well as operational characteristics of voltage variation and efficiency factor at different type of loads and variation of secondary voltage for different secondary currents. The obtained results are compared to the experimental data from the producer. Finite Element Method (FEM) is implemented for calculation of distribution of magnetic flux density in transformer cross section for different operating regimes: no-load and rated load. FEM calculations are based on data obtained from transformer design. They enable electromagnetic processes inside the machine to be analyzed and accurate calculation of magnetic flux density to be done.**

Keywords – distribution transformer, operational characteristics, FEM, magnetic flux density

1. Introduction

Transformer design is always a challenging task with respect to proper definition and calculation of parameter and operational characteristics. The transformer design should be carried out with economical usage of the available materials in order to achieve low costs, low weight, small size and good operational performance. During the designing stage, emphasis should be put on reduction of costs by saving materials. Time-consuming design calculations should be reduced by using adequate software programs. The developed design should be satisfactory with respect to dielectical strength and mechanical endurance. In addition, the transformer windings must withstand dynamical and thermal stresses in the event of short-circuit. This paper presents methodology for designing the distribution transformer type ETN 50-10/0.4 (Fig.1) product of “EMO” company with respect to the calculation of all transformer parameters as well as operational characteristics. For that purpose computer program is developed in Matlab, which enables automated computation of transformer design. Obtained results are compared with data from producer in order for their accuracy to be verified. By using the calculated parameters, a transformer model suitable for FEM analysis is developed [1]. The model uses the transformer actual dimensions and enables magnetic flux distribution and density to be calculated in transformer cross-section for different operating regimes: no-load and rated load [2]. Transformer rated data are presented in Table 1.

Table 1. Transformer rated data

Transformer type	ETN 50-10/0.4
Rated power -S_n	50 kVA
Transformation ratio	10/0.4 kV/kV
Rated load losses-P_{cun}	1050 W
No-load losses-P_{FE}	190 W
Number of phases	3
Connection group	Yzn5
Short-circuit voltage -U_k	4 %

Figure 1. Layout of transfer type ETN 50-10/0.4

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2. Transformer design

Transformer design is consisted of several parts: design of magnetic core with calculation of core losses, design of the low and high voltage winding with calculation of resistances and copper losses. Afterwards, are done calculations of reactance on low and high voltage side, calculation of short circuit voltage, no-load current as well as dynamic stresses in windings due to the short-circuit. For easier representation of transformer design calculation of parameters of equivalent circuit is done. In the last stage of the design, heat transfer calculation of the windings and the transformer tank are done as well as calculations of the operational characteristics.

Flux density in magnetic core is found from:

$$B_m = \frac{e_{1k}}{4.44 \cdot f \cdot S_{FE}} \tag{1}$$

e_{1k} is the induced voltage in one turn of low voltage winding. Cross section of magnetic core is S_{FE} and f is the supply frequency

$$e_{1k} = \frac{U_{2n}}{\sqrt{3} \cdot W_2} \tag{2}$$

U_{2n} is the rated voltage of secondary winding and W_2 is the number of turns in secondary winding. Magnetic flux density in cross-section of transformer core is found to be B_m= 1.54 T. The detailed layout of transformer core is presented in Fig.2.

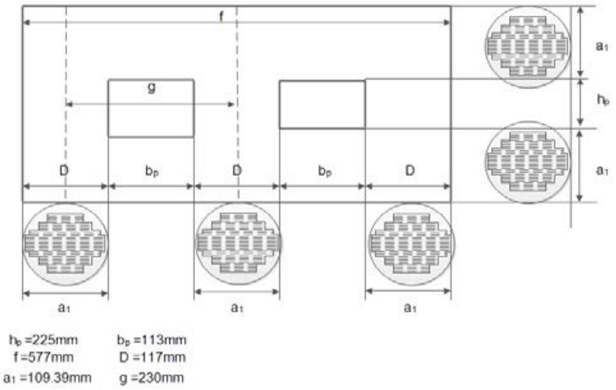


Figure 2. Cross-section of transformer core

Transformer equivalent circuit is presented in Fig. 3. and the calculated parameters of equivalent circuit are presented in Table 2.

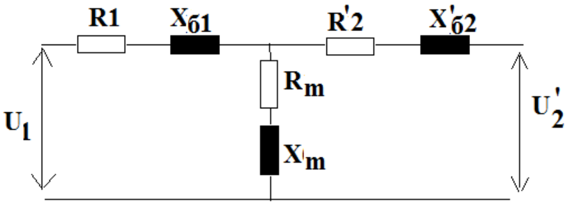


Figure 3. Equivalent circuit of transformer

Table 2. Equivalent circuit parameters

Primary winding active resistance R_1 [Ω]	27.22
Primary winding reactive resistance $X_{\sigma 1}$ [Ω]	45.71
Magnetizing active resistance R_m [Ω]	14648
Magnetizing reactive resistance X_m [Ω]	26154
Secondary winding active resistance R_2' [Ω]	14.0625
Secondary winding reactive resistance $X_{\sigma 2}'$ [Ω]	45

Based on the calculated parameters, the copper losses in primary and secondary winding are calculated respectively:

$$P_{cu1} = 3 \cdot k_f \cdot R_1 \cdot I_1^2 \tag{3}$$

$$P_{cu2} = 3 \cdot k_f \cdot R_2 \cdot I_2^2 \tag{4}$$

where k_f-is Field coefficient, I_1 and I_2 are phase currents in primary and secondary winding.

No-load losses mainly consisted of core losses are calculated from the dimensions of magnetic core (Fig.2):

$$P_{FE} = \Gamma_{FE} \cdot m_{FE} \tag{5}$$

Where m_{FE} is the mass of magnetic core found from core dimensions and specific weight of iron. Γ_{FE} is specific core losses.

Short circuit voltage-U_k its active-U_{ka} and reactive component-$U_{k\sigma}$ are calculated respectively from:

$$U_k = \sqrt{U_{ka}^2 + U_{k\sigma}^2} \tag{6}$$

$$U_{ka} = \frac{P_{cun}}{S_n} \cdot 100\,[\%] \tag{7}$$

$$U_{k\sigma} = \frac{X_k \cdot I_1}{U_1} \cdot 100\,[\%] \tag{8}$$

$$X_k = X_{\sigma 1} + X_{\sigma 2}^{'}; \tag{9}$$

$$P_{cun} = P_{cu1} + P_{cu2} \tag{10}$$

S_n is the transformer rated power.

Calculated values of losses and short circuit voltage are compared with data from experiment showing reasonable agreement.

Table 3. Comparison of calculated and experimental data

	Calculat. data	Experimental data
Copper losses P_{cun} [W]	1010.8	1050
No-load losses P_{FE} [W]	163	190
Short circuit voltage	4.48	4

Based on calculated parameters, the transformer operational characteristics are calculated. Voltage variations of transformer for different loads, inductive and capacitive are calculated from:

$$\Delta U(\%) = \varepsilon_1 + \frac{\varepsilon_2^2}{200} \tag{11}$$

$$\varepsilon_1 = \beta\left(U_{ka}\cos\varphi_2 \pm U_{k\sigma}\sin\varphi_2\right) \tag{12}$$

$$\varepsilon_2 = \beta\left(U_{k\sigma}\cos\varphi_2 \pm U_{ka}\sin\varphi_2\right) \tag{13}$$

$$\beta = \frac{I}{I_n} \tag{14}$$

For different loads i.e., β=1, 0.8 and 0.6 or for rated load, 80% and 60% of the rated load as well as for different power factors (inductive and capacitive) calculated voltage variation are presented in Tables 4., 5. and 6.

Table 4. Load factor β=1

$\cos\varphi_2$	ΔU[%] inductive	ΔU[%] capacitive
1	2.1026	2.1026
0.8	4.1297	-0.7768
0.6	4.5132	-2.0028
0.4	4.5706	-2.8928
0.2	4.3864	-3.5320
0	4.0441	-4.0033

Table 5. Load factor β=0.8

$\cos\varphi_2$	ΔU[%] inductive	ΔU[%] capacitive
1	1.6691	-3.644
0.8	3.2881	-0.113
0.6	3.5975	-1.975
0.4	3.6469	-2.733
0.2	3.5029	-3.265
0	3.2320	1.643

Table 6. Load factor β=0.6

$\cos\varphi_2$	ΔU[%] inductive	ΔU[%] capacitive
1	2.749	-2.735
0.8	2.633	-0.088
0.6	2.936	-1.482
0.4	3.019	-2.049
0.2	2.938	-2.449
0	1.219	1.219

Based on data in Tables 4., 5. and 6., the characteristics of voltage variation are presented in Fig. 4.

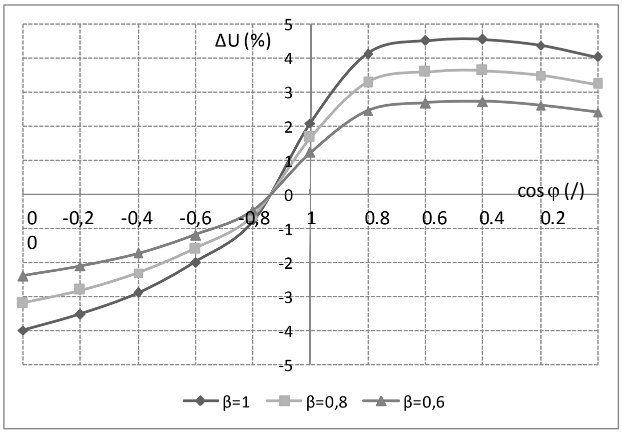


Figure 4. Variation of transformer voltage for different loads

Variation of transformer secondary voltage with respect to the transformer secondary current for rated load β=1 is calculated from:

$$U_2 = U_{2n} = \left(1 - \frac{\Delta U(\%)}{100}\right) \tag{15}$$

$$U_2 = \frac{U_{2n}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V \tag{16}$$

Calculated data from (15) and (16) are presented in Table 7.

Table 7. Variation of secondary voltage

	$\cos\varphi_2$	ΔU(%)	ΔU[V]	U_2-ΔU
inductive	1	2.1026	4.8555	226.0845
	0.8	4.1297	9.5371	221.4029
	0.6	4.5132	10.4228	220.5172
capacitive	0.8	-0.776	-1.7939	232.7339
	0.6	-0.002	-4.6253	235.5653

Characteristics of variation of secondary voltage for rated load and different power factors are presented in Fig. 5.

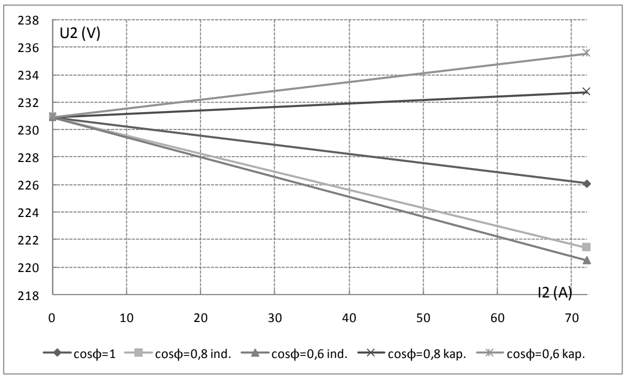


Figure 5. Variation of voltage for rated load and different power factors

Transformer efficiency factor-η as a function of output power P_2 at constant power factor is calculated from:

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_1 + \sum P_\gamma} = \frac{S_n \cos\varphi_2}{S_n \cos\varphi_2 + P_{FE} + P_{cun}} \qquad (17)$$

Where P_1 is transformer input power and Pγ is the sum of all transformer losses. Efficiency factor is calculated for different loads-β. Since efficiency factor is calculated for constant voltage, the core losses will be constant as well. Copper losses will vary in accordance to the load-$P_{cu}=\beta^2 P_{cun.}$ Efficiency factor becomes:

$$\eta = \frac{\beta \cdot S_n \cdot \cos\varphi_2}{\beta \cdot S_n \cdot \cos\varphi_2 + P_{FE} + \beta^2 P_{cun}} \qquad (18)$$

Table 8. Efficiency factor for different loads

β	$\cos\varphi_2=1$	$\cos\varphi_2=0.8$
-	η(%)	η(%)
0.2	98.00	97.52
0.4	98.40	98.01
0.6	98.27	97.85
0.8	98.01	97.53
1	97.71	97.15
1.2	97.37	96.74

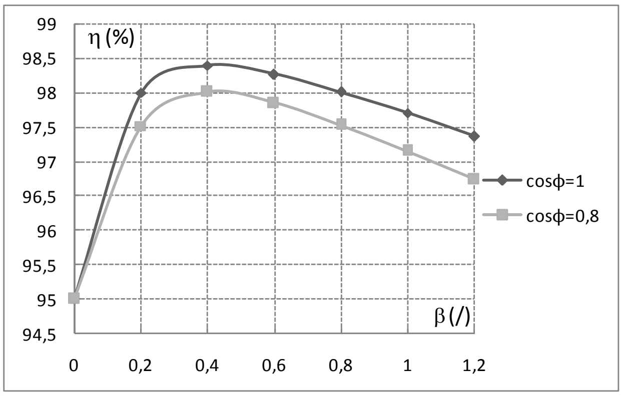


Figure 6. Characteristics of transformer efficiency factor

3. FEM model of the transformer

Finite Element Method (FEM) is a numerical method used for solving relatively complex electromagnetic problems where material nonlinearity and anisotropy is included in the analyzed domain [3]. The method involves discretization of the whole analyzed domain in small triangle surfaces, which are called finite elements. By applying Maxwell's equations into FEM it is possible to calculate the distribution of magnetic field inside electrical devices [4]. The FEM analysis is divided into three parts: pre-processing, processing and post-processing part. In pre-processing part are defined the object geometry, as well as boundary conditions. For all machines are chosen Dirichlet boundary conditions e.g. **A**=0. The most common use of Dirichlet-type boundary conditions in magnetic problems is to define magnetic vector potential **A**=0 along a boundary to keep the magnetic flux from crossing the boundary. Properties of all materials are input in the model. The magnetization curve of magnet material-B=f(H) (Fig. 7.) is input in the transformer model as well as the lamination of the magnet material according to Figure 8 (a.) and the fill factor of the magnet core [5]. The result of this model is that one can account for laminations with hysteresis and eddy currents. In order, for the value of the magnetic vector potential **A** to be determined it is necessary for the whole domain i.e. object cross-section to be divided into a certain number of elements.

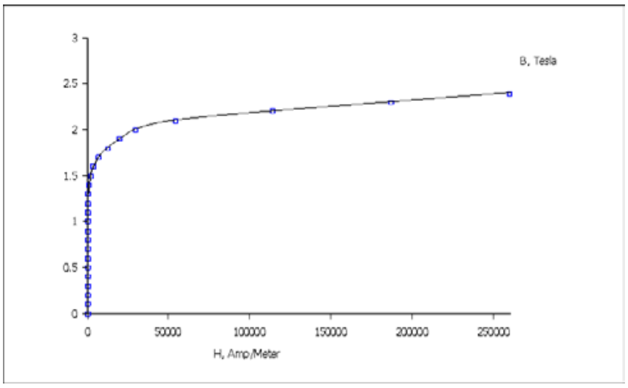
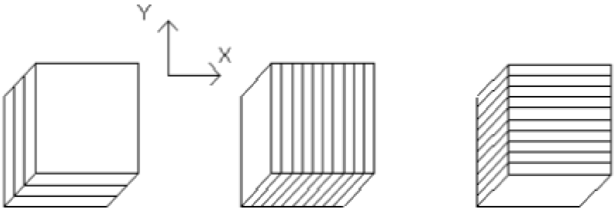


Figure 7. Characteristic of magnetization of transformer core



(a) (b) (c)

Figure 8. Lamination of magnetic core

The number of elements is problem dependent and for transformer model the finite element mesh is consisted of N=17833 nodes and E=35089 elements (Fig. 9.).

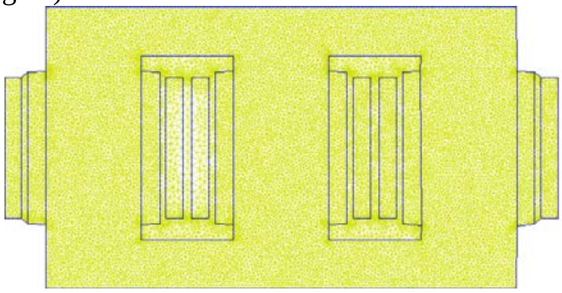


Figure 9. FEM mesh in transformer cross-section

Magneto-static problems are problems in which the fields are time-invariant [6]. In this case, magnetic field intensity H and flux density B must obey:

$$\nabla x H = \mathbf{J} \tag{19}$$

$$\nabla x B = 0 \tag{20}$$

subject to a constitutive relationship between B and H for each material:

$$B = \mu H \tag{21}$$

FEM goes about finding a field that satisfies (19)-(21) via a magnetic vector potential. The flux density is written in terms of vector potential **A** as:

$$B = \nabla x A \tag{22}$$

For the magneto-static problem and the non-linear B-H relation FEM solves the equitation:

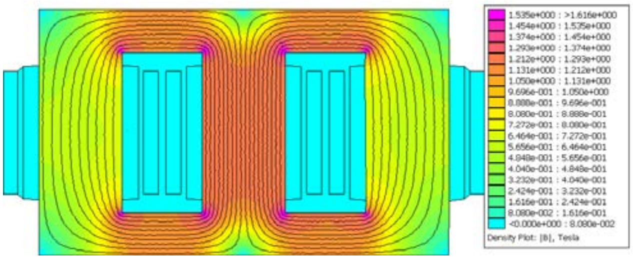
$$\nabla x \left(\frac{1}{\mu(B)} \nabla x \mathbf{A} \right) = \mathbf{J} \tag{23}$$

The advantage of using the vector-potential formulation is that all the conditions to be satisfied have been combined into a single equation. If magnetic vector potential **A** is found, *B* and H can be deduced by differentiating **A**.

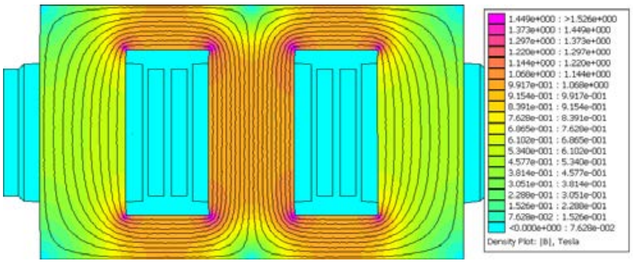
When analyzing induction machines, considering their AC excitation the air gap magnetic field is always a time-varying quantity. In materials with non-zero conductivity eddy currents are induced, consequently the field problem turns into magnetodynamic i.e. non-linear time harmonic problem [7,8]. Consequently following partial equation is going to be solved numerically:

$$\nabla x \left(\frac{1}{\mu(B)} \nabla x \mathbf{A} \right) = -\sigma \dot{\mathbf{A}} + \mathbf{J}_{\mathbf{src}} - \sigma \nabla V \tag{24}$$

where $\mathbf{J}_{\mathbf{src}}$ represents the applied current sources. The additional voltage gradient ∇V in 2-D field problems is constant over the conduction body. Strictly speaking, the permeability μ should be constant for harmonic problems. However, FEM retains a nonlinear relationship in the harmonic formulation, allowing the program to approximate the effects of saturation on the phase and amplitude of the field distribution. FEM also allows for the inclusion of complex and frequency-dependant permeability in time harmonics [9,10]. These features allow the program to model materials with thin laminations and approximately model hysteresis effects. Program is run at constant frequency f=50 Hz. For the transformer model current density is input in primary and secondary winding in all three phases and the problem is analyzed at frequency 50 Hz. Distribution of magnetic flux density in transformer cross-section for no-load and rated load is presented in Fig. 10.



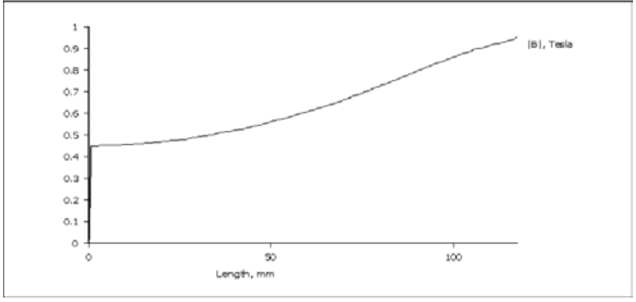
(a) no-load



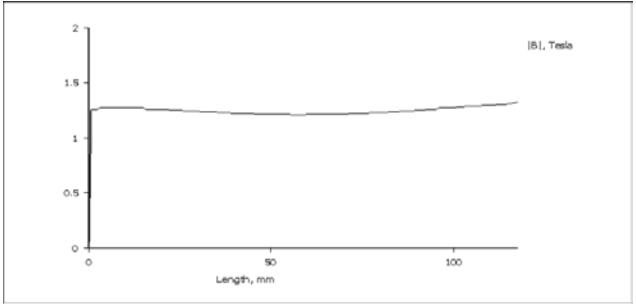
(a) rated load

Figure 10. Distribution of magnetic flux density

Presented numerical results of magnetic flux density in Fig. 10. are adequate to the calculated maximal value of magnetic flux density B_m=1.54 T. As expected magnetic flux density has the maximal value at no-load operating regime of transformer and it is slightly decreased in case of rated load because of transformer operational principle. The similarity of empirically calculated magnetic flux density and numerically found one proves the accuracy of the developed FEM model of the transformer. Distribution of magnetic flux density in transformer first and second leg for no-load and rated load operating condition is presented in Figs. 11. and 12.

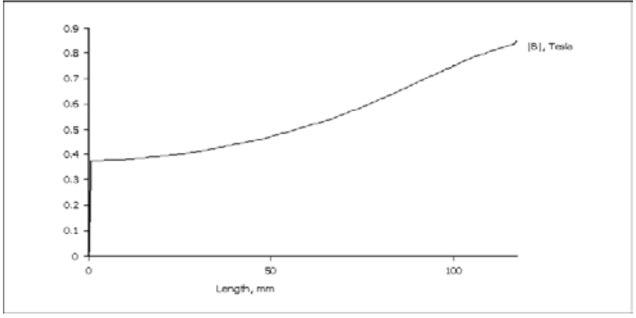


(a) first leg

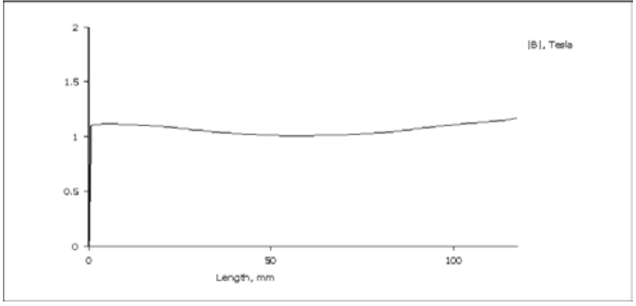


(b) second leg

Figure 11. Distribution of magnetic flux density at transformer legs , no-load



(a) first leg



(b) first leg

Figure 12. Distribution of magnetic flux density at transformer legs , rated load

4. Conclusion

In this paper, the object of design and analysis is a three phase distribution transformer 10/0.4 kV, product of “EMO” company. The design procedure is carried out by applying analytical and numerical methods. Computer program in Matlab is developed for the calculation of parameters and the characteristics of the transformer. Calculated values of losses as well as short circuit voltage are compared with data from the producer. Namely, copper loses are calculated to be 1010 W, core losses 163W and short circuit voltage of 4.48 %. They are compared with values of copper losses 1050 W, core losses 190 W and short circuit voltage of 4 % from experiment. This proved the proposed design to be accurate enough for implementing the transformer parameters and constructive dimensions in FEM model of the transformer. FEM model enables magnetic flux density in transformer cross-section to be calculated for different operating regimes: no-load and rated load. Calculated maximal value of magnetic flux density of 1.5 T from FEM model corresponds to the empirically calculated value of flux density of 1.54 T, thus proving the accuracy of developed FEM model. Further authors’ research will be focused on calculation of transformer reactance by applying numerical methods. Optimization of transformer design with respect to maximization of efficiency as a result of decreased transformer losses is also part of authors’ further research which should result with improved transformer design.

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