Determining of the Optimal Device Lifetime using Mathematical Renewal Models

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***Abstract* – Paper deals with the operations and equipment of the machine in the process of organizing production. During operation machines require maintenance and repairs, while in case of failure or machine wears it is necessary to replace them with new ones. For the process of replacement of old machines with new ones the term renewal is used. Qualitative aspects of the renewal process observe renewal theory, which is mainly based on the theory of probability and mathematical statistics. Devices lifetimes are closely related to the renewal of the devices. Presented article is focused on mathematical deduction of mathematical renewal models and determining optimal lifetime of the devices from the aspect of expenditures on renewal process.**

***Keywords* – Renewal theory, renewal models, cost function, optimal lifetime.**

1. Introduction

Renewal theory is a method included into operating research, which is dealing with economic problems related to replacement and operational abilities of the machines and the technical devices. In general terms renewal means process of continual object replacement (devices, components, etc.), what represents results of failure analyzes during the operation [1] [2] [3].

In terms of modeling renewal, processes are distinguished processes divided into two main principles:

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- continual (possibility to create model and monitor the process in the optional time),
- determinate interval.

Practical model applications are often modeled as discrete renewal processes [1]. Practical experience provides the possibility to reject selected objects before final failure. In this case the problem is in optimization of the renewal moment, what represents determination of the optimal lifetime. Lifetime is one of the most significant terms in modeling renewal processes. Lifetime (trouble-free processes) is according [4] period from establishment of the object to rejection. Lifetime of any device represents an ability of the device to perform the required function in order to reach the ultimate limit state in designed system of prescribed maintenances and repairs.

The aim of the presented article is to deduce mathematical renewal models and to set optimal lifetime of chosen devices – conveyor belts, which are an essential element of belt conveyors.

Belt conveyance has wide range of use in transportation of different material and is exposed to the environmental conditions [5][6]. Conveyor belt is consider as a most stressed and vulnerable part of the system. The belt during operation time is stressed by various fatigues, which induce the process of wears and damage [7]. According to [5] lifetime of conveyor belt represents the ability to realize required functions to reach limit state in defined system prescribed by maintenance and repairs. The lifetime of conveyor belts from the aspect of a selected parameter is analyzed in thesis [8]. The topic issue of conveyor belt damage is analyzed in studies [9][10]]11] and the problems of optimal lifetime of conveyor belts by renewal theory in studies [12][13][14][15].

2. Renewal models

We must realize that operation of all objects (machines and devices) cause wearing of the object, but on the other side dependence on time production is also increasing costs on maintenance and repairs.

The cost approach to the lifetime examination uses the convex function $C(t)$ consisting of two functions $A(t)$ and $L(t)$. The $A(t)$ function is the increasing function of costs that depend on the duration of the object's operation. It represents the amount of costs on maintenance and repairs of the objects per a time unit that occur due to wear during their operation. The $L(t)$ function is descending and can express the losses caused by maintenance and down-times, or the costs of depreciation in various service live durations. It can also be a separate lifetime function. The literature describes plenty of approaches to the structure of these functions. The differences between individual specific models of optimal lifetime are mainly represented by different definitions of the $L(t)$ function [3, 4].

The $C(t)$ function represents the function of total costs of the object. In optimization of the object lifetime it is the determination of the optimal lifetimet_{opt} which is related to the search for the minimum of the objective function of the total costs $C(t)$. Value t_{opt} determines the time of optimal renewal of the object [1].

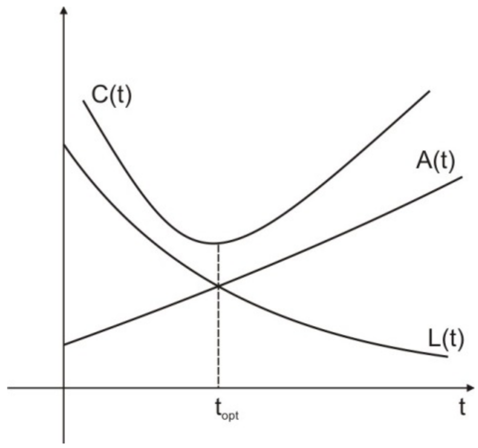

Figure 1.The total cost function

Using cost approach is also considering costs of maintenance and repairs, purchase price and costs. Following models consider with statement, that object is functional until the rejection and replacement. During lifetime of maintenance and common repairs only replace wears of individual parts without the influence on performance and lifetime.

3. Model I

In the simplest version of the renewal model, the $L(t)$ equals the average price of the object per a time unit. It applies that

$$L(t) = \frac{C_P - C_R}{t} = \frac{C_D}{t}, \tag{1}$$

where$C_P$is the purchase cost of the object, C_R is the residual cost of the object at depreciation and$C_D$is the difference between the purchase cost and the residual cost of the object.

If we know the costs of maintenance C_M in the basic period of object operation and the coefficient of cost increaseαin each following period, then the $A(t)$ function is determined by the relation $A(t) = \alpha t C_M$. The total cost function per a time unit has then the following structure

$$C(t) = \frac{C_P - C_R}{t} + \alpha t C_M = \frac{C_D}{t} + \alpha t C_M. \tag{2}$$

Optimal lifetime t_{opt} of the object can be determined by searching the local minimum of the $C(t)$ function from the equation

$$\frac{dC(t)}{dt} = -\frac{C_D}{t^2} + \alpha C_M = 0, \tag{3}$$

which can serve as the base for determination of the stationary point

$$t_{opt} = \sqrt{\frac{C_D}{\alpha C_M}} \cdot \tag{4}$$

As for$t > 0$,it applies that

$$\frac{d^2C(t)}{dt^2} = \frac{2C_D}{t^3} > 0, \tag{5}$$

the $C(t)$ function is coherent convex function that in time$t = t_{opt}$ reaches the local minimum.

The t_{opt} value determines the time of optimal object renewal, which means that in practice there is constant decrease in unit costs per a time unit of the operation time until the moment$t_{opt}$and after this period of time the costs begin to increase [1, 4]. The formula (4) is very affected by the method of maintenance cost definition and leads to the results with rather short lifetime periods.

4. Model II

More exact renewal model divides the maintenance costs$C_M$into the constant costs$C_K$and the increasing costsC_I. Average maintenance costs assigned to one period are then determined by the formula

$$C_M = C_K + (t-1)\frac{C_I}{2} \cdot \tag{6}$$

The total costs per time unit $C(t)$ are then determined by the relation

$$C(t) = \frac{C_P - C_R}{t} + C_K + (t-1)\frac{C_I}{2} =$$

$$= \frac{C_D}{t} + C_K + (t-1)\frac{C_I}{2} \cdot \qquad (7)$$

Optimal lifetime t_{opt} of the object can be determined from the equation

$$\frac{dC(t)}{dt} = -\frac{C_D}{t^2} + \frac{C_I}{2} = 0, \qquad (8)$$

from which the stationary point is

$$t_{opt} = \sqrt{\frac{2C_D}{C_I}} = \sqrt{\frac{2(C_P - C_R)}{C_I}} \cdot \qquad (9)$$

As for $t > 0$, it applies that

$$\frac{d^2C(t)}{dt^2} = \frac{2C_D}{t^3} > 0, \qquad (10)$$

the $C(t)$ function is a coherent convex function which in time $t = t_{opt}$ reaches the local minimum [1, 5]. The relation (9) indicates that the constant maintenance costs C_K does not affect the optimal lifetime.

5. Model III

A more complicated renewal model is the model in which we assume the dependence of total maintenance costs in time, determined by the equation

$$C_0(t) = A + Bt + Ct^D, \qquad (11)$$

where A, B, C, D are constants for which it applies that $A > 0,\ B > 0,\ C > 0$ and $D > 1$, whereas these constants must be estimated in a proper way [4, 16].

The total costs per time unit are then determined by the relation

$$C(t) = \frac{C_0(t)}{t} = \frac{A}{t} + B + Ct^{D-1}. \qquad (12)$$

Optimal lifetime t_{opt} of the object can be determined by searching the local minimum of the function $C(t)$ from the equation

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = -\frac{A}{t^2} + C(D-1)t^{D-2} = 0. \qquad (13)$$

After the adjustment will bring

$$t_{opt} = \sqrt[D]{\frac{A}{(D-1)C}} \cdot \qquad (14)$$

As the value

$$\frac{d^2C(t_{opt})}{dt^2} = \frac{2A}{t_{opt}^3} + C(D-1)(D-2)t_{opt}^{D-3} =$$

$$= \left[\frac{A}{C(D-1)}\right]^{\frac{-3}{D}} \cdot D \cdot A > 0, \qquad (15)$$

the $C(t)$ function is a coherent convex function and in time $t = t_{opt}$ it reaches the local minimum.

In general, the A constant can be regarded as equal to the amount of purchase costs C_P of the object, i.e. $A = C_P$. The B constant can be placed as equal to the constant maintenance costs C_K and the C constant can be approximated as the initial status of variable costs C_I. These costs can be estimated from the specific data on the object sample.

After adjustment, the optimal lifetime can be determined by the relation

$$t_{opt} = \sqrt[D]{\frac{C_P}{(D-1)C_I}}, \qquad (16)$$

where the D coefficient represents the exponent determining the progressive cost increase depending on the lifetime duration. The relation shows that the constant maintenance costs C_K do not impact the optimal lifetime.

Estimation of the D constant was made using the method of least squares which requires the following

$$S(D) = \sum_{i=1}^{n}\left(A + Bt_i + Ct_i^D - C_i\right)^2 \rightarrow \text{min.}, \qquad (17)$$

where C_i are values of costs related to the operation of a specific belt in the period i, $A = C_P$, $B = C_K$, $C = C_I$. The B constant was put as equal to the constant costs C_K which do not affect the optimal lifetime [12].

The D value can be found by determination of the local minimum of the function $S(D)$

$$\frac{\mathrm{d}S(D)}{\mathrm{d}D} =$$

$$= 2\sum_{i=1}^{n}\left(A + Bt_i + Ct_i^D - C_i\right)Ct_i^D \ln t_i = 0, \qquad (18)$$

from where

$$A\sum_{i=1}^{n} t_i^{D}\ln t_i + B\sum_{i=1}^{n} t_i^{D+1}\ln t_i +$$

$$+C\sum_{i=1}^{n} t_i^{2D}\ln t_i - \sum_{i=1}^{n} C_i t_i^{D}\ln t_i = 0. \qquad (19)$$

The equation is a non-linear equation with the unknown quantity *D* which can be solved using the numerical methods, e.g. bisection.

6. Models in practice

Belt conveyers belong to wide spread means of transport to deliver different types of materials in machinery, mining, metallurgy, construction and other ways of industries. The most important part of the conveyors is belt and its optimal use has significant effect on process efficiency, to reduce cost and to decrease the environmental loads. In the fact of this is necessary to monitor conveyors belt lifetime. The question of conveyor´s belt lifetime is very complex. Lifetime of conveyor´s belt is influenced by several parameters and factors, which include mainly quality construction conveyor belt, optimal design, manufacture and properties of the conveyor belt, a suitable solution of dunes during transport routes, rational maintenance and repair quality conveyor belts [5]. Derivate models that determine the optimal service lifetime of conveyor belts were tested on a series of conveyor belts from practice [14]. Obtained optimum lifetimes were compared with an operating lifetime (real lifetime).

The recovery model for calculating the optimum service lifetime of the conveyor belts areC_P purchase costs, $C_R$residual price of the conveyor belt on the rejection and coefficient C_D is the difference between the acquisition cost and the residual value of the conveyor belt. The calculations can be based on the assumption that the residual price of conveyor belt is represent by the value of the coefficient $C_R = 0$.

The value $C_I$can be approximated as an initial condition variable costs of the conveyor belt (these costs can be estimated from a sample of specific data on conveyor belts).

In all cases it was found, that the differences between the operational lifetime and the best lifetime of obtained models are not statistically significant.

For evaluating the accuracy of models the factor RMS (Root Mean Square) was used. The analyses results show that the least accurate result was obtained by model I, the best results in comparison with real lifetime shows the model III.

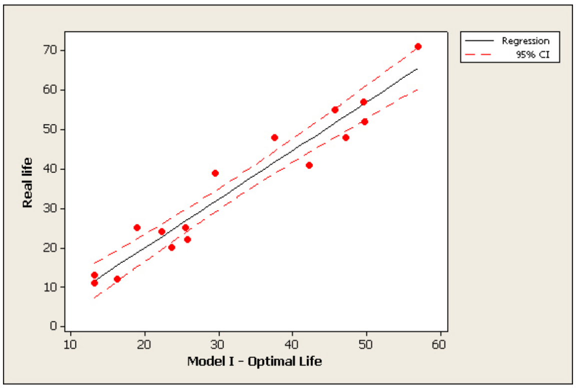

Figure 2.Dependence of operational lifetime - Model I

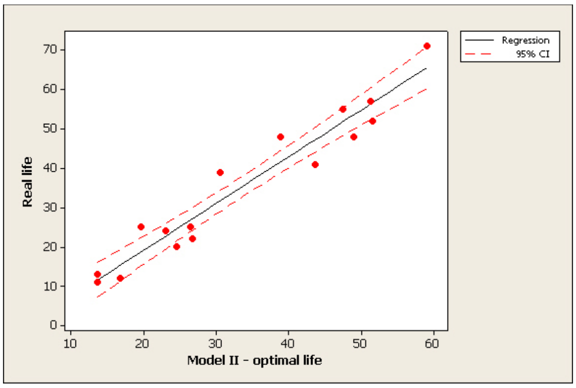

Figure 3.Dependence of operational lifetime - Model II

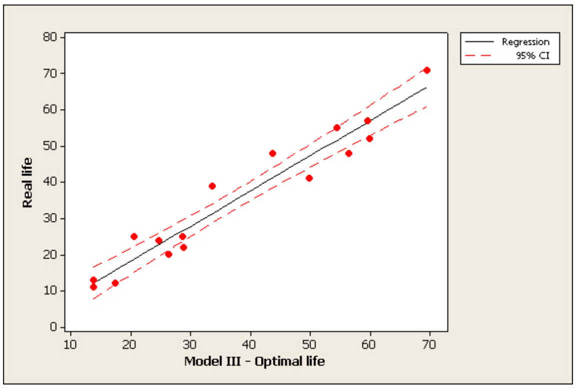

Figure 4.Dependence of operational lifetime - Model III

The use of MRS was used to obtain very strong linear dependence. Graphical dependences shown in figures 2-4 represent operational lifetime depend on the optimal lifetime.

7. Summary

One of the important strategic problems in production organization is operating machinery and technical equipment. Machinery used within the operation requires either repair or maintenance in case of failure or when completely worn out, to be replaced with a new machine. The theory of renewal deals with modeling the mathematical process of renewal.

The problem of optimal defining the lifetime is in terms of mathematical modeling very complex issue. There are many different factors that directly affect lifetime of the objects, reasons of wear and the length of each cycle of renewal.

Lifetime of conveyor belts is very complex. Conveyor belt lifetime is influenced by many factors, which include mainly quality construction of conveyor belts, optimal design, manufacture and properties of the conveyor belt, a suitable solution of dunes during transport routes, rational maintenance and quality repair of conveyor belts. Belt conveyors are high performance transport system with a wide application in practice; therefore the problem of technical and economic level of conveyors is highly current and relates to increased durability and operational reliability of conveyor belts, which are the most sensitive element of the conveyor belt.

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