Investigating the Misconceptions of 15-Year-Old Pupils when Solving Geometric Problems at Different Levels of Mathematical Competences

Veronika Bočková [1], Gabriela Pavlovičová [1]

[1] *Department of Mathematics, Constantine the Philosopher University in Nitra, Nitra, Slovakia*

Abstract **– OECD´s Programme for International Student Assessment (PISA) describes mathematical competencies on three levels – reproduction, connection and reflection. This article presents the research results of creating and analyzing solutions to geometric problems at three different competence levels. The most common misconceptions when solving these problems were identified. At the same time, the connection of geometric knowledge among pupils at the end of primary education in Slovakia with the solution of more complex problems were investigating. The research results show that with increasing difficulty of the problems, the percentage of incorrectly solved problems increases, and pupils are more likely not even to start solving the problem. The qualitative analysis of the problem solutions shows that pupils have significant problems connecting geometric knowledge and solving more complex problems at the level of reflection.**

Keywords **– Mathematical competencies, geometrical tasks, elementary school, qualitative analysis.**

DOI: 10.18421/TEM144-70
https://doi.org/10.18421/TEM144-70

Corresponding author: Gabriela Pavlovičová,
Department of Mathematics, Constantine the Philosopher University in Nitra, Nitra, Slovakia
Email: gpavlovicova@ukf.sk

Received: 05 February 2025.
Revised: 10 September 2025.
Accepted: 15 September 2025.
Published: 27 November 2025.

1. Introduction

The acquisition, development and mastering of competencies in the educational process is a current topic for teachers at all education levels. Teachers are constantly trying to increase the effectiveness of the teaching process and are looking for ways to improve the personal and professional development of the students. Therefore, teachers of individual subjects are trying to create universal knowledge that is not tied only to a specific subject, rather knowledge that students could use in everyday life and various work areas.

The term competence is derived from the French word "compétence", which means the ability or capability to perform a certain task. The content of the term competence in teaching is defined by the behavioral definition of Schoonover Associates [1], according to which "Competence is behavior (activity or complex of activities) that characterizes excellent performance in some area of activity". According to [2], competencies represent knowledge, skills, abilities, and ways of human behavior demonstrated in practical activities. Competence can generally be understood as the intersection of acquired knowledge, abilities and skills, human attitudes, value orientations and motives for activity.

1.1. Mathematical Competencies

According to the European Parliament, mathematics focuses on developing students' mathematical competencies, which defines mathematical competence as “the ability to develop and use mathematical thinking to solve a variety of problems in everyday situations.” As is stated by [3], mathematical competence is the ability to understand, assess and perform a variety of mathematical and non-mathematical situations while using mathematics.

According to [4], mathematical competences are "a non-hierarchical list of general mathematical knowledge, skills, abilities and attitudes" that students use to solve tasks, mathematical modeling, as well as to solve everyday problems.

There are various definitions of mathematical competencies and their division in the literature. OECD PISA [5] uses eight mathematical competencies to assess the ability to use mathematics in different situations:

1. Thinking and reasoning,
2. Argumentation,
3. Communication,
4. Modelling,
5. Problem posing and solving,
6. Representation,
7. Using symbolic, formal and technical language and operations,
8. Use of aids and tools.

For example, a description of the eight mathematical competencies at individual levels is mentioned in OECD PISA [5] on page 107. Individual competence levels are not isolated but form a continuum.

OECD PISA [6] describes mathematical competencies based on three levels:

1. Level of reproduction – The lowest class of competencies, based on elementary knowledge, skills and abilities. Students at the level of reproduction can repeat learned material, perform routine calculations and solve simple problems.
2. Level of connection – A class of competencies based on the connection of mutually different ideas of the problem being solved or areas of mathematics. Students at the level of connection can solve problems that are not routine using integration, connection, a combination of known methods, simple addition of acquired knowledge and modeling.
3. Level of reflection – The highest class of competencies concerning knowledge, abilities, skills and attitudes. Students at the level of reflection penetrate the essence of mathematics, plan strategies for solving more complex tasks and require an original mathematical approach as well as a combination of different methods. Students at this level use developed reasoning, argumentation, abstraction, and modeling skills in various unfamiliar situations.

Solving a task may require the mathematization of real problems, the subsequent connection of known methods and procedures, as well as the use of routine calculations. It is evident that a task requiring competencies at the level of reflection is more demanding, but not all competencies from the previous levels are necessary for its solution.

Mathematical competencies divided according to OECD PISA [5], [6] and many other (Slovak or foreign) authors are challenging to research individually since several competencies are often used simultaneously when solving different tasks.

Paying attention to the development of geometric competencies in mathematics is also essential. Competencies in geometry are understood as a set of those abilities, knowledge, and attitudes that provide success in solving theoretical and practical geometric problems. As shown by [7], geometric competencies include the following abilities and skills:

- The ability to generalize and abstract,
- The ability to construct,
- The art of observing, creating connections,
- The ability to argue and prove.

2. Empirical Researches Dealing with Mathematical Competencies

Various research studies exist on the mathematical competence of children and students at all levels of education. Much research confirms that early mathematical competence is the foundation for mastering more complex mathematical understanding [8], [9], [10]. The mathematical competence of children in kindergarten varies considerably. To improve opportunities for all children, regardless of their family background, kindergartens need to specifically support mathematics and provide children with learning opportunities that meet their diverse learning needs [11], [12]. Children best develop mathematical competence intuitively using personally meaningful activities while facilitating the learning process [13], [14].

Developing mathematical competence in pupils is essential for their intellectual growth and success in various areas of life [15]. Various studies have addressed effective pedagogical strategies for developing mathematical competence in elementary school pupils. The Khan Academy educational platform improves pupils' mathematical competence and effectively improves academic performance [16]. The study [17] confirms that dynamic geometry software (especially GeoGebra) has a positive impact on the development of pupils' mathematical competences. Pupils developed visualization processes and modeling while using dynamic geometry software. Significant progress was made in developing mathematical competencies in pupils with lower mathematical competencies. Based on research conducted with primary and secondary school students, teachers and future teachers, authors state that programming can be used to develop mathematical competencies [18].

Research also confirms that modern technology and applications, such as augmented reality, 3D printing, and the Lumen application (an ad-hoc solution designed and developed to visualize and combine mathematical surfaces in 3D) develop all 8 types of mathematical competencies listed in OECD PISA 2009 [19]. Authors in [20] mention that 3D printing can effectively link the two areas - tangible models and computer simulations, along with active student involvement in the learning process.

Research on mathematical competencies also deals with teachers or future mathematics teachers. The research in [21] examines whether high school prospective mathematics teachers can identify the mathematical competencies required to solve a specific mathematical task. They confirm that the awareness of future teachers is low, and future teachers have difficulty distinguishing between mathematical competencies and the process. Author in [22] answers the question: "What does it mean to be a good mathematics teacher?" According to his opinion, a good mathematics teacher can effectively support the development of mathematical competencies in their pupils and must also have them themselves. They state that every mathematics teacher should develop curriculum, teaching, uncovering of learning, assessment, collaboration, and professional development competencies. Research by [23] confirms that experienced teachers (teaching for more than 5 years) also focus more on the competencies of connecting mathematical knowledge, while novice teachers focus more on understanding and applying the acquired knowledge. The findings suggest that teacher professional development programs should emphasize the importance of planning activities by pupils' cognitive development and the development of mathematical competencies.

3. Materials and Methods

Geometry is one of the problem areas in teaching mathematics at all levels of education in Slovakia. The results of national testing over the past 10 years [24], as well as other studies, confirm that pupils and students face issues understanding the geometry curriculum and often have only formal knowledge. For the reasons mentioned above, the presented research focuses on examining mathematical competencies in geometry among pupils in the final phase of primary education.

Research goals: The following research goals were stated:

- To create geometric tasks at three levels of mathematical competences according to PISA.
- To analyze the success of solving tasks at individual levels of competences.
- To identify the most common mistakes in solving the tasks typical for pupils in the final stage of primary education.

Research sample: The research sample consisted of 781 ninth-grade pupils from 29 different primary schools in Slovakia, mostly aged 15. The research was conducted in May - June, i.e., at the end of the school year, and each pupil had mastered all the geometry curriculum that they were supposed to have mastered by the end of primary school in Slovakia.

Research tools: For the research, a set of geometric tasks was designed to identify the mathematical competencies possessed by ninth-grade pupils in geometry. Six tasks were created at three levels of mathematical competencies: the first pair of tasks at the level of reproduction, the second pair at the level of connection, and the third pair at the level of reflection.

The individual tasks can be briefly characterized as follows:

- Task1 (TI): The task focuses on calculating the length of a side of a rectangle if the length of its other side and its area are known. To solve the task, it is necessary to know the formula for calculating the area of a rectangle and express the unknown from the given formula.
- Task 2 (T2): The task deals with calculating the circumference of a circle if the length of a circular arc corresponding to a specific central angle is identified. To solve the task, pupils must identify how many times a circular arc is located on the circle and know the size of the central angle corresponding to the entire circle.
- Task 3 (T3): The task is to calculate the area of a triangle, which is depicted in a rectangular coordinate system. To solve the task, it is necessary to identify the length of the triangle's base and its height from the rectangular coordinate system, which is then substituted into the formula for calculating its area.
- Task 4 (T4): The task focuses on identifying the most significant possible length of a side of a triangle, given two known side lengths. To solve the task, it is necessary to be aware of the triangle inequality and then identify the longest possible length of the third side of the triangle.
- Task 5 (T5): The task focuses on calculating the size of an angle using knowledge about adjacent and vertex angles. To solve the task, it is necessary to use knowledge about adjacent vertex angles and the sum of the sizes of the angles in a triangle.
- Task 6 (T6): The goal of the task is to identify what percentage of a rectangular tile is made up of its pattern composed of right-angled triangles. To solve the task, pupils must calculate the area of the tile, the area of the tile pattern, and identify what percentage of the tile is made up of its pattern.

Solutions to the tasks were evaluated from four aspects: the pupil solved the task incorrectly, the pupil partially solved the task correctly, the pupil solved the task correctly, and the pupil did not start solving the task. The solutions of individual tasks were also divided into several steps the pupils could follow when solving.

4. Data Analysis and Interpretation of Results

All tasks were solved correctly by 48 (6%) pupils, five tasks were solved correctly by 92 (12%) pupils, only one task was solved correctly by 231 (30%) pupils, and not even one of the tasks was solved by 84 (11%) pupils.

The first pair of tasks at the level of reproduction was solved correctly by 291 (37%) pupils, the second pair at the level of connection by 184 (24%) pupils, and the last pair of tasks at the level of reflection was solved correctly by 116 (15%) pupils.

The success of solving individual tasks is shown in Figure 1. As visible from Figure 1, pupils solved tasks at the level of reproduction (T1, T2) with the greatest success, tasks at the level of connection (T3, T4) with the least success, and tasks at the level of reflection (T5, T6) with the least success. The success of solving groups of tasks results from the increasing demands placed on mathematical competencies, and the particular success of the solution also reflects the expectations.

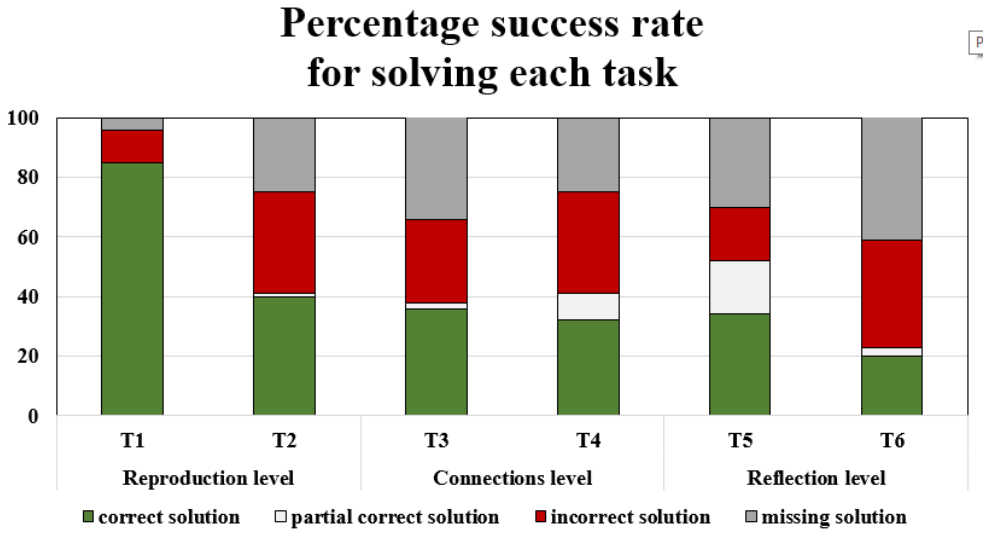

Figure 1. Geometric tasks success rate

The tasks at the level of reproduction were the easiest for the pupils, as they used basic theoretical knowledge to solve them - the formula for calculating the area of a rectangle and the length of a circle if the length of the arc of a circle corresponds to a mentioned central angle is known. The percentage of correct answers is 85% in T1 and 40% in T2. The tasks at the level of connection were more challenging for the pupils, and the percentage of correct answers was 36% in task T3 and 32% in task T4. The results of the pupils' solutions, confirmed by statistical evaluation, showed that the tasks were compiled correctly and graded, and therefore, the success rate of their solutions gradually decreased. The findings identify that the pupils have theoretical knowledge in geometry, but they have a significant problem connecting and integrating the acquired knowledge. The last two tasks at the level of reflection were the most challenging for the pupils. The success rate of legal answers is 34% in task T5 and only 20% in task T6. Task T5 was less demanding for pupils than task T4 at the level of connection.

The procedure for solving task T5 is familiar to pupils, as similar tasks also occur in the national testing T9 in Slovakia, or in tasks for secondary school entrance exams. Task T6 is one of the irregular tasks, the solution of which also requires critical thinking from pupils and they do not immediately know the solution procedure. Therefore, pupils had the most significant problems with solving this task.

4.1. Content Analysis of Pupils' Solutions to Geometric Problems

Note: In Slovak the area of a figure is denoted by S, the perimeter and circumference of a circle by o, the radius of a circle by r, the lengths of the sides by a, b, the height by v.

Task 1: If the area of a rectangle is 48 cm^2 and the length of the rectangle is 6 cm, what is the rectangle's width?

Steps for task solving:

- Identifying the formula for calculating the area of a rectangle.
- Substituting into the relation for calculating the area of a rectangle. (If the pupil passed the first two steps of the solution correctly, then his solution was partially correct.)
- Identifying the unknown length of the side of the rectangle.

The highest success rate of the solution characterizes task T1. If the pupil correctly solved the formula for calculating the area of a rectangle, then the pupil correctly substituted it into the particular formula and subsequently identified the length of the other side of the rectangle.

The most common misconceptions: The identified mistake in pupils solving task T1 was that they used the wrong formula to calculate the area of a rectangle. Pupils most often (1.2%) used the formula to calculate a rectangle's perimeter or the triangle's area (Figure 2). It could also be observed in the solutions that pupils considered the area of a rectangle to be the length of its side (Figure 3).

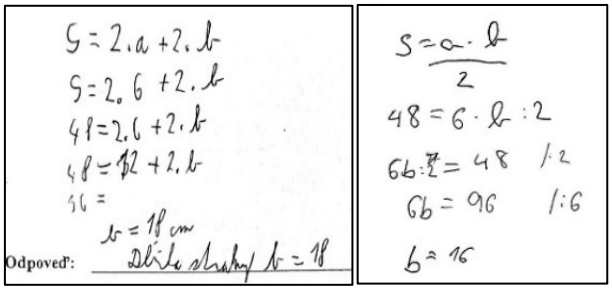

Figure 2. Using the formula to calculate the perimeter of a rectangle (formula S = 2.a + 2.b) or the area of a triangle (formula S =$\frac{a.b}{2}$)

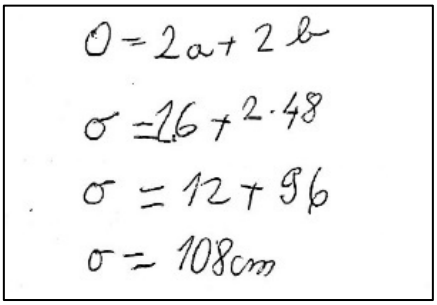

Figure 3. The area of a rectangle is considered to be the length of the rectangle side

Task 2: The arc of the circle *c* with a central angle $\alpha = 18°$ is $28\ cm$ long. Determine the length of the whole circle *c*.

Steps for task solving:

- Calculate the radius of a circle or identify how many times an arc of a circle corresponding to a stated central angle is located on a circle. (If the pupil passed the first step of the solution, then the solution was partially correct.)
- Calculating the circumference of a circle.

The most significant issue for pupils was to decide how to solve the issue. If pupils realized that they had to calculate the radius of a circle or how many times the arc of a circle corresponding to a central angle is located in a circle (44.2%), then 39.9% of pupils solved the problem correctly. This shows that understanding the problem was key to its calculation.

***The most common misconceptions*:** There were several misconceptions among pupils in solving the problem. The most common mistake made by 62 (8%) pupils was that they considered the length of a circular arc to be the radius of the circle (Figure 4). Pupils also state the circumference of a circle as twice the length of the circular arc or as a π-multiple of it. The second serious mistake pupils made was identifying the size of the central angle of the whole circle. 2% of pupils identified that the size of the central angle of the whole circle was 100° or 180° (Figure 5).

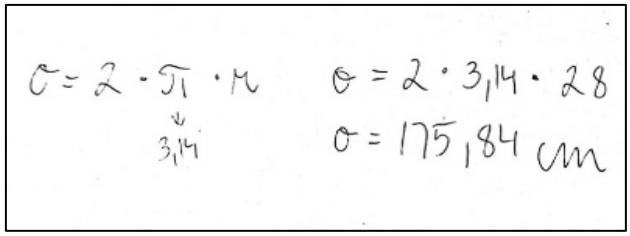

Figure 4. Calculating the circumference of a circle with a radius equal to the length of a circular arc

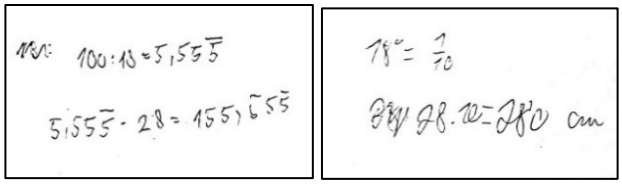

Figure 5. Calculating the length of a circular arc for a central angle of 100° (solution on the left side) and 180° (solution the right side)

Pupils also tried calculating the length of a circle using the formula for calculating the length of a circular arc. They often used the correct formula but incorrectly inserted the particular data (Figure 6) or wrote the wrong formula, thus miscalculating the circumference (Figure 7).

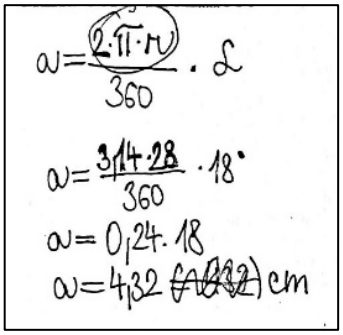

Figure 6. Incorrect substitution in the formula for the calculation of the length of a circular arc

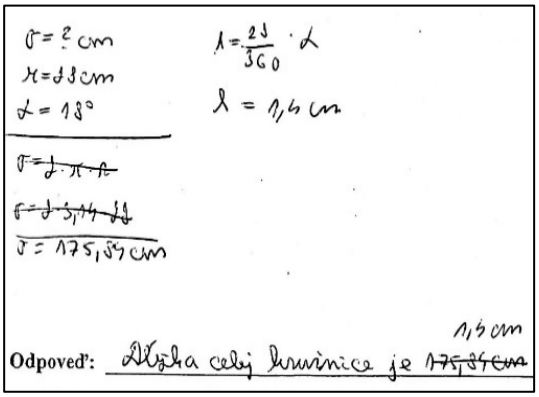

Figure 7. Incorrect formula for calculating the length of a circular arc

In this task, pupils were incorrectly confused about the formulas for calculating the circumference and area of a circle (Figure 8), considering the circle's radius to be the length of the arc of the circle.

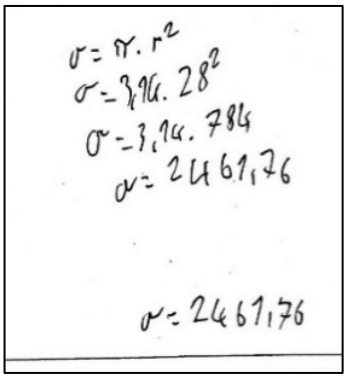

Figure 8. Confusing the circumference and area formulas by interpreting the radius – r as the given arc length of the circle.

Task 3: The triangle *ABC* (Figure 9) is drawn in the (orthonormal) Cartesian coordinate system. Determinate how many square units is the area of a triangle if one length unit is the unit either on the *x* or *y* axis.

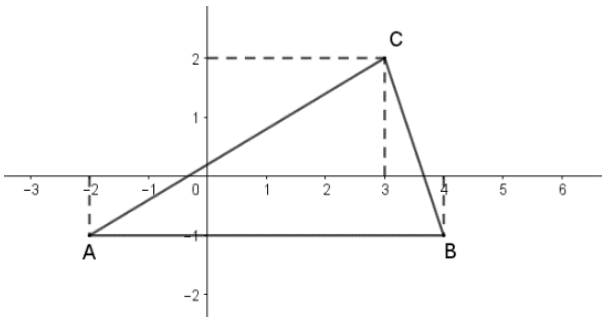

Figure 9. Triangle for the task 3

Steps for task solving:

- Identifying the length of the side of a triangle and the length of the height to this side given in the Cartesian coordinate system.
- (If the pupil passed the first step of the solution, then the solution was partially correct.)
- Substituting the identified data into the formula for calculating the area of a triangle.
- Calculating the area of a triangle.

The success of solving this problem depended on correctly identifying the length of the side and the triangle's height. If the pupils correctly identified the particular lengths, only 1.5% did not use the correct formula to calculate the triangle area or made a numerical error in the solution.

The most common misconceptions: In the solutions to the problem, two types of mistakes were observed. The first type was the use of an incorrect formula for calculating the area of a triangle. Pupils used the formula to calculate the area of a parallelogram (Figure 10) or other incorrect formulas, such as $A = a.b.c$, $A = a^3$. The second type of mistake observed in 55 (7%) pupils, was incorrect identification of the height and the side of the triangle (Figure 11). Pupils identified the length of the side of the triangle from $4\,cm$ to $7\,cm$ and the triangle's height from $3\,cm$ to $5\,cm$.

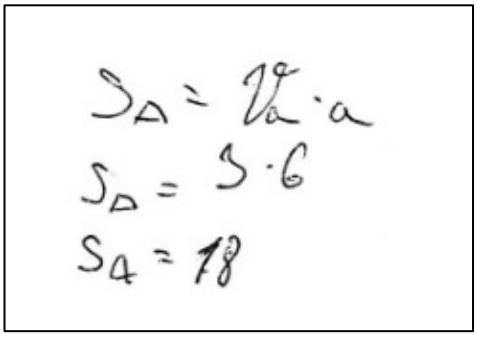

Figure 10. Using the formula for calculating the area of a parallelogram

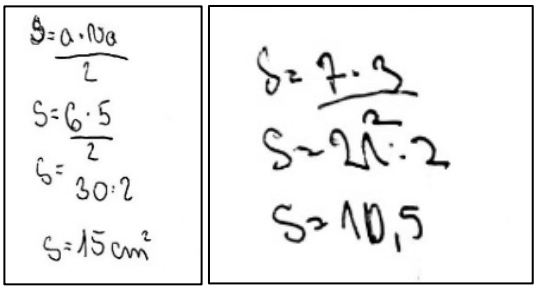

Figure 9. Incorrect identification of the height and length of the side of a triangle

Task 4: The lengths of the sides of the triangle *ABC* are three natural numbers. The two shorter sides have lengths: $c = 6\ cm$, and $b = 8\ cm$ Determine the third side's length so that the triangle's perimeter is as large as possible.

Steps for task solving:

- Identifying a triangular inequality (generally or with specific numbers).
  (If the pupil passed the first step of the solution, then the solution was partially correct.)
- From the triangular inequality, correctly calculate the length of the third side.

41% of pupils could use the triangular inequality in the task, but only 32.1% could state the correct conclusion. This means 9% of pupils could not connect their knowledge and correctly solve task T4.

Using the triangle inequality, 36 (5%) pupils identified that the length of the third side should be $14\ cm$. In their solutions, pupils also mentioned that the length of the third side should be more than $14\ cm$, i.e., $15\ cm$. Some pupils identified using the triangle inequality that the length of the third side should be less than $14\ cm$, but they mentioned their result as a decimal number, e.g., $13.9\ cm$.

The most common mistakes: In addition to the mistakes mentioned above resulting from incorrect use of the triangle inequality, several misconceptions were observed in the pupils' solutions. 60 (8%) pupils used the Pythagorean theorem to identify the unknown side length (Figure 12). The pupils did not realize that the triangle in the task was general and not right-angled.

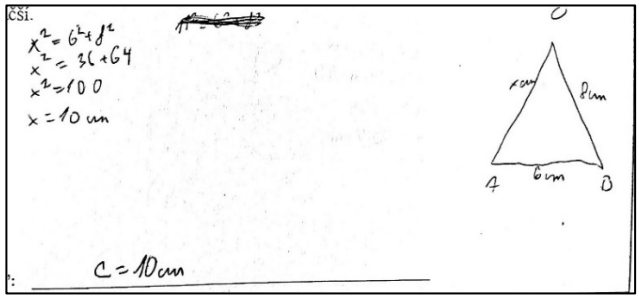

Figure 10. Solving the problem using the Pythagorean theorem

45 pupils (6%) identified the unknown length of the triangle based on the sequence of numbers 6 and 8. Pupils also identified, without providing a reason, that the length of the third side is 7, 8 or 9 cm. Some pupils tried to solve the issue by drawing or combining the numbers 6 and 8 or mentioned that the length of the third side could be any regular number.

Task 5: In the picture (Figure 13), there are two isosceles triangles - triangle *ABC* with base *AB* and triangle *CDE* with base *CD*. Determine the size of the angle α.

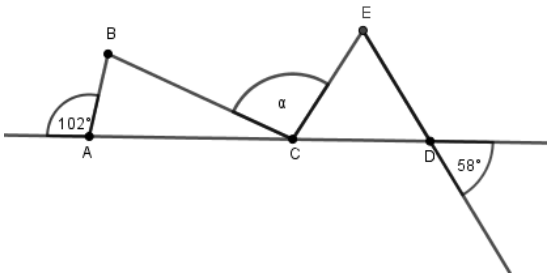

Figure 11. Graphical assignment for the Task 5

Steps for task solving:

- Identifying the size 78° of the adjacent angle to the 102° angle at vertex A and the size 58°of the vertex angle to the 58° angle at vertex D. (If the pupils passed the first step of the solution, then the solution was partially correct).
- Identifying the sizes of the interior angles at the bases of isosceles triangles.
- Calculating the size of angle α.

It was observed in the solutions that the pupils know about vertex and adjacent angles, about the sum of angles in a triangle, and that at the base of an isosceles triangle, the sizes of the angles are identical. 97 (12.5%) pupils correctly determined the vertex and adjacent angle to the particular angles. However, these pupils could not identify which two angles in a triangle are identical in size (Figure 14). The pupils did not realize which sides are the base of an isosceles triangle, even though this was stated in the task assignment.

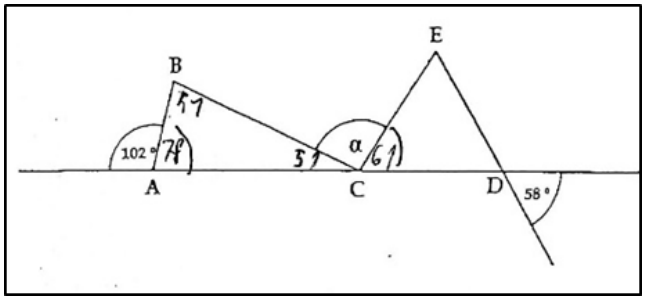

Figure 12. Incorrect identification of the angle sizes at the bases of isosceles triangles

***The most common misconceptions*:** There were observed various mistakes in the pupils' solutions. The pupils made several numerical mistakes due to the lack of attention or, despite correctly identifying the angles at the bases of isosceles triangles, incorrectly identified the size of the third angle in the triangle. Without additional counting, 2% of the pupils decided that the angle α is right. In their solutions, the pupils also stated the angle α as the difference between the right angle and only one angle at the vertex C, and not as the difference between the right angle and the sum of both angles (Figure 15). Several pupils also mentioned that the adjacent angle to the angle of size 102° is also 102°, or that the sum of the angles in one half-plane is 360° and not 18 ° (Figure 16). Some pupils tried to solve the issue by adding another triangle to the picture. However, the pupils did not solve the problem this way and made mistakes in the calculation.

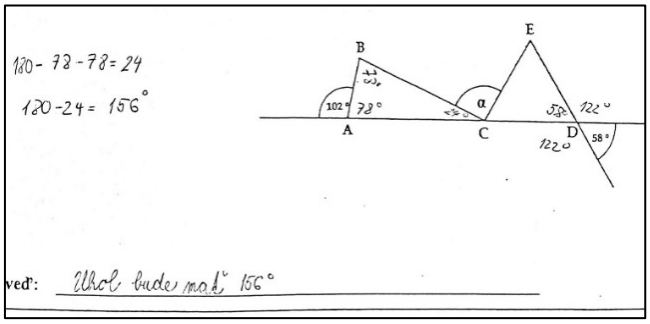

Figure 13. Incorrect calculation of the size of angle α by not subtracting the size of angle DCE

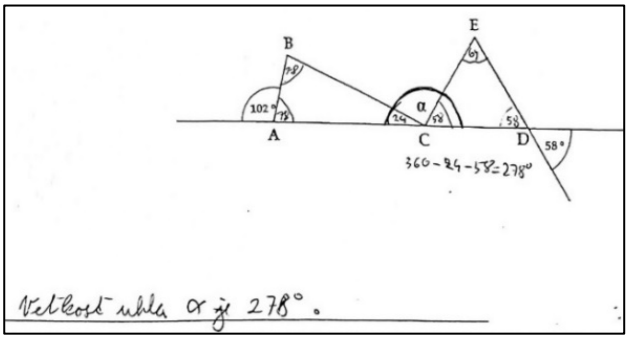

Figure 14. Incorrect identification of the right angle size

Task 6: A pattern (Figure 17) is composed of four congruent right-angled triangles, which are in the rectangular tile. Determine the percentage representation of this pattern in the rectangular tile. There are given the values of the sizes of the rectangular tile in Figure 17.

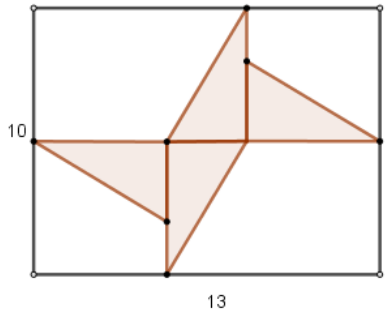

Figure 15. Graphical assignment for the Task 6

Steps for task solving:

- Calculating the area of one right-angled triangle from a mosaic.
- Calculating the area of four identical triangles. (If the pupil passed the first two steps of the solution, then his solution was partially correct.)
- Calculating the area of a rectangle (the entire tile).
- Calculate the percentage of the pattern area that is the area of the entire tile.

The analysis of the solution to this task presents that 184 (23.6%) pupils could correctly identify the area of one triangle in the mosaic. 172 (22%) pupils identified the area of the entire tile pattern, 20 (2.5%) pupils did not calculate what percentage the area of the pattern is of the area of the entire rectangular tile. These pupils faced an issue with calculating percentages; therefore, their problem was not geometric.

***The most common misconceptions*:** In addition to numerical mistakes, it was observed that pupils only estimated the correct result. 25 (3%) pupils identified by such an estimate that the pattern forms half or a quarter area of the tile. Some pupils identified by calculation that four triangles can be placed in a quarter of the tile. There were also solutions in which pupils incorrectly divided the tile into several, according to them, identical, right-angled triangles (Figure 18).

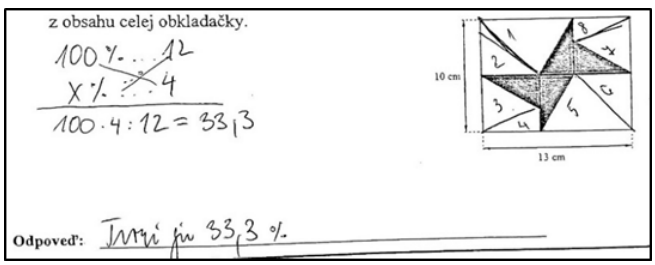

Figure 16. Incorrect placement of pattern triangles in the tile

In some solutions (1.6%), pupils stated the correct answer as the percentage of the content of one triangle out of the content of the entire tile. The pupils may have made this mistake due to the lack of attention, as the pupils' steps would have been correct if they had multiplied by the number of triangles in the tile pattern.

It was also observed in the solutions that pupils identificate different side lengths of the triangles, which led to incorrect calculations of the area of the triangles forming the tile pattern. The incorrectly calculated areas of the pattern are shown in Figure 19 and Figure 20.

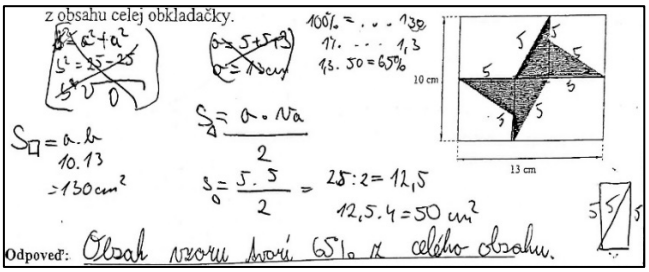

Figure 17. Incorrect identification of the lengths of the sides of a triangle – the triangle is isosceles

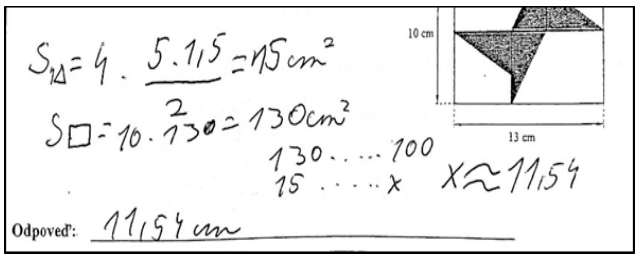

Figure 20. Incorrect identification of the triangle side lengths – the length of the cathetus is 1.5 *cm*

5. Conclusion

The research results demonstrated that the created geometric tasks were correctly classified into the three levels of mathematical competence according to PISA. Therefore, they can be used in school education to prepare pupils for national or international testing. The stated misconceptions in the geometric curriculum identified specific lacks of pupils' understanding of geometric concepts and relationships. The identified misconceptions were of a procedural and conceptual nature. They pointed to the acquisition of formal knowledge by pupils, which leads to more serious problems in their further studies in secondary school. Only 6% of pupils involved in the research solved all the tasks correctly. An unfavorable conclusion was that 11% of pupils did not correctly solve a single task from the suggested set. The content analysis of incorrect solutions to tasks at the level of reproduction showed that when solving them, pupils often do not know or confuse the formulas for calculating the area and perimeter of geometric shapes.

Learning formulas by heart leads to the inability of pupils to derive the necessary relationships, and therefore, they cannot calculate the tasks even by logical reasoning. In solving tasks, it was observed that pupils also have problems determining the values to be inserted into the formulas. The analysis of incorrect solutions to tasks at the level of connection showed that many pupils could not connect their knowledge and solve the tasks correctly. Even though pupils knew the necessary mathematical relationships and correctly determined the mutual position of geometric shapes in the task assignment, they could not obtain other necessary information for their solution. In solving tasks at the level of reflection, in comparison with other tasks, there is an estimate of the solution to the task. In incorrect solutions to tasks, it was observed that pupils often cannot plan a comprehensive and complex procedure necessary for their solution. They often calculated only individual parts of the tasks, which could not connect and come to the correct conclusion of the task solution. In all tasks, numerical errors were observed, as well as errors due to inattention.

Identifying misconceptions early is crucial to prevent students from forming inaccurate models of mathematical concepts that could affect their ability to understand and apply mathematics in other subjects. The identified problems in the solution of geometry tasks are not only in Slovakia but also abroad Therefore, it is necessary for mathematics teachers to be able to diagnose problems in students' understanding of mathematical concepts in a timely manner and to have effective tools for the development of students' mathematical competences.

Acknowledgements

This work was supported by Scientific Grant Agency Ministry of Education, Science, Research and Sport of the Slovak Republic and Slovak Academy of Sciences VEGA 1/0532/23.

References:

[1]. Schoonover Associates. (2001). *Competency Q&A.* Schoonover. Retrieved from: https://www.schoonover.com/faq/ [accessed: 20 January 2025].

[2]. Turek, I. (2005). *Inovácie v didaktike: príspevok k realizácii projektu Milénium vo vyučovacom procese na základných a stredných školách* (2nd ed.). Metodicko-pedagogické centrum.

[3]. Niss, M., & Højgaard, T. (2019). *Mathematical competencies in mathematics education*.

[4]. Sekerák, J. (2008). *Diagnostikovanie a rozvíjanie kľúčových kompetencií v matematickom vzdelávaní*. Univerzita Pavla Jozefa Šafárika v Košiciach.

[5]. OECD. (2009). *PISA 2009 Assessment Framework: Key Competencies in Reading, Mathematics and Science*. OECD Publishing.

[6]. OECD. (2003). *The PISA 2003 Assessment Framework. Mathematics, Reading, Science and Problem Solving Knowledge and Skills.* OECD Publishing.

[7]. Kmeťová, M. (2009). O zložkách geometrických kompetencií. *Acta Mathematica 12*, 139–143. Fakulta prírodných vied, Univerzita Konštantína Filozofa v Nitre.

[8]. Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study. *Developmental psychology*, *47*(6), 1539.

[9]. Jordan, N. C., et al. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850 – 867.

[10]. Nguyen, T., et al. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement?. *Early childhood research quarterly*, *36*, 550-560.

[11]. Vogt, F., et al. (2018). Learning through play – pedagogy and learning outcomes in early childhood mathematics. *European Early Childhood Education Research Journal, 26*(4), 589–603.

[12]. Säfström, A. I. (2018). Preschoolers exercising mathematical competencies. *Nordic Studies in Mathematics Education*, *23*(1), 5-27.

[13]. Chatzaki, M. A., et al. (2024). Exploring the potential of a game-based preschool assessment of mathematical competencies. *Frontiers in Education*,*9.*

[14]. Hirsh-Pasek, K., et al. (2009). *A mandate for playful learning in preschool: Presenting the evidence*. Oxford University Press.

[15]. Quispe, J., et al. (2024). Effectiveness of an educational program in the development of mathematical competencies in Peruvian elementary school students. *Journal of Educational and Social Research*, *14*(6), 118.

[16]. Condori, L. C. T., et al. (2024). Evaluation of the Impact of an Online Educational Platform on the Development of Mathematical Competencies in Elementary School Students. *Revista Iberoamericana de Tecnologías del Aprendizaje: IEEE-RITA*, *19*(1), 397-406.

[17]. Albaladejo, I. M. R., López, M. D. M. G., & Sánchez, A. C. (2015). Developing Mathematical Competencies in Secondary Students by Introducing Dynamic Geometry Systems in the Classroom. *Education and Science*, *40*(177), 43-58.

[18]. Brandsaeter, A., & Berge, R. L. (2025). Promoting mathematical competence development through programming activities. *Educational Studies in Mathematics*, *119*(2), 225-247.

[19]. Ruiz Loza, S., et al. (2022). Facilitating mathematical competencies development for undergraduate students during the pandemic through ad-hoc technological learning environments. *Frontiers in Education*, *7*.

[20]. Bímová, D., et al. (2023). Developing students' visuospatial abilities in geometry using various tangible and virtual 3D geometric models. *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*. Alfréd Rényi Institute of Mathematics; ERME.

[21]. Güler, H. & Arslan, C. (2019). Mathematical competencies required by mathematical literacy problems. *MOJES: Malaysian Online Journal of Educational Sciences, 7(2), 57-70.*

[22]. Niss, M. (2011). The Danish KOM project and possible consequences for teacher education. *Cuadernos de Investigación y Formación en Educación Matemática*, *6*(9), 13-24.

[23]. Qi, C., Liang, H., Zuo, S., & Li, R. (2024). Comparing competency-oriented student activities between expert and novice teachers in China: Insights from an epistemic network analysis (ENA). *Education and Information Technologies*, *29*(12), 15375-15402.

[24]. Národný inštitút vzdelávania a mládeže. (n.d.). *Testovanie 9 – školský rok 2023/2024*. NUCEM. Retrieved from: https://www2.nucem.sk/sk/merania/narodne-merania/testovanie-9/roky/2023-2024 [accessed: 02 February 2025]