Moments of Macrodiversity SC Receiver with Two Microdiversity EGC Receivers over Gamma Shadowed Rician Multipath Fading Channel

Ilija Temelkovski [1], Predrag Milačić [1], Danijela Aleksić [2], Danijel Đošić [3], Stanislav Veljković [1]

[1] Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia
[2] Faculty of Technical Sciences, Svetog Save 65, 32000 Čačak, Serbia
[3] Faculty of Natural Science and Mathematics, Ive Lole Ribara 29, 38220 Kosovska Mitrovica, Serbia

Abstract – **SC macrodiversity system with dual EGC microdiversity receivers operating over shadowed multipath fading environment is considered. Received signal experiences correlated Gamma long term fading and Rician short term fading. SC macrodiversity receiver reduces Gamma large scale fading effects, while EGC microdiversity receivers mitigate short term fading effects on system performance. Closed form expressions for arbitrary order output moments of macrodiversity structure output signal is evaluated. Numerical results, obtained for arbitrary order output moments are presented graphically to show Gamma long term fading severity effect, correlation coefficient effects and Rician factor effect.**

Keywords – **equal gain combining (EGC); Gamma shadowed; Rician fading; selection combining (SC).**

1. Introduction

Long term fading and short term fading degrade system performance and limit channel capacity. Reflections, refraction, diffraction and scattering cause short term fading and terrain profile deviation and large obstacles cause long term fading resulting in system performance degradation. It is important to determine how multipath fading and shadowing effect outage probability and bit error probability wireless communication system. There are more distribution that could be used to describe signal envelope variation depend on existence of line-of-sight (LOS) component, the number of clusters in propagation environments, nonlinearity of environment, inequality of quadrature components power and variation of signal envelope average power. The most frequently used statistical models to describe multipath fading are Rayleigh, Rician, Nakagami-*m*, Nakagami-*q*, Weibull and α-μ distribution [1], [2], [3].

Rayleigh distribution can be used to describe small scale signal envelope variation in linear, non line-of-sight multipath fading environment with on cluster. In line-of-sight multipath fading channels, signal envelope variation can be described by using Rician distribution. Nakagami-*m* distribution describes signal envelope variation in environment with two or more clusters. Nakagami-*q* distribution describes fading in environment where powers of in phase component and quadrature component are different [4], [5].

There are several combining techniques can be used the reduces multipath fading effects on system performance. The most frequently used combining techniques are maximum ratio combining (MRC), equal gain combining (EGC) and selection combining (SC). MRC combining enable the best performances. EGC technique processes all diversity branches and sums received equally-weighted replicas from each one to produce output statistics. EGC combining technique requires separate receiver chain for each branch of the diversity system, which increase its complexity [6]. Selection combining (SC) technique receiver is much simpler for practical realization (trade-off with obtained reception performances), in opposition to these combining techniques, since it processes one of the received signal replicas. However, this combining technique still requires the simultaneous and continuous monitoring of all the channels. Namely, in general, SC, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR), that is, the branch with the strongest signal [7], [8].

However, the usage of diversity techniques applied at single base station (micro-diversity combining) alleviates only influence of short-time fading. In order to combat with overall channel degradation, when shadowing (long-term fading) simultaneously occurs with multipath, combining between base stations (macro-diversity combining) has to be also applied. In that way is assured that different long-term fading is experienced, by signals received at two or more base stations (BS).

In [9], macrodiversity SC receiver with two microdiversity MRC receivers operating over Gamma shadowed Nakagami-*m* multipath fading channel is analyzed. Average level crossing rate of proposed signal is calculated closed form expression. Level crossing rate and average fade duration of macrodiversity system in the presence calculated Gamma long term fading and Rician multipath fading is considered.

In this paper, macrodiversity system with macrodiversity SC receiver and two microdiversity EGC receivers is analyzed. Desired signal experiences Rician short term fading resulting in signal envelope variation and Gamma long term fading resulting in signal envelope power variation. Microdiversity reception is used to mitigate small scale Rician fading, while macrodiversity selection reduces Gamma large scale signal envelope power variation. Closed form expressions for arbitrary order output moments of macrodiversity structure are calculated. To the best authors' knowledge such contribution has not been yet reported in open technical literature.

2. Rician Random Variable Moments

Moments are important characteristics of random variables. Rician random variable follows distribution:

$$p_x(x) = \frac{2(1+k)x}{e^k \Omega} \exp\left(-\frac{(1+k)x^2}{\Omega}\right) \times \times I_0\left(2\sqrt{\frac{k(k+1)}{\Omega}}x\right), \quad (1)$$

where k is Rician factor which can be calculated as ratio of dominant component power and scattering components power Ω is average power of Rician random variable x and $I_0()$ is modified Bessel function of the first kind. Then the moment of nth order of random variable x is given with:

$$m_{nx} = \int_0^\infty dx x^n p_x(x) = \frac{1}{e^k} \frac{\Omega^{\frac{n}{2}}}{(1+k)^{\frac{n}{2}}} \times \times \sum_{i=0}^{\infty} \frac{\Gamma\left(i+1+\frac{n}{2}\right)}{(i!)^2}. \quad (2)$$

where $\Gamma(x)$ represents special Gamma function.

3. EGC Receiver Output Signal Envelope Moments

Let us consider dual branches gain combiner (EGC) subjected to independent Rician multipath fading. Signal envelopes at inputs of EGC are denoted with x_1 and x_2.

EGC receiver output signal envelope is denoted with x. moment nth order of EGC output signal envelope is:

$$m_x^n = \overline{x^n} = \overline{(x_1+x_2)^n} = \sum_{i=0}^{n} \binom{n}{i} \overline{x_1^{n-i}} \cdot \overline{x_2^i} = = \sum_{i=0}^{n} \overline{x_{1(n-i)}} \cdot \overline{x_{2i}}, \quad (3)$$

where $\overline{x_{1(n-i)}}$ and $\overline{x_{2i}}$ is with:

$$\overline{x_{1(n-i)}} = \frac{1}{e^k} \frac{\Omega^{\frac{n-i}{2}}}{(1+k)^{\frac{n-i}{2}}} \sum_{i_1=0}^{\infty} \frac{\Gamma\left(i_1+1+\frac{n-i}{2}\right)}{(i_1!)^2}, \quad (4)$$

$$\overline{x_{2i}} = \frac{1}{e^k} \frac{\Omega^{\frac{i}{2}}}{(1+k)^{\frac{i}{2}}} \sum_{i_2=0}^{\infty} \frac{\Gamma\left(i_2+1+\frac{i}{2}\right)}{(i_2!)^2}. \quad (5)$$

After substituting (4) and (5) in (3), the expression for moment nth order of EGC output signal envelope becomes:

$$m_x^n = \frac{1}{e^{2k}} \frac{\Omega^{\frac{n}{2}}}{(1+k)^{\frac{n}{2}}} \sum_{i=0}^{n} \binom{n}{i} \times \times \sum_{i_1=0}^{\infty} \frac{\Gamma\left(i_1+1+\frac{n-i}{2}\right)}{(i_1!)^2} \sum_{i_2=0}^{\infty} \frac{\Gamma\left(i_2+1+\frac{i}{2}\right)}{(i_2!)^2}. \quad (6)$$

4. Macrodiversity SC Receiver Output Signal Envelope Moments

Macrodiversity reception realized by using macrodiversity SC receiver and two microdiversity EGC receivers is considered. Received signal experiences Rician short term fading and Gamma long term fading resulting in system performance degradation. Macrodiversity SC reception is used to reduce large scale fading effects and microdiversity EGC reception mitigates small scale fading effects. In this paper microdiversity EGC receivers have two branches. Microdiversity EGC receiver output signal envelopes are denoted with x_1 and x_2. Macrodiversity SC receiver output signal envelope is denoted with x. signal envelope powers at inputs of microdiversity SC receivers are denoted Ω_1 and Ω_2. Random variables Ω_1 and Ω_2 fallow correlated Gamma distribution:

$$p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)=\frac{1}{\Gamma(c)\left(1-\rho^2\right)\rho^{\frac{c-1}{2}}\Omega_0^{c+1}}\times$$
$$\times\sum_{j_1=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2j_1+c-1}\times \quad , \quad (7)$$
$$\times\frac{1}{j_1!\Gamma\left(j_1+c\right)}\Omega_1^{j_1+c-1}\Omega_2^{j_1+c-1}e^{-\frac{\Omega_1+\Omega_2}{\Omega_0\left(1-\rho^2\right)}}$$

where ρ is correlation coefficient Ω_0 is power Ω_1 or Ω_2 and c is order of Gamma distribution. Macrodiversity SC receiver selects microdiversity EGC receiver with higher power at input to enables the serve to user. There the moment *nth* order of macrodiversity SC receiver output signal envelope is:

$$m_x^n=\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}d\Omega_2 m_{nx_1/\Omega_1}p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)+$$
$$\int_0^{\infty}d\Omega_2\int_0^{\Omega_2}d\Omega_1 m_{nx_2/\Omega_2}p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)= \quad , \quad (8)$$
$$=2\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}d\Omega_2 m_{nx_1/\Omega_1}p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)$$

where m_{nx_1/Ω_1} and m_{nx_2/Ω_2} are given with (6).

After substituting (6) and (7) in (8), with respect to [10, Eq. 3.381.1] the expression for moment nth order of macrodiversity SC receiver output signal envelope can be expressed as:

$$m_x^n=\frac{2}{\Gamma(c)\left(1-\rho^2\right)\rho^{\frac{c-1}{2}}\Omega_0^{c+1}}\frac{1}{e^{2k}}\times$$
$$\times\frac{1}{(k+1)^{\frac{n}{2}}}\sum_{i=0}^{n}\binom{n}{i}\sum_{i_1=0}^{\infty}\frac{\Gamma\left(i_1+1+\frac{n-i}{2}\right)}{\left(i_1!\right)^2}\times$$
$$\times\sum_{i_2=0}^{\infty}\frac{\Gamma\left(i_2+1+\frac{i}{2}\right)}{\left(i_1!\right)^2}\sum_{j_1=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2j_1+c-1}\times$$
$$\times\frac{1}{j_1!\Gamma\left(j_1+c\right)}\left(\Omega_0\left(1-\rho^2\right)\right)^{j_1+c}\times$$
$$\times\int_0^{\infty}d\Omega_1\Omega_1^{j_1+c-1+\frac{n}{2}}e^{-\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}}\gamma\left(i_1+c,\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}\right)=$$
$$=\frac{2}{\Gamma(c)\left(1-\rho^2\right)\rho^{\frac{c-1}{2}}\Omega_0^{c+1}}\frac{1}{e^{2k}(k+1)^{\frac{n}{2}}}\times$$
$$\times\sum_{i=0}^{n}\binom{n}{i}\sum_{i_1=0}^{\infty}\frac{\Gamma\left(i_1+1+\frac{n-i}{2}\right)}{\left(i_1!\right)^2}\times$$
$$\times\sum_{i_2=0}^{\infty}\frac{\Gamma\left(i_2+1+\frac{i}{2}\right)}{\left(i_1!\right)^2}\sum_{j_1=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2j_1+c-1}\times \quad (9)$$
$$\times\frac{1}{j_1!\Gamma\left(j_1+c\right)}\left(\Omega_0\left(1-\rho^2\right)\right)^{j_1+c}\cdot I_1$$

with γ (a,x) denoting the lower incomplete Gamma function. Capitalizing on [10, Eq.8.331.1] integral I_1 can now be solved as:

$$I_1=\frac{1}{i_1+c}\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)^{i_1+c}}$$
$$\sum_{j_2=0}^{\infty}\frac{1}{\left(i_1+c+1\right)_{(j_2)}}\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)j_2}$$
$$\times\int_0^{\infty}d\Omega_1\Omega_1^{j_1+2c-1+\frac{n}{2}+j_2}e^{-\frac{2\Omega_1}{\Omega_0\left(1-\rho^2\right)}}d\Omega_1=$$
$$=\frac{1}{i_1+c}\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)^{i_1+c}}\times$$
$$\times\sum_{j_2=0}^{\infty}\frac{1}{\left(i_1+c+1\right)_{(j_2)}}\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)j_2}\times$$
$$\times\left(\frac{\Omega_0\left(1-\rho^2\right)}{2}\right)^{j_1+2c-1+\frac{n}{2}+j_2}\times$$
$$\times\Gamma\left(j_1+2c-1+\frac{n}{2}+j_2\right) \quad . \quad (10)$$

Obtained infinite series expressions convergence rapidly, since only 10-15 farms shoved by summed to achieve accuracy at 5th significant digit for each values at Gamma shadowing and Rician multipath fading parameters.

5. Numerical Results

The first moment, the second moment and the third moment versus correlation coefficient ρ of long term Gamma fading is plotted on Figure 1. for some values of Gamma shadowing severity parameter *c*. Moment of the first order or mean increases when Gamma shadowing severity parameter increases. The system performance is better for higher values of Gamma long term fading severity. As known from theory, important system performance measure, outage probability (OP) decreases as the first moment increases. When Gamma shadowing severity parameter and Rician factor go to infinity, Gamma showed Rician multipath fading channel becomes no fading channel. The mean has higher values as Rician factor increases Rician factor increases when dominant component power increases or scattering components power decreases. Output moments decrease as the correlation coefficient increases. This indicates that better values for OP could be obtained when correlation coefficient is lower. In Figure 2., the first moment is shown in terms of Gamma shadowing severity *c*.

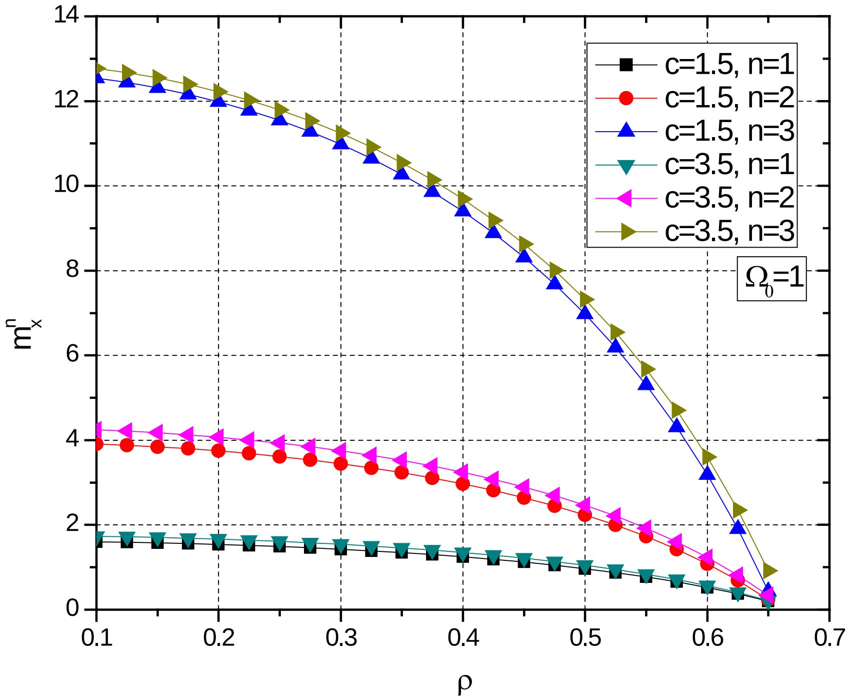

Figure 1. Moments of macrodiversity SC receiver versus correlation coefficient for different values severity parameter c.

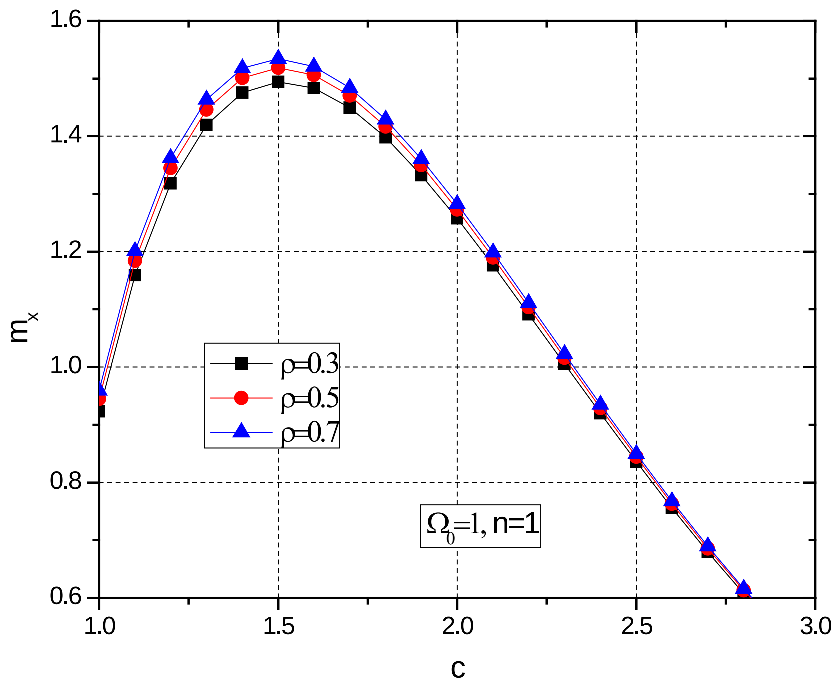

Figure 2. First moment of macrodiversity SC receiver output signal versus shadowing severity c.

6. Conclusion

In this paper macrodiversity system reducing simultaneously long term fading and short fading effects on system performance is analyzed. Dual SC macrodiversity system with two EGC microdiversity receivers operating over Gamma shadowed Rician multipath fading environment has been considered. Closed form expressions for macrodiversity structure arbitrary order output moments are calculated, and discussed in the function of system parameters.

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