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Mathematical Model of Thermo Hydraulic Processes in Venturi Apparatus with Rectangular Cross Section

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***Abstract* – The cleaning processes of polluted gases in high velocity gas streams based on water flushing, as well as the heat and mass transfer mechanisms in Venturi apparatus, are well known in engineering investigations. Flowing processes in Venturi apparatus with rectangular cross section are also not unknown process in scientific literature.**

The modeling of flow process in Venturi apparatus with rectangular cross section with real shape and dimensions are related in this paper. Thereby, special attention is paid to the reviewing of the flowing processes in gas-flow path, particularly in the confusor, the throat and the diffuser of the Venturi apparatus.

***Keywords* – fluid flow, heat and mass transfer, Venturi, mathematical model, numerical solution.**

1. Construction processes and overview

The aspect of the real-size shaped Venturi apparatus is given in Fig. 1. The manufacturing material for this construction is steel sheet having well-known mechanical characteristics and heat conductive coefficient.

The confuser's function is to increase gas velocity at the inlet to the throat, where the cleaning process with water injection into the throat takes place. Complex processes happen in the diffuser as well, where the cleaning process of the polluted gas goes on, with intensive heat and mass transfer.

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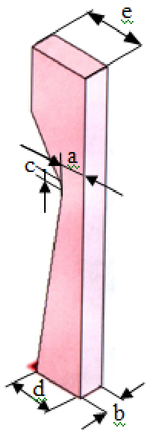

	mm
a	*40*
b	*40*
c	10
d	82
e	100

Fig. 1 Real size of Venturi aparatus

From thermo–hydraulic aspect and conditioned by the apparatus dimensions, certain assumptions and simplifications are introduced leading to the following conclusions:

- polluted heated gas flows into the diffuser, the flowing process is stationary, two dimensional, two-phased, having more components (the gas contain dust particles, gas components such as SO_2, NOx, CO_2 etc.),

-the flowing process in the throat can be reduced down to a stationary one. The cleaning process using water takes place here, actually being a three-phased flow (gas, dust, water) with a multi-component flow occurring here, which due to the small dimension of the throat can be viewed as one dimensional flow along the direction of the general flow. Intensive mass and heat transfer processes happen in the throat possessing a certain cleaning efficiency. Chemical reactions which actually take place in the throat are not taken into consideration within this investigation.

Next to the throat is the diffuser, which takes over the cleaned gas, within which intensive and complex processes unfold because the cleaning process keeps on going there. The same assumptions are valid here as well as for the confusor.

2. Postulation of the mathematical model

Based on the general Navier – Stokes equations relating to three dimensional, unsteady flow of real viscous fluid, and reducing them down to the real processes taking place in the Venturi apparatus, a mathematical model has been postulated. When postulating the model, simplifications and assumptions have been introduced making allowances for the basic physical laws of mass conversion, heat and quantity of motion in respect to the thermo–hydraulic processes unfolding in the Venturi apparatus during the stationary flow regime.

2.1. Mathematical model for flowing processes in confuser of the Venturi apparatus

Based on postulated simplifications and assumptions, the mathematical model for gas flow in confusor is reduced down to the following system of equations:

- *Equation of continuity:*

$$\frac{\partial}{\partial x}\left(\rho_g \cdot u_g\right)+\frac{\partial}{\partial y}\left(\rho_g \cdot \mathrm{v}_g\right)=0 \tag{1}$$

- *Quantity of gas motion in x direction:*

$$u_g \rho_g \frac{\partial u_g}{\partial x}+\mathrm{v}_g \rho_g \frac{\partial u_g}{\partial y} -\frac{\partial}{\partial x}\left(\eta_{ef} \frac{\partial u_g}{\partial x}\right)-\frac{\partial}{\partial y}\left(\eta_{ef} \frac{\partial u_g}{\partial y}\right)=-\frac{\partial p_g}{\partial x}+\rho g_x \tag{2}$$

- *Quantity of gas motion in y direction:*

$$u_g \rho_g \frac{\partial \mathrm{v}_g}{\partial x}+\mathrm{v}_g \rho_g \frac{\partial \mathrm{v}_g}{\partial y} -\frac{\partial}{\partial x}\left(\eta_{ef} \frac{\partial \mathrm{v}_g}{\partial x}\right)-\frac{\partial}{\partial y}\left(\eta_{ef} \frac{\partial \mathrm{v}_g}{\partial y}\right)=-\frac{\partial p_g}{\partial y}+\rho g_y \tag{3}$$

Thereby, the system is reduced down to three equations with three unknowns. It is necessary to obtain the velocity field in any node of the generated grid, as well as the pressure field.

Starting from the well-known numerical mathematical methods for solving of the two dimensional fluid flow thermal processes in the confusor, the Laplace formulation of primary equation for continuity is used. Due to the simple shape of the confusor, and the possibilities for generating simple grid with quadrangular elements in order to solve the problem numerically with satisfactory accuracy, the technique of finite elements can be used.

The subject technique is elaborated in more detail in [9-11], where solving of the mathematical model of the plate shaped module part is expounded. Thereby the mathematical model written in discrete form gets the following form:

- *Equation of continuity:*

$$\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}+\frac{\mathrm{v}_{i,j+1}-\mathrm{v}_{i,j-1}}{2\Delta y}=0 \tag{4}$$

- *Quantity of gas motion*
  in x direction

$$u_{i,j}\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}+\mathrm{v}_{i,j}\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}=\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta x}+ +\nu\left(\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{\Delta x^2}-\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta y^2}\right) \tag{5}$$

in y direction

$$\mathrm{v}_{i,j}\frac{\mathrm{v}_{i+1,j}-\mathrm{v}_{i-1,j}}{2\Delta y}+u_{i,j}\frac{\mathrm{v}_{i,j+1}-\mathrm{v}_{i,j-1}}{2\Delta x}=\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta y}+ +\nu\left(\frac{\mathrm{v}_{i+1,j}-2\mathrm{v}_{i,j}+\mathrm{v}_{i-1,j}}{\Delta y^2}-\frac{\mathrm{v}_{i,j+1}-2\mathrm{v}_{i,j}+\mathrm{v}_{i,j-1}}{\Delta x^2}\right) \tag{6}$$

Where ν is the kinematic viscosity of the gas.

In flowing conditions where there isn't any heat transfer, enegretic equation is out of question.

The Laplace formulation of the basic equation is:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0 \tag{7}$$

Thereby, the for defined geometry of the confusor, and having known initial and boundary conditions, values of velocity in all nodes of generated grid are obtained:

$$u_{i,j}=\frac{\psi_{i,j+1}-\psi_{i,j-1}}{2\Delta y}$$
$$\mathrm{v}_{i,j}=-\frac{\psi_{i+1,j}-\psi_{i-1,j}}{2\Delta x} \tag{8}$$
$$\mathrm{w}_{\mathrm{i,j}}=\sqrt{u_{i,j}^2+\mathrm{v}_{i,j}^2}$$

The distribution of pressure in any given node according to the previously defined model, is obtained by solving the equation:

$$p_{i,j}^{n+1}=\frac{1}{4}\left(p_{i+1,j}^n+p_{i-1,j}^n+p_{i,j+1}^n+p_{i,j-1}^n+2\Delta x^2\psi_{i,j}^n(d\cdot e-f^2)\right) \tag{9}$$

While the coefficients are calculated according to:

$$d=\frac{\psi_{i+1,j}-2\psi_{i,j}+\psi_{i-1,j}}{\Delta x^2}$$
$$e=\frac{\psi_{i,j+1}-2\psi_{i,j}+\psi_{i,j-1}}{\Delta y^2} \qquad (10)$$
$$f=\frac{\psi_{i+1,j+1}-\psi_{i+1,j-1}-\psi_{i-1,j+1}+\psi_{i-1,j-1}}{4\Delta x\Delta y}$$

which is contingent on the velocity potential in any given node of the grid.

2.2. Mathematical model relative to the throat

The processes happening in this part of construction are complex ones, taking into consideration that right here an interaction between the gas and the water takes place, thus creating a cleaning process, whereby the fluid flow velocities are maximum in conditions of developed turbulent streaming. Because of the throat dimensions – small cross section and small length - the flowing of the gas can be viewed as one dimensional along the gas flow direction with satisfactory accuracy with all its complexity from aspect of heat and mass transfer processes.

When modeling, the fluid can be considered as a multi-component one with adopted ten fractions of dust having defined dimensions. After the water injection into the gas at the throat, due to the mutual interaction, the flow becomes multi-component, three-phased (gas- dust-water) with a phase change (evaporation-condesation), thus bringing about additional proceses of cleaning the gas from dust and gas components by means of adhesion to water and absorption. The chemical reactions that actually happen in such flowing conditions, followed by forming weak solutions, are not subject of this investigations.

Regarding the cleaning gas processes from dust (solid particles), as well as the process of gas cleaning from gas components, a lot of theory exists in the literature [1], [4,5]. The disintegrating of the droplets with defined sizes in disperse structure, has been a subject of many scientific investigations [2,3], [5-7].

The model in one dimensional form along the gas-flow direction, for adopted local coordinate z, rerlative to the throat of the Venturi apparatus, is reduced down to:

- Mass of gas:

$$\frac{\partial}{\partial z}\left(\rho_g\cdot \mathrm{v}_{g,z}\right)=-\sum m_{SV} \qquad (11)$$

- Mass of water:

$$\frac{\partial}{\partial z}\left(\rho_W\cdot \mathrm{v}_{W,z}\right)=\sum m_{SV} \qquad (12)$$

- Quantity of gas motion:

$$v_{g,z}\frac{\partial \mathrm{v}_{g,z}}{\partial z}=-\frac{\partial p_g}{\partial z}-f_z \qquad (13)$$

- Gas enthalpy:

$$\frac{\partial}{\partial z}\left(\rho_g\cdot \mathrm{v}_{g,z}\cdot i_g\right)=-\sum q_{SV} \qquad (14)$$

- Water enthalpy:

$$\frac{\partial}{\partial z}\left(\rho_W\cdot \mathrm{v}_{W,z}\cdot i_W\right)=\sum q_{SV} \qquad (15)$$

- Moisture content in the gas:

$$\frac{\partial}{\partial z}\left(\rho_g\cdot \mathrm{v}_{g,z}\cdot C_g\right)=-m_{SVIC} \qquad (16)$$

- Concentration of the components in the gas:

$$\frac{\partial}{\partial z}\left(\rho_g\cdot \mathrm{v}_{g,z}\cdot C_{g,i}\right)=-m_{SVAP,i} \qquad (17)$$

- Concentration of dusts in the gas:

$$\frac{\partial}{\partial z}\left(\rho_g\cdot \mathrm{v}_{g,z}\cdot C_{CT,j}\right)=-m_{SVCT,j} \qquad (18)$$

whereby:

i – stands for the number of components in gas, $i = 1 \div n_1$

j – stands for the number of fractions in disperse dust structure, $j = 1 \div n_2$

As to the suggested model, a substitution can be introduced in accordance with:

$$m_{SAg} = \rho_g \cdot \mathrm{v}_g \quad ; \quad m_{SAw} = \rho_w \cdot \mathrm{v}_w \qquad (19)$$

As for the one dimensional case, when quantities are changed only in z direction, the partial derivations are converted in simple form.

Due to the change performed, the model ordered and given the developed form, can be comprehended as a system of differential equations:

$$\frac{dm_{SAg}}{dz}=-\sum m_{SV} \qquad (20)$$

$$\frac{dm_{SAW}}{dz}=\sum m_{SV} \qquad (21)$$

$$\mathrm{v}_{g,z}\frac{d\mathrm{v}_{g,z}}{dz}=-\frac{dp_g}{dz}-f_z \qquad (22)$$

$$\frac{di_g}{dz} = \frac{-\sum q_{SV} + i_g \cdot \sum \dot{m}_{SV}}{m_{SAg}} \quad (23)$$

$$\frac{di_W}{dz} = \frac{\sum q_{SV} - i_W \cdot \sum \dot{m}_{SV}}{m_{SAW}} \quad (24)$$

$$\frac{dx_g}{dz} = \frac{-m_{SVIC} - x_g \cdot \sum m_{SV}}{m_{SAg}} \quad (25)$$

$$\frac{dC_{g,i}}{dz} = \frac{-m_{SVAP,i} - C_{g,i} \cdot \sum m_{SV}}{m_{SAg}} \quad (26)$$

$$\frac{dC_{CT,j}}{dz} = \frac{-m_{SVCT,j} - C_{CT,j} \cdot \sum m_{SV}}{m_{SAg}} \quad (27)$$

Thereby:

$$\sum m_{SV} = m_{SVIC} + \sum m_{SVAP,i} + \sum m_{SVCT,j} \quad (28)$$

$$\sum q_{SV} = q_{SVIC} + \sum q_{SVAP,i} + \sum q_{SVCT,j} \quad (29)$$

The number of differential equations in the proposed model is 6+n1+n2, and is applicable according to the number of components in the gas mixture, as well as to the number of fractions in the dispersed structure of the dust.

The system of differential equations having known initial conditions is solved using one of the known mathematical methods.

In this case, a programmed solution has been worked out by using of standard Runge – Kuta model featuring a fourth order of accuracy.

When solving the differential equations, a need emerges of defined local values for the specific values of flows of mass and heat (Σm_{SV} и Σq_{SV}) along the whole length of surveyed space, in any given node relative to which the system of differential equations is being solved.

The defining of the local parameters of Σm_{SV} and Σq_{SV}, involves also the defining of the correlated dependencies of the local parameters of water and gas, also along all the length of the surveyed space.

Basically, exhaust (outlet) gasses are gas mixtures comprising more components and dispersed dust too.

The characteristics of the outlet exhaust gases depend primarily on the fuel composition, the conditions and the regime of combustion processes.

The outlet exhaust gasses from the pollution source can be considered as inlet gases in reviewed construction. The gases are mixture of the dust and other components such as O_2, CO_2, SO, SO_2, NOx, water steam. As a main component the nitrogen is being adopted. The mathematical model for the gas mixture is postulated and programmed solution is created therefore, [9-11].

As inlet values, the temperature and the pressure of the gas, the contained steam in the gas or the relative humidity are assigned, as well as the mass concentrations of the of gas-mixture components.

Outlet parameters are the following ones: the calculated physical values of the gas, as well as the physical values of the gas-mixture components.

The impact of dust presence in the gas mixture upon its characteristics at low polluted gases can be ignored.

In the programmed solution, the dust is taken into account setting up a dispersed composition containing ten fractions. Inlet dates are: average diameter of the particles, the mass rate in the whole mass per fractions, and the density of the material. In the model, the number of the fractions can be varied.

To reckon the water parameters, data and dependencies quoted in the literature [8,10] are used. The used tabular data are further worked out, and the functional dependencies therefore are derived by approximation and interpolation.

The mathematical models and the programmed solutions for calculation of the gas and water parameters are integrated into the ultimate model and the programmed solution.

This paper does not treat the transport characteristics of water droplets in conditions of dewy space, the processes of gas moisturizing or gas drying, the absorption of gas components in water, as well as water-flushing of the gas from dust since they are subject to another investigation.

2.3. Mathematical model for the diffuser

Owing to the geometrical shape of the diffuser, which has been aforementioned, the flowing in this part of the Venturi apparatus is two dimensional.

The flowing in the diffuser as the following segment after the throat, by its very nature is much intricate one, conditioned by a large number of impact parameters and values. In order to postulate a more comprehensive and accurate mathematical model for the diffuser, the following assumptions have been taken into account:

- The process of the water droplets dispersing continues in the diffuser, with possibilities of secondary dispersing emerging depending on the local conditions and impact values.
- The motion of the water droplets is conditioned by the gas velocity, and therefore it ought to be treated separately.
-

On account of these introduced assumptions, a large number of the relations in the mathematical models which apply to the throat of the Venturi apparatus are valid for the diffuser too.

More precisely, the differential equation for water drop motion along the gas-flow is recorded in two-dimensional form. The velocity gas field akin to the solution in the confuser is obtained by solving Laplace equation. Thus, the local values of the velocity in any given node of the diffuser, determine the velocity of the water droplets in x, y directions, respectively the resulting velocity.

To define the local velocity, the water droplet motion differential equation within i-designated fraction of the water-droplets dispersed structure of in gas stream is used in the following form:

-in x direction:

$$\frac{d\mathrm{u}_{\mathrm{K,i}}}{d\tau} = -C_{FK,i} \cdot \frac{\rho_G}{\rho_W} \cdot \frac{3}{4 \cdot d_{K,i}} \cdot \left| \mathrm{u}_{K,i} - \mathrm{v}_G \right| \cdot \left(\mathrm{u}_{K,i} - \mathrm{v}_G \right) \tag{30}$$

-in y direction:

$$\frac{d\mathrm{v}_{K,i}}{d\tau} = -C_{FK,i} \cdot \frac{\rho_G}{\rho_W} \cdot \frac{3}{4 \cdot d_{K,i}} \cdot \left| \mathrm{v}_{K,i} - \mathrm{v}_G \right| \cdot \left(\mathrm{v}_{K,i} - \mathrm{v}_G \right) \tag{31}$$

Whereby:

$$C_{FK,i} = k_{FK,i} \cdot \left[\frac{24}{\mathrm{Re}_{K,i}} + 0.22 \cdot \left(1 + \sqrt{1 + \frac{220}{\mathrm{Re}_{K,i}}} \right) \right] \tag{32}$$

obtains two values of velocity in x and y directions for different values od the local Re numbers:

$$\mathrm{Re}^x{}_{K,i} = \frac{\mathrm{u}_{\mathrm{K,i}} \cdot d_{k,i} \cdot \rho_g}{\eta_g}$$

$$\mathrm{Re}^y{}_{K,i} = \frac{\mathrm{v}_{\mathrm{K,i}} \cdot d_{k,i} \cdot \rho_g}{\eta_g} \tag{33}$$

Wherefore:

For $Re = 10 \div 250$ $\quad k_F = 0.55$
For $Re = 250 \div 5000$ $\quad k_F = ln\ (Re / 100)$

The path which the droplet crosses at the defined value of its velocity can be calculated as follows:

$$Z^x{}_{K,j} = \int_{\tau_0}^{\tau} \mathrm{u}_{K,j} \cdot d\tau$$

$$Z^y{}_{K,j} = \int_{\tau_0}^{\tau} \mathrm{v}_{K,j} \cdot d\tau \tag{34}$$

In the model are taken into account all formerly reviewed relations in order to define and figure out the separate parameters and values.

By solving the differential equation for the two-dimensional motion of the water droplet, the velocities of the water droplets within i-designated fraction of dispersed structure of the same droplets as a time function are being obtained.

As in one-dimensional model for flowing of water droplet in the throat of the Venturi apparatus, as well as in the diffuser having known local resultant velocity of the water droplets for i-designated fraction of the dispersed structure of the droplets, another local parameters can be reckoned which are characteristic for dewy space (number of droplets in time unit, volume in time unit per volume unit, area of droplets in time unit and etc.), which can be also included in the model. The model for dust cleaning proposed for the throat is used for the diffuser likewise. The changing of the resultant velocity of the water droplets and of the gas in two-dimensional form is taken into account as well.

The differential equations in x and y direction are solved by with procedure using the Runge Kuta method of fourth order of accuracy. Thereby we note that the local values of the gas velocity in the two directions taken as an input date in the differential equations are obtained beforehand by solving the Laplace equation related to two-dimensional flow.

Thus, by postulating such a mathematical model, a distribution of the fluid velocities and droplets at the inlet, outlet, that is, in the cross sections on flow path of the gas, are obtained.

By getting the local values of the velocities in any given node of the generated grid, the other necessary parameters can be calculated, such values being characteristic for this part of the Venturi apparatus.

By solving of Laplace equation in one form only, the data for the local values of the resultant velocities per streams in the diffuser are attained. Thereby, the model for one-dimensional flowing in the throat of Venturi, is applied as well to the flow conditions in the diffuser, the action being repeated as many times as the number of steams amounts to, adopting the diffuser division. Thus, the local values of the velocity are changed in the cross-sections normal to the flow direction. In this manner, the method is repeated following defined procedures, depending on the initial and boundary conditions.

Since a great deal of the relations in the mathematical models for throat and confusor apply as well to the flowing in diffuser, and in order not to overburden the material, it is mentioned that in the integrated programmed mathematical model all relations concerning the flow conditions in this part of the segment are taken into account.

3. Conclusion

The aim of the postulated mathematical models is to get the values of the respective parameters defining fluid-flow processes and the gas cleaning processes in high-velocity polluted gas flow through the Venturi apparatus having rectangular cross section.

The obtained results can be used as comparative ones in relation to the data at experimental investigations on real created construction which is foreseen as a next stage in the research.

References

[1]. Daniel Mussatti, Paula Hemmer: *Wet Scrubbers for Particulate Matter, Chapter 2, EPA/452/B-02-001,* July 2002 http://www3.epa.gov/ttncatc1/dir1/cs6ch2.pdf

[2]. Schifftner, K.C. and H.E. Hesketh. *Wet Scrubbers: A Practical Handbook.* Lewis Publishers, Chelsea, Michigan. 1986. http://www3.epa.gov/ttncatc1/dir1/cs6ch2.pdf

[3]. Wet Scrubbers, http://www3.epa.gov/ttn/caaa/t1/reports/sect5-4.pdf

[4]. Venturi Tubes, Flow Tubes, and Flow Nozzles, ftp://ftp.unicauca.edu.co/Facultades/FIET/DEIC/Materias/Instrumentacion%20Industrial/Instrument_Engineers__Handbook_-_Process_Measurement_and_Analysis/Instrument%20Engineers'%20Handbook%20-%20Process%20Measurement%20and%20Analysis/1083ch2_29.pdf

[5]. Thermal Fluid Laboratory, https://eng.najah.edu/sites/eng.najah.edu/files/labs/thermal-fluid-manual.pdf

[6]. Chuen-Jinn Tasi, Chia-Hung Lin and Yu-Mon Wang, Institute of Environmental, Taiwa of China, *An Efficient Venturi Scrubber System to Remote Submicron Particles in Exhaust Gas,* Technical Paper, ISSN 1047-3289 J.Air&Waste Mamage. Assoc. 55:319-325 http://www.unc.edu/courses/2007spring/envr/754/001/VenturiScrubber_Tsai_2005.pdf

[7]. Ahmet Baylar, M. Cihan Aydin, Mehmet Unsal and Fahri Ozkan: *Numerical Modeling Of Venturi Flows For Determining Air Injection Rates Using* Fluent V6.2, Mathematical and Computational Applications, Vol. 14, No. 2, pp. 97-108, 2009, Association for Scientific Research http://mcajournal.cbu.edu.tr/volume14/Vol14No2p97.pdf

[8]. M. A. M. Costa, P. R. Henrique, J. A. S. Gonçalves and J.R.Coury, Departamento de Engenharia Química, Universidade Federal de São Carlos: Droplet Size in a Rectangular Venturi Scrubber, Brazilian Journal of Chemical Engineering, Vol. 21, No. 02 pp. 335 - 343, April - June 2004, ISSN 0104-6632 http://www.scielo.br/pdf/bjce/v21n2/a24v21n2.pdf

[9]. Lapidus L., Pinder G.F. (1982). *Numerical Solution of Partial Differential Equations in Science and Engineering*, Princeton University, John Wiley&Sons.

[10].Numerical Solution of Navier-Stokes equation using FEMLAB, Umea University, Department of Physics, Staffan Grundberg, 2002.

[11].Dimitrieska B. C., Stavreva S. (2015). Mathematical Model for Fluid Flow and Heat Transfer Processes in Plate Exchanger, Tem Journal, Vol.4, No.4, November 2015, ISSN: 2217-8309 (Print). eISSN: 2217-8333 (Online).