Power Parameters and Efficiency of Class B Amplifier Operating with Resistive Load and Random Signal

Hristo Zhivomirov [1]

[1]*Technical University - Varna, Str. Studentska 1, Varna, Bulgaria*

***Abstract* – The work presented in this paper provides a theoretical analysis and estimation of the most important power parameters of the class B amplifiers operating with random signals and resistive loads, i.e. loudspeaker with impedance compensation. Some parameters of the random signals are exposed and energy relation between the sine-wave signals and the random signals are given via the coefficient of ARV-compliance. Analytical expressions of the power relations are derived for operation with random signals with Gaussian distribution. The simulation results are given that confirm the theoretical statements, as well as data of real-world amplifier measurements with NI USB-6211.**

***Keywords* – class B amplifier, random signal, resistive load, power parameters.**

1. Introduction

The analysis of the audio power amplifiers is usually performed under the assumption that they work with sine-wave signals as shown in [1], [2], [3], [4], [5], [6] and [7]. The audio power amplifiers, however, operate exclusively with music and speech signals – musical program, speech, acoustic noise, etc. Therefore, the analysis should be done under these conditions. The enumerated audio signals, in a number of parameters and characteristics can be considered as random (stochastic) signals [8], [9], [10], [11], [12].

In the literature the problem of operation with random signals is less affected. In [8], [13] and [14] are given only general guidelines that the power parameters of the amplifier will undergo some reduction. In [8] and [15] the expressions are given about the power parameters binding them with the probability density function of the signals. According to [8], [9], [10], [11], [12] and [13], in the general case, the probability distribution of the music signals is Gaussian.

In [8] it is mentioned that the analytical expressions can be derived with difficulty, and in [15] the analysis is made by means of a numerical integration of empirical data. If one assumes that the loudspeaker impedance is compensated, it is possible to produce analytical expressions about the power parameters of the class B amplifiers. This entails a number of consequences about the choice of amplifier components, the design of the heat sink, the power supply, etc.

2. Theoretical analysis

General description and characteristics of the stochastic processes

In the analysis and experimental research it is necessary to deal with a sufficiently long realization (sample) of the observed signal. That allows the results to have probative value and thus more general and valid conclusions to be made. The condition for sufficient length T is determined by [16]:

$$T >> \tau_R , \tag{1}$$

where: τ_R is the correlation time.

The parameters of the random signal which are of particular interest are: the mean value of the signal (the DC component of the signal) which is given by [16], [17]:

$$\mu_x = \frac{1}{T}\int_0^T x(t)dt , \tag{2}$$

and the standard deviation (the root-mean-square value) of the signal [16], [17], recorded as:

$$\sigma_x = x_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(x(t) - \mu_x\right)^2 dt} . \tag{3}$$

Peak-factor and Form-factor

The crest-factor (also known as peak-factor in the signal processing theory) is defined as [18]:

$$PF = \frac{max\left(\left|x(t)\right|\right)}{x_{rms}} , \tag{4}$$

For a sine-wave signal $PF_{\sin} = \sqrt{2}$ (3 dB) [19] and for random signals $PF_{rand} = 3 \div 10$ (10 ÷ 20 dB), according to [8], [12] and [18].

The form-factor is the ratio of the root-mean-square value (RMS-value) of the signal to its averaged rectified value (ARV) x_{DC} [19]:

$$FF = \frac{x_{rms}}{x_{DC}}, \quad (5)$$

where:

$$x_{DC} = \frac{1}{T}\int_0^T |x(t)| dt. \quad (6)$$

For a sine-wave signal $FF_{\sin} = \frac{\pi}{2\sqrt{2}} \approx 1{,}11$ [19], and for random signals with Gaussian distribution $FF_{rand} = \sqrt{\frac{\pi}{2}} \approx 1{,}25$ [20].

Energy relation between sine-wave signal and random signal with Gaussian distribution

By means of Eq. (5), the ARV of the current drawn from the power supply when operating with random signal is defined as:

$$I_{DCrand} = \frac{I_{rms\,rand}}{FF_{rand}}, \quad (7)$$

where: $I_{rms\,rand}$ is the RMS-value of the current drawn from the power supply.

The ARV of a current caused by sine-wave signal is:

$$I_{DC\sin} = \frac{I_{rms\sin}}{FF_{\sin}}, \quad (8)$$

where: $I_{rms\sin}$ is the RMS-value of the signal current.

By means of Eqs. (7) and (8) the following coefficient can be derived:

$$\chi = \frac{I_{DCrand}}{I_{DC\sin}} = \frac{I_{rms\,rand}}{I_{rms\sin}} \cdot \frac{FF_{\sin}}{FF_{rand}} = \frac{\sqrt{\pi}}{2} \approx 0{,}8862. \quad (9)$$

This is a coefficient of compliance between ARV of sine-wave and Gaussian noise with one and the same RMS-value (coefficient of ARV-compliance).

Let the coefficient of effective use of the power supply voltage is assumed to be:

$$\xi = \frac{U_{out\,m}}{U_{cc}}. \quad (10)$$

where: $U_{out\,m}$ is the amplitude of the sine-wave signal;

U_{cc} – power supply voltage.

Generally, the coefficient of effective use of the power supply voltage can also be defined as:

$$\xi = \frac{\max\left(|x(t)|\right)}{U_{cc}}, \quad (11)$$

that is convenient when the power amplifier operates with a random signal.

The author proposes a replacement of the random signal $x(t)$ with a RMS-value $U_{out\,rms\,rand}$ with an equivalent sine-wave with the same RMS-value $U_{out\,rms} = U_{out\,rms\,rand}$ and amplitude $U_{out\,m\,rand}$. Using Eq. (4) and Eq. (10), the coefficient of effective use of the power supply voltage ξ_{eq} when operating with equivalent sine-wave can be rewritten as:

$$\xi_{eq} = \frac{U_{out\,m}}{U_{cc}} = \frac{U_{out\,rms} \cdot PF_{\sin}}{U_{cc}} = \\ = \frac{U_{out\,rms\,rand} \cdot PF_{\sin}}{U_{cc}} = \frac{U_{out\,m\,rand} \cdot PF_{\sin}}{U_{cc} \cdot PF_{rand}} \\ = \frac{PF_{\sin}}{PF_{rand}}\xi. \quad (12)$$

From Eq. (12) follows:

$$\xi_{eq} = \varepsilon \cdot \xi \quad (13)$$

$$\varepsilon = \frac{\sqrt{2}}{PF_{rand}}, \quad (14)$$

i.e. the random signal can be replaced with an equivalent sine-wave signal, so the coefficient of effective use of the power supply voltage decreases and becomes ξ_{eq}.

Energy parameters of audio power amplifiers, operating with random signal

As already mentioned in the previous paragraph, the data show that $PF_{rand} = (10 \div 20)$ dB, or average 15 dB [8], [13], [15] (Fig. 1).

If the calculations of the power parameters for a class B amplifier operating with sine-wave signals are available, it is also possible to make the same in the case of the random signals.

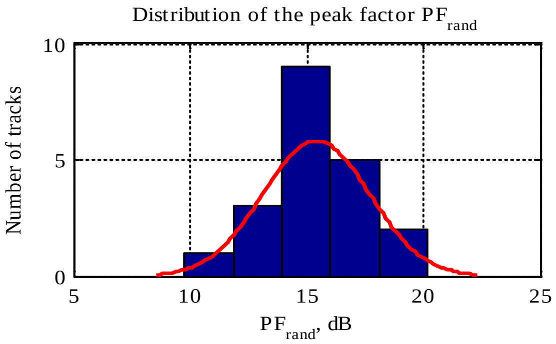

Figure. 1. Probability distribution of the PF_{rand} of 20 investigated music tracks

The equations describing the power parameters of class B amplifier when operating with sine-wave signal are [3], [4], [5], [21], [22]:

$$P_{DC} = \frac{2U_{cc} \cdot U_{out\,m}}{\pi R_L} = \frac{2U_{cc}^2}{\pi R_L}\xi\,, \qquad (15)$$

$$P_L = \frac{U_{out\,m}^2}{2R_L} = \frac{U_{cc}^2}{2R_L}\xi^2 \qquad (16)$$

$$P_D = \frac{U_{cc}^2}{R_L}\left(\frac{2\xi}{\pi} - \frac{\xi^2}{2}\right) \qquad (17)$$

$$\eta = \frac{P_L}{P_{DC}} = \frac{\pi}{4}\xi\,, \qquad (18)$$

where: R_L is the load resistance;

P_{DC} – power drawn from the power supply;

P_L – power delivered to the load resistance;

P_D – power dissipation of the amplifier;

η – efficiency.

Using the coefficients χ (9) and ξ_{eq} (12), Eqs. (15) ÷ (18) take the form:

$$P_{DCrand} = \frac{2U_{cc}^2}{\pi R_L}\xi_{eq}\chi \qquad (19)$$

$$P_{Lrand} = \frac{U_{cc}^2}{2R_L}\xi_{eq}^2 \qquad (20)$$

$$P_{Drand} = \frac{U_{cc}^2}{R_L}\left(\frac{2\xi_{eq} \cdot \chi}{\pi} - \frac{\xi_{eq}^2}{2}\right) \qquad (21)$$

$$\eta_{rand} = \frac{P_{Lrand}}{P_{DCrand}} = \frac{\pi}{4}\frac{\xi_{eq}}{\chi}. \qquad (22)$$

The expressions which allow a relation between the energy parameters of the amplifier working with sine-wave signals and random signals based on Eqs. (13), (15) ÷ (18) can be written as:

$$\frac{P_{DCrand}}{P_{DC}} = \varepsilon \cdot \chi \qquad (23)$$

$$\frac{P_{Lrand}}{P_L} = \varepsilon^2 \qquad (24)$$

$$\frac{P_{Drand}}{P_D} = \frac{\varepsilon(4\chi - \varepsilon\pi\xi)}{4 - \pi\xi} \qquad (25)$$

$$\frac{P_{Drand}}{P_{D\max}} = \frac{\varepsilon\pi\xi(4\chi - \varepsilon\pi\xi)}{4} \qquad (26)$$

$$\frac{\eta_{rand}}{\eta} = \frac{\varepsilon}{\chi} \qquad (27)$$

The graphical representations of the dependencies (23) ÷ (27) are given in Fig. 2 ÷ Fig. 6.

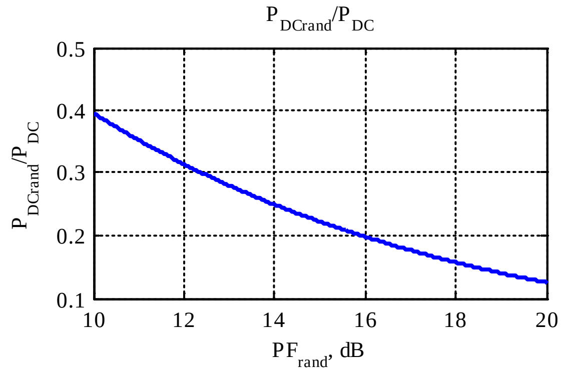

Figure 2. The P_{DCrand}/P_{DC} relation, depending of PF_{rand}

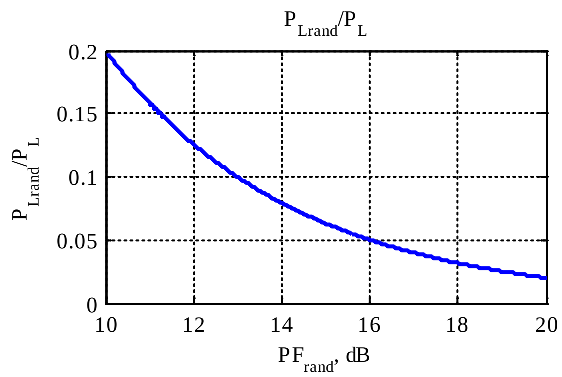

Figure 3. The $P_{Lrand}/P_{L\max}$ relation, depending of PF_{rand}

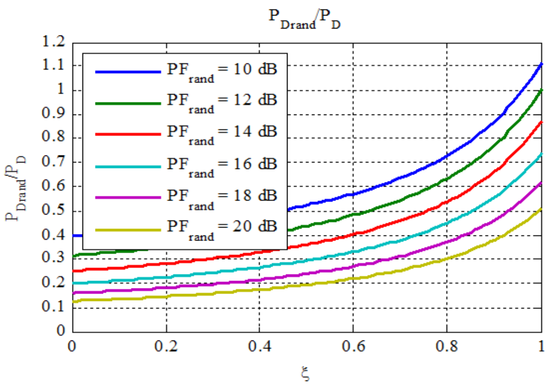

Figure 4. The P_{Drand}/P_D *relation, depending of* PF_{rand} *and* ξ

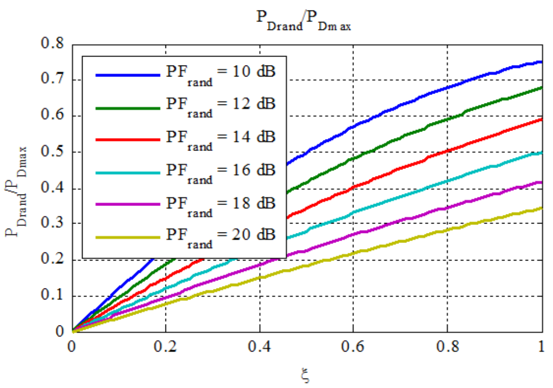

Figure 5. The $P_{Drand}/P_{D\max}$ *relation, depending of* PF_{rand}

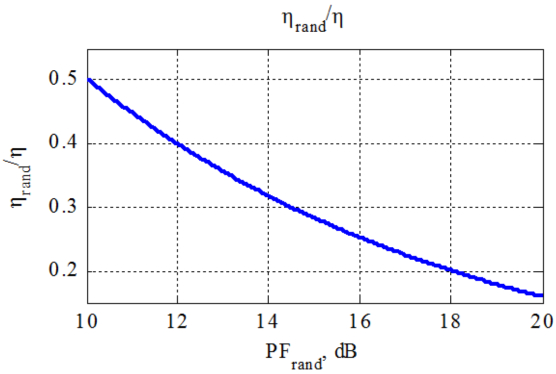

Figure 6. The η_{rand}/η *relation, depending of* PF_{rand}

3. Experimental results

Experimental investigations have been performed to confirm the theoretical analysis.

A computer simulation with Matlab® implementation is provided according the author mathematical model shown in [23], [24]. The results about P_{Drand} are shown in Fig. 7 and Fig. 8.

Further measurements have been performed on real amplifier prototype with IC LM3886. The measurement setup with data acquisition system NI USB-6211 is shown in Fig. 9. The measurements have been conducted under the following conditions:

- power supply voltage: ± 12 V;
- coefficient of effective use of the power supply voltage ξ: 0,75;
- load: 8 Ω (purely resistive).
- pink noise test signal:
  - duration: T = 30 s;
  - $U_{rms\,rand} = 2$ V;
  - $PF_{rand} \approx 14$ dB (≈ 5);
  - frequency band: 0 Hz ÷ 22 kHz;

The measurement results are shown in Fig. 10 and Fig. 11.

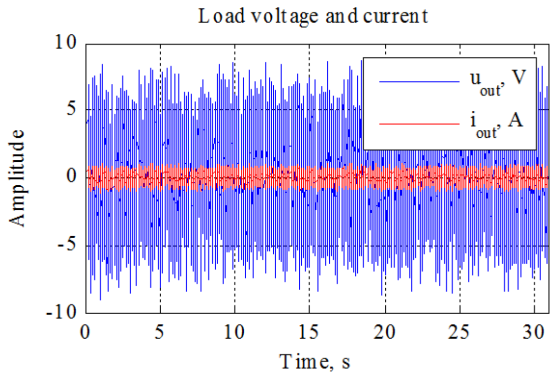

Figure 7. Pink noise output voltage and current of the class B power amplifier (Matlab® simulation)

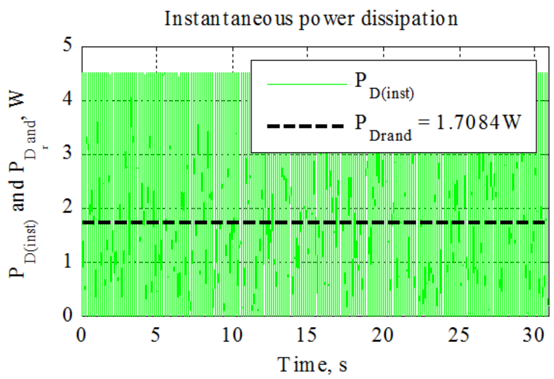

Figure 8. Instantaneous $P_{D(inst)}$ *and average* P_{Drand} *power dissipation of the class B power amplifier operating with pink noise (Matlab® simulation)*

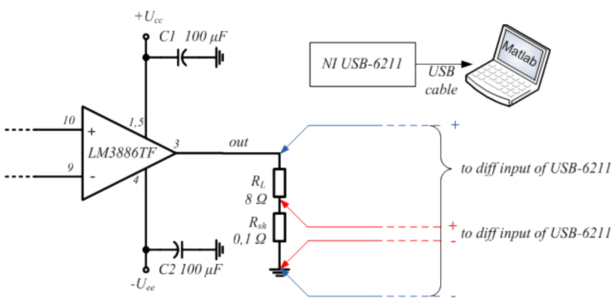

Figure. 9. Measurement setup for measurement of real-world class B push-pull solid state audio power amplifier

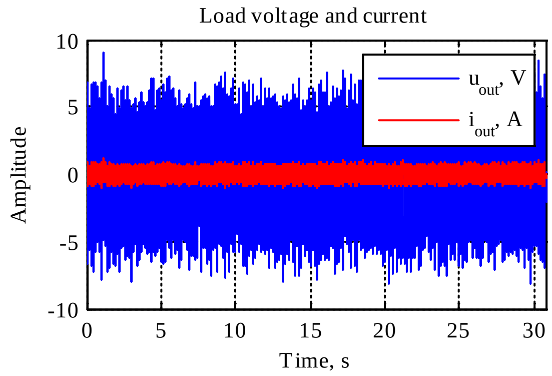

Figure 10. Pink noise output voltage and current of the class B power amplifier (measurement)

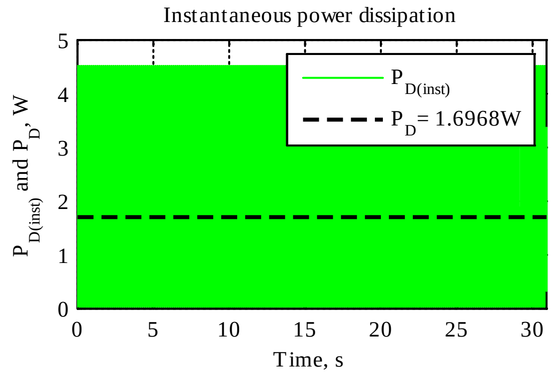

Figure 11. Instantaneous $P_{D(inst)}$ and average P_{Drand} power dissipation of the class B power amplifier operating with pink noise (measurement)

The results from the analysis, simulation and measurements are summarized in Tab. 1.

Table 1. Results from analysis, simulation and measurements

Exploration	P_{DCrand}, W	P_{Lrand}, W	P_{Drand}, W	η_{rand}, %
Analytical	2,127	0,397	1,730	18,66
Matlab® simulation	2,128	0,420	1,708	19,75
Real measurement	2,110	0,413	1,697	19,57

The obtained results are a proof of the consistency of the proposed analytical expressions.

4. Discussion

The analysis of the Eqs. (19) ÷ (22) shows that when operating with random signals:

1) The average power drawn from power supply P_{DCrand} decreases (Fig. 2) as compared to that calculated for a sine-wave signal P_{DC}, due to:

-the lower current consumption when operating with random signal that has the same RMS-value as a sine-wave;

- reduction of the coefficient of effective use of the power supply voltage.

2) The average power delivered to the load P_{Lrand} decreases (Fig. 3) as compared to that calculated for a sine-wave signal P_L due to:

- reduction of the coefficient of effective use of the power supply voltage.

3) The average power dissipation P_{Drand} of the power amplifier changes (Fig. 4) as compared to that calculated with a sine-wave signal P_D.

4) The efficiency of the amplifier η_{rand} decreases (Fig. 6) as compared to that calculated with a sine-wave signal η due to:

- reduction of the coefficient of effective use of the power supply voltage.

The quantitative results of the reviewed dependencies for several values PF_{rand} are summarized in Tab. 2.

Table 2. Values of the considered quantities for several meanings of PF_{rand}

Peak-factor	$\frac{P_{DCrand}}{P_{DC}}$	$\frac{P_{Lrand}}{P_L}$	$\frac{P_{Drand}}{P_D}$ $\xi = 0,9$	$\frac{\eta_{rand}}{\eta}$
$PF_{rand} = 10$ dB	0,396	0,200	0,870	0,505
$PF_{rand} = 15$ dB	0,223	0,063	0,608	0,284
$PF_{rand} = 20$ dB	0,125	0,020	0,379	0,160

5. Conclusion

This work presents a theoretical analysis of the power parameters of class B audio power amplifiers, operating with random signals. The coefficients χ and ξ_{eq} are proposed in Eqs. (9), (14). Eqs. (20) ÷ (23) are derived to calculate the power parameters when dealing with random signals, and Eqs. (24) ÷ (28) give the relations between the parameters of operation with sine-wave and random signals.

Computer simulation with Matlab® and real measurements verified the proposed power relations.

The results obtained can be discussed in several aspects:

1) The power consumption from the power supply is reduced to about 40 % as compared to that calculated when operating with sine-wave signal with

$PF_{rand} = 10\,\text{dB}$. This facilitates the construction of the power supply and leads to reduction of its weight, size and cost.

2) The power delivered to the load is 5 to 50 times lower than the one calculated when the amplifier operates with a sine-wave signal. The latter must be taken into account in the design of the amplifier and in the placement of requirements thereto.

3) The power dissipated in the form of heat losses decreased to average 60 % of P_D. This circumstance makes it possible to alleviate the cooling of the components in the power amplifier and reduce the size, weight and cost of the heat sinks.

4) The efficiency of the amplifier reduces by at least half of its maximum value, taking values in the range 0,12 ÷ 0,4.

The obtained results can be successfully used in the design of audio power amplifiers, especially if there is an impedance compensation of the loudspeaker(s) impedance.

This paper is a continuation and a final stage of the exploration of the power relations and the efficiency of the class B amplifiers operating with complex load and random signal.

Acknowledgements

The author would like to acknowledge the financial support from the project "Examination of power parameters and efficiency of class B power amplifier operating with complex load and random signal", sponsored by the Bulgarian Ministry of Education and Science, 2014.

References

[1]. Sirakov, E. (2002). The Instantaneous Dissipation of a Class AB Amplifier. RADIOELEKTRONIKA 2002, May 14 – May 16 2002, Bratislava, Slovak Republic. *Proceedings of Papers*, pp. 457-460.

[2]. Sirakov, E. (2001). Models of push-pull transistor power amplifier. RADIOELEKTRONIKA 2001, May 10 – May 11 2001, Brno, Czech Republic. *Proceedings of Papers*, pp. 330-333.

[3]. Leach, M., Jr. (2010). *Introduction to Electroacoustics and Audio Amplifier Design*. Dubuque, Iowa, Kendall/Hunt Publishing.

[4]. Amos, S., & James, M. (2000). *Principles of Transistor Circuits*. Oxford, Newness.

[5]. Kazimierczuk, M. (2015). *RF Power Amplifiers*. Chichester, West Sussex, John Wiley and Sons.

[6]. Self, D. (2009). *Audio Power Amplifier Design Handbook*. Oxford, Focal Press.

[7]. Bortoni, R., Filho, S., & Seara, R. (2002). On the Design and Efficiency of Class A, B, AB, G, and H Audio Power Amplifier Output Stages. *JAES* Vol. 50, No. 7/8, pp. 547-563.

[8]. Zee, R. (1999). *High Efficiency Audio Power Amplifiers*. Nijmegen, Philips Semiconductors.

[9]. Tashev, I., & Acero, A. (2010). Statistical Modeling of the Speech Signal. *IWAENC 2010*, August 30 - September 2 2010, Tel Aviv, Israel.

[10]. Mijic. M., Masovic, D., Pavlovic, D., & Petrovic, M. (2009). Statistical properties of music signals. 126th AES Convention, May 7 – May 10, Munich, Germany. *Proceedings of papers*, p. 7702.

[11]. Hansler, E., & Schmidt, G. (2004). *Acoustic Echo and Noise Control A Practical Approach*. Hoboken NJ, John Wiley & Sons Inc.

[12]. Zhivomirov, H. & Plamenova, D. (2012). Study of some parameters of the audio signals (in Bulgarian*). Journal of Acoustics*, year XIV, № 14, pp. 89-95.

[13]. Benjamin, E. (1994). Audio Power Amplifiers for Loudspeaker Loads. *JAES*, Vol. 42, No. 9, pp. 670 - 683.

[14]. Dymond, H., & Mellor, P. (2010). An Active Load and Test Method for Evaluating the Efficiency of Audio Power Amplifiers. *JAES* Vol. 58, № 5, pp. 394-408.

[15]. Raab, F. (1986). Average efficiency of class-G power amplifiers. *IEEE Transactions on Consumer Electronics*, Vol. CE-32, № 2, pp.145-50.

[16]. Nachev, N. & Sinapov, P. (2011). Measurement of the characteristics of random processes: Part 1 (in Bulgarian). *International Virtual Journal MTM*, year V, № 1, pp. 3-8.

[17]. Manolakis, D., & Ingle, V. (2011). *Applied Digital Signal Processing*. Cambridge, Cambridge University Press.

[18]. Hartmann, W. (1998). *Signals, Sound and Sensation*. New York, Springer.

[19]. Boylestad, R. (2012). *Introductory Circuit Analysis*. Upper Saddle River, New Jersey, Prentice Hall.

[20]. Horowitz, P., & Hill, W. (2012). *The Art of Electronics*. Cambridge, Cambridge University Press.

[21]. Sedra, A., & Smith, K. (2010). *Microelectronic Circuits*. Oxford, Oxford University Press.

[22]. Zhivomirov, H. (2011). Instantaneous and average power dissipation of Class B amplifier operating with complex load (in Bulgarian). *Journal of Acoustics*, year XIII, № 13, pp. 71-77.

[23]. Zhivomirov, H. & Vasilev R. (2014). Power parameters and efficiency of class B amplifier operating with complex load and random signal. *UNITECH'2014 Conference Proceedings*, Vol. II, pp. II-53–II-58.

[24]. Zhivomirov, H. & Sirakov, E. (2014). On Instantaneous Power Dissipation in Class B Amplifier. *TEM Journal, 3*(2), pp. 101-105.

Corresponding author: Hristo Zhivomirov
Institution: Technical University – Varna, Bulgaria
E-mail: hristo_car@abv.bg