Mathematical Model for Fluid Flow and Heat Transfer Processes in Plate Exchanger

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***Abstract* – Within the analytical solution of the system of equations which solve fluid flow and heat transfer processes, the elliptical and parabolic differential equations based on initial and boundary conditions is usually unfamiliar in a closed form. Numerical solution of equation system is necessarily obtained by discretization of equations.**

When system of equations relate to estimation of two dimensional stationary problems, the applicable method for estimation in basic two – dimensional form is recommended.

***Keywords* – fluid flow, heat transfer, mathematical model, numerical solution.**

1. Introduction

Regarding the methods for energy and process engineering investigation, most often we mean models and modeling, as the most common method used.

Mathematical modeling, of course as an indivisible part of it, has commonly used equations, like derivation, integration and other mathematical equations, [7], [8].

In this paper, the focus will be more on the results that comes from modeling and explain the reason why they derive from these equations. Explaining these mathematical equations systematically is not a purpose of this paper, and they are not part thereof, unless it is necessary. It is considered that these equations and their solutions should be related and well known by previous knowledge.

When systems of equations are related to engineering problems, especially for two-dimensional stationary fluid – flow processes in a heat plate exchanger, using of the Poisson formulation for streamline – vortex ψ, ω, [3], [5], [6], the elliptical differential equations in basic two dimensional form are recommended.

In that manner of accessing the problem, the substitution of velocity components of the vortex ω and streamline ψ is required.

The vector of the vortex is defined like, [3]:

$$\omega = \nabla \times \mathbf{u} \qquad (1)$$

2. Solution of the model

By using the Cauchy - Riemann conditions of the velocity potential, [3]:

$$\frac{\partial \psi}{\partial x} = -\mathrm{v}$$

$$\frac{\partial \psi}{\partial y} = u \qquad (2)$$

For two – dimensional stationary fluid flow regardless of vortex existing, by using two-dimensional continuity equation, for scalar vortex and streamline following equations are obtained:

$$u\frac{\partial \omega}{\partial x} + \mathrm{v}\frac{\partial \omega}{\partial \mathrm{y}} = \nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \qquad (3)$$

Equation (3) is parabolic differential equation, well known vortex-transport equation, [3].

By ordering of (2) and (3), streamline equation is obtained:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad \Delta \psi = -\omega \qquad (4)$$

As a result of variable substitute, pressure elimination of base system is possible (which is often problem), and then second step is dividing of elliptic-parabolic incompressible equations on one parabolic and one elliptic equation. Those equations are mostly calculated by using the following procedure:

1. Ordering of initial values for ω and ψ;
2. Calculation of vortex – transport equation;
3. Calculation of Poisson's equation for ψ in all points, by using of new values for ω in interior points;
4. Determination of boundary value of ω by using of values for ω and ψ in interior points;
5. Back to step 2 if solution in no converge able;
6. Determination of velocity component values, u and v.

Idem means setting for next estimation, getting values of pressure and temperature distribution, for estimated values of velocity in x and y direction, in area of definition, under known boundary conditions.

For system equation solution, on that way of problem formulation, two methods are known. The direct methods, where Gauss method of elimination belongs and Cramer rule are characterized with large number of arithmetical operations and a lot of time for calculation for getting results. On the other hand, iterative methods are more simplified for programing, the calculation takes short time with satisfactory accuracy, [2], [3].

According to [5], [6] the known iterative methods for numerical solution of system of elliptical differential equations and calculation of stream - velocity potentials are: Jacobi iterative method, point iterative method Point Seidel, linear iterative method, Line Siedel and ADI method (Alternating Direct Implicit Method).

Under defined initial and boundary conditions for area of streaming, for problem estimation, ADI method was used, which is more useful and adequate for rectangular cross section channel with defined contour.

Based on ADI method, for calculation of two dimensional problems, estimation of system of elliptical equations is similar with estimation of system of parabolic equations. Estimation of Poisson equation is postulated on approximation of derivation of the stream line ψ of second order, based on method of finite differences:

$$\frac{\partial^2\psi}{\partial x^2} \approx \frac{\psi_{i-1,j} - 2\psi_{i,j} + \psi_{i+1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2\psi}{\partial y^2} \approx \frac{\psi_{i,j-1} - 2\psi_{i,j} + \psi_{i,j+1}}{(\Delta y)^2} \tag{5}$$

By summing equations in (5) and identification with (4), according condition $\Delta x = \Delta y$, final form of Poisson equation is related:

$$\psi_{i,j} = \frac{\psi_{i-1,j} + \psi_{i,j-1} + \psi_{i+1,j} + \psi_{i,j+1} + \omega_{i,j}\Delta x\Delta y}{4} \tag{6}$$

Thereby, values of stream line ψ in node i, j depend on values of neigh board nodes in its area, Fig. 1 and values of the vortex ω in the node. Gauss – Seidel iteration and TDMA method (Tridiagonal Matrix Algorithm Method) are used to find solution of the problem.

In the TDMA form, the equation (6) is reduced as:

$$\psi_{i,j-1} - 4\psi_{i,j} + \psi_{i,j+1} = -\psi_{i-1,j} - \psi_{i+1,j} - \omega\Delta x\Delta y \tag{7}$$

Registered in general form, by introduction of main variable u, equation (7) get form:

$$a_i\mathbf{u}_{i-1} + b_i\mathbf{u}_i + c_i\mathbf{u}_{i+1} = d_i \tag{8}$$

Where a_i, b_i, c_i and d_i are coefficients of the finite elements differences.

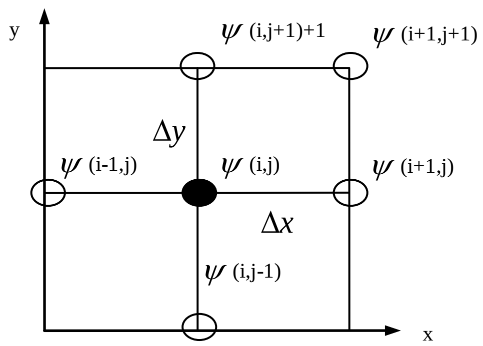

Figure 1.Scheme of 2D finite differences for Poisson formulation

That method is similar with Gauss method elimination, but it is more accurate. By introduction of dependence, [3]:

$$\mathbf{u}_{i-1} = e_i\mathbf{u}_i + f_i \tag{9}$$

Whereupon:

$$e_{i+1} = \frac{-c_i}{a_ie_i + b_i} \qquad f_{i+1} = \frac{d_i - a_i f_i}{a_ie_i + b_i} \tag{10}$$

The procedure is reduced to numerical estimation and getting of values for ψ, ω in any interior node in area of interest. Starting from line i=2, mention that constant *di* in three agonal equation is composed of known values for $\psi_{1,j}$ and estimated values for $\psi_{3,j}$. By using of TDMA new value for $\psi_{2,j}$ is obtained.

Procedure are repeated for i=3, by using of values for $\psi_{2,j}$ for assumed of last estimated value for $\psi_{4,j}$. The procedure is repeated step by step, line by line, until the end.

During estimation, criteria for convergation from old (stara) to new (nova) value must be kept, based on definied value of allowed error ε:

$$\sum_{i,j} \left| \psi_{nova} - \psi_{stara} \right| < \varepsilon \tag{11}$$

Based on the same analogy, values of vortexes in all nodes of generated grid are estimated. But have in mind that, stream line and vortexes are depended between each other.

3. ADI Method

Based on ADI method, the calculation of the main parameters, velocity and pressure are reduced on arithmetical calculation of the new system of equations.

In order to calculate the velocities, two equations are used:

$$\begin{aligned} u_{i,j} &= \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta y} \\ \mathrm{v}_{i,j} &= -\frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x} \end{aligned} \tag{12}$$

for related value of initial velocity in the module of examination.

For the pressure estimation next equations are in use, [3], [5], [6]:

$$\begin{aligned} p_{i,j}^{n+1} &= \frac{1}{4}\left(p_{i+1,j}^{n} + p_{i-1,j}^{n} + p_{i,j+1}^{n} + p_{i,j-1}^{n}\right) + \\ &+ \frac{1}{4}\left(2\Delta x^2 \psi(i,j)(d \cdot e - f^2)\right) \end{aligned} \tag{13}$$

therefore:

$$\begin{aligned} d &= \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2} \\ e &= \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2} \\ f &= \frac{\psi_{i+1,j+1} - \psi_{i+1,j-1} - \psi_{i-1,j+1} + \psi_{i-1,j-1}}{4\Delta x \Delta y} \end{aligned} \tag{14}$$

for known base value of pressure p, for given temperature t.

Estimation of energetic equation, concerning two – dimensional temperature fields, is in relation with explicit knowing of velocity values in x and y direction in any node of the generated grid. The energetic equation is parabolic differential equation, whose estimation is well known in many engineering problems in literature, [3] and [4].

Two – dimensional temperature fields are related to values of local convective heat transfer coefficients, local velocity values and local Re number in any node of the generated grid. For known input fluid temperature and inside plate temperature (both temperatures are measurable), termophysic characteristics are obtained, Pr number is calculated, then Nu number, and the convective coefficient. A lot of empirical and semi empirical relations exsist about forced turbulent flow of incompressible heated fluid (20-300 ^{o}C) in chanel with rectangular cross section between two parallel plates, [1], [3], but in this paper next relations are in use:

-local Re number for channel flowing:

$$\mathrm{Re}(i,j) = \frac{\mathrm{w(i,j)}\delta}{\nu(t_o)} \tag{15}$$

Therefore, w(i,j) is a resultant local velocity of the fluid in any node, δ is characteristic destination between parallel plates, $\nu(t_o)$ is kinematic fluid viscosity for initial input temperature t_o:

-local Nu number base on [1], [3]:

$$Nu(i,j) = 0.021\varepsilon_l \,\mathrm{Re}(i,j)^{0.8}\,\mathrm{Pr}(t_o)^{0.43}\left(\frac{\mathrm{Pr}(t_o)}{\mathrm{Pr}_z(t_{oz})}\right)^{0.25} \tag{16}$$

Therefore, ε_l is correction related with channel geometry and Re number, $\mathrm{Pr}(t_o)$ for input temperature t_o, $\mathrm{Pr}(t_{oz})$ for fluid temperature from interior and exterior side of the plate. Therefore, for known thermo physic characteristics of the fluid for wall temperature, $\mathrm{Pr}(t_{oz})$ number is defined in compliance with:

$$\mathrm{Pr}_z(t_{oz}) = \frac{c_p(t_{oz})\eta(t_{oz})}{\lambda(t_{oz})} \tag{17}$$

Where c_p means specific heat under constant pressure, η is dynamic viscosity, λ heat conductivity for fluid temperature on the interior side of the plate.

Local heat transfer coefficient is estimated according:

$$\begin{aligned} \alpha(i,j) &= 0.021\varepsilon_l \,\mathrm{Re}(i,j)^{0.8}\,\mathrm{Pr}(t_o)^{0.43} \cdot \\ &\cdot \left(\frac{\mathrm{Pr}(t_o)}{\mathrm{Pr}_z(t_{oz})}\right)^{0.25} \frac{\lambda(t_o)}{\delta} \end{aligned} \tag{18}$$

For known measurable values of fluid and plate temperatures on interior and exterior sides of the module, as well for known thickness and heat conductivity of the plate material, constant heat transfer flux is defined:

$$q = \alpha_v(i,j)(t_v(i,j) - t_z^v) = const$$

$$q = \frac{\lambda_m}{\delta_m}(t_z^v - t_z^n) = const \tag{19}$$

$$q = \alpha_n(i,j)(t_z^n - t_n(i,j)) = const$$

Heat transfer is in directon according:

$$t_v(i,j) > t_z^v > t_z^n > t_n(i,j) \tag{20}$$

Therefore, $\alpha_v(i,j)$ is local heat convective coefficient of the heated gas on the interior side of the plate; $\alpha_n(i,j)$ is the local convective coefficient of heat transfer from the exterior side of the plate in direction of the gas with lower temperature; $t_v(i,j)$, is the temperature of the fluid on the interior side with the higher temperature where hot gas streaming; $t_n(i,j)$ is the temperature of the fluid with lower temperature; λ_m is the conductive coefficient of the material – sheet metal; δ_m is thickness of the material – sheet metal.

For initial values of the inlet fluid temperature, on the interior or exterior side, meaning the temperatures of the plate from both sides, under formerly estimated local values of the coefficients of heat transfer, the local values of the fluid temperature in any nodes on the both sides are calculated.

Because estimation starts with assumption of the inlet fluid temperature, with numerical iterative calculation, step by step, by the corection, real values of the temperature in the nodes will be related.

Under flowing on the flat plate, local heat transfer coefficients are estimated:

$$k(i,j) = \left(\frac{1}{\alpha_v(i,j)} + \frac{\delta_m}{\lambda_m} + \frac{1}{\alpha_n(i,j)} \right)^{-1} \tag{21}$$

By using the known values of the local heat transfer coefficients and know temperature difference on internal and external side of the plate, local values for the heat power of the heat exchanger in any nodes of the grid are:

$$q(i,j) = k(i,j)\left[\ t_v(i,j) - t_n(i,j)\ \right] \tag{22}$$

Based on presented procedure for calculation of the local velocity values, local pressure values, temperature in any nodes of the generated grid from the both sides of the module can be estimated.

4. Conclusion

The postulated mathematical model in discredited form is in adequate numerical form for program solution.

The local values of the temperature in any node of the grid, for known velocity field and pressure values are estimated according known procedure. For known values of Re and Nu numbers, there for defined local values of heat transfer coefficients, under known thermo physical fluid characteristics and sheet metal for temperature values in inlet, of interior and interior side of the module – performed like heat plate exchanger, and for wall temperatures from the other sides, temperature distribution in nodes are estimated.

The procedure is repeated with iterations, until satisfactory accuracy of the temperature values are obtained. The related dates can be used for convective heat transfer estimation, on both sides of the plate exchanger. This model can be adjusted for pipe wall or plate. Depending on pipe or plate material and conductivity coefficient, the value of heat flux can be obtained.

By solving the postulated mathematical models, the main values related to the thermo hydraulic processes are obtained in two dimensional ways. It represents the base for three dimensional calculations.

In this way, main parameters which defined fluid flow processes are related.

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