Multi-Criteria Fuzzy Inference System for Objective Assessment in Higher Education Institutions

Boryana Uzunova-Dimitrova [1]

[1] *Faculty of Mathematics and Computer Science, Konstantin Preslavsky University of Shumen, 115 Universitetska str, Shumen, Bulgaria*

Abstract **- The accurate assessment of learners' acquired knowledge and skills is a crucial component of successful learning. Unbiased assessment tools can be used to model inference systems to support the decision-making process. The article presents a fuzzy inference system built from the FAHP model for knowledge assessment considering the factors influencing the preparation of learners.**

The development of an advanced methodology that will serve to create a multi-criteria system for choosing assessment technology in higher education institutions, with the ability to apply it to studies of various scopes, including the selection of learning technology, the evaluation of the effectiveness of the teaching method (online, hybrid, or face-to-face), the impact of the number of student visits, and the evaluation of social and economic factors. These factors affecting the objective evaluation of students, together with their sub-criteria, were used to determine their priority weights using the Fuzzy Analytical Process Method (FAHP).

In several cases, fuzzy logic can outperform linear, partially linear, or look-up table approaches in terms of control performance.

Keywords **- Fuzzy logic, fuzzy inference system, FAHP, decision making, criteria, Chang.**

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Corresponding author: Boryana Uzunova-Dimitrova,
Faculty of Mathematics and Computer Science, Konstantin Preslavsky University of Shumen, 115 Universitetska str, Shumen, Bulgaria
Email: b.uzunova@shu.bg

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1. Introduction

The modern educational process is global in nature; at present, there is not a single developed country in the world that has not tried to set itself the task of making changes in the education system and taking certain actions to increase its efficiency [1], [2]. Simultaneously with the modernization of the education system, the approaches to the system for assessing the knowledge of students are also being reviewed. Standardization and integration of efforts, especially in the field of university education, are the only chance to create competitive curricula and training programs in the world market in accordance with good world practices. Education also performs important social functions by transferring knowledge, skills, competencies, habits, attitudes, and values from one direction to another, as well as from generation to generation [3], [4].

A trend in recent years has been the lurch towards a remote form of learning and testing, caused by the past Covid wave of distancing. Optimistic expectations for increasing the efficiency of the educational process and the quality of education through the introduction of new, innovative forms based on modern information technologies are not always justified in practice [5], [6].

The study conducted is how the number of visits, the form of teaching, and the number of submitted assignments and projects affect the measured knowledge of the learners, who are the object of the study. The factor for the form of teaching focuses on the economic aspects of the investments made by the learners. There is a compromise between teaching and testing, namely the so-called hybrid learning [7].

The aim of the study is to develop a fuzzy model for selecting the optimal method for assessing the acquired knowledge, skills, and competencies through the use of effective and basic criteria, which are the objectives of the study. At the same time, the subjective judgments of the educators, who ultimately have the last word, are taken into account.

The proposed model is based on the fuzzy analytical hierarchical process (FAHP) [8], [9], which is used in determining the weights of the criteria by the teachers [10], [11], [12]. In 1996 [8] launched this new method for processing FAHP, using triangular fuzzy numbers for the pairwise comparison scale and their immediate analysis using the rank analysis method for synthetic values.

2. FAHP Methodology

Using the FAHP degree introduced by [8], [9], [13], [14], [15], put X = {x_1, x_2, x_3, ..., x_n} be an object set and G={g_1, g_2, g_3, ..., g_n} be a set of goals. Each object is taken, and extent analysis for each goal is performed according to Chang's degree analysis approach. Thus, for each object, the m-level values can be used for analysis:

$$M_{gi}^1, M_{gi}^2, \dots, M_{gi}^m, \quad i = 1, 2, \dots, n,$$

where M_{gi}^j $(j = 1, 2, \dots, m)$ are all TFN [10], [11]. The following steps can be considered in Chang's degree analysis [8]:

• *Step 1:* With respect to the relation of the i-th object, the value of fuzzy synthetic extent is defined as [8],[9], [13], [16]:

$$S_i = \sum_{j=1}^m M_{gi}^j \otimes \left[\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j\right]^{-1}, \quad (1)$$

For example, the fuzzy operation of adding m values for power analysis for a given matrix derived is $\sum_{j=1}^m M_{gi}^j$

$$\sum_{j=1}^m M_{gi}^j = \left(\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j\right), \quad (2)$$

results in $\left[\sum_{j=1}^n \sum_{j=1}^m M_{gi}^j\right]^{-1}$, which is the operation for fuzzy addition of M_{gi}^j $(j = 1, 2, \dots, m)$values, which is performed as follows:

$$\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j = \left(\sum_{i=1}^n l_i, \sum_{i=1}^n m_i, \sum_{i=1}^n u_i\right), \quad (3)$$

and then the vector inverse of the above is defined, such as:

$$\left[\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^n u_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n l_i}\right), \quad (4)$$

• *Step 2:* When $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as:

$$V(M_2 \geq M_1) = \sup_{y \geq x}[\mu_{M1}(x), \mu_{M2}(y)], \quad (5)$$

and can also be expressed as follows [13], [14], [15], [17]:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M2}(d) = \begin{cases} 1 & , if\ M_2 \geq M_1 \\ 0 & , if\ l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & , otherwise \end{cases}, \quad (6)$$

Equation (6) representing d (Figure 1) is the ordinate of the highest intersection point D between $\mu_{M1}(x)$and $\mu_{M2}(y)$is necessary to compare M_1 and M_2, which in turn requires the calculation of the values $V(M_1 \geq M_2)$and $V(M_2 \geq M_1)$.

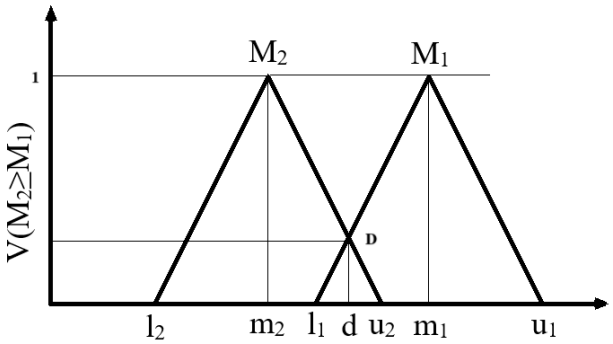

Figure 1. The point where M1 and M2 cross

• *Step 3:* The degree of probability that one convex fuzzy number exceeds k convex fuzzy number M_i (i=1, 2, ..., k) numbers can be determined by:

$$V(M \geq M_1, M_2, \dots, M_k) = V\begin{bmatrix} (M \geq M_1) \\ and\ (M \geq M_2) \\ and \dots and \\ (M \geq M_k) \end{bmatrix} = \min V(M \geq M_i), \ i = 1, 2, 3, \dots, k. \quad (7)$$

It is assumed that $d(A_i) = \min V(S_i \geq S_k)\ for\ k = 1, 2, \dots, n\ and\ k \neq i.$

Then the weight vector is given by:

$$W' = \left(d'(A_1), d'(A_2), \dots, d'(A_n)\right)^T, \quad (8)$$

where A_i (i=1, 2, …, n) are n elements.

• *Step 4:* Via normalization, the normalized weight vectors are:

$$W = \left(d(A_1), d(A_2), \dots, d(A_n)\right)^T, \quad (9)$$

where W is a non-fuzzy number.

Using the FAHP methodology, it becomes clear that the sustainable knowledge of the learners depends mainly on factors such as initial knowledge in the field, number of visits during the training, number of submitted works (practical tasks or projects) [13], [14], and last but not least, the form of training.

It is considered to be based on fuzzy multi-criteria factors when making a decision about the acquired knowledge, skills, and competencies of the learners, including the relationship between the form of training, the number of visits, and the satisfaction among the learners for predicting the final results of their training in the course [18].

For methods for assessing the sustainability of the acquired knowledge and competencies, it is related to attendance and the impact on the knowledge of the implementation of the tasks set. Based on the formative method and fuzzy logic principles, this study creates a model for evaluating critical competencies. The approach, known as FLFCA (Fuzzy Logic-Based Formative Competence Assessment), integrates quantitative and qualitative metrics to monitor students' development in a way that is more adaptable, long-lasting, and customized.

The main objectives of the study are:

(1) To develop a model for training learners and maintaining the sustainability of their knowledge using a fuzzy rule-based system.

(2) Integrating FLFCA with educational decision-making factors in learning, thereby improving the sustainable absorption of acquired knowledge [19].

(3) Recognizing factors related to learning to contribute to long-term sustainability using the proposed model.

(4) Applying assessment metrics with influencing factors to assess the acquired knowledge and skills by learners.

3. Research Methodology

The need to provide efficient and flexible assessment techniques that accurately represent students' actual knowledge, abilities, and competencies is emerging in the contemporary educational setting. The fluidity and complexity of the learning process, as well as the impact of outside social and economic forces, are frequently overlooked by current systems [1], [3]. The goal of the current study is to propose a methodological framework that integrates fuzzy logic concepts with evaluation criteria for the relevance, sustainability, and adaptability of the learnt material. For a more precise understanding of the variables affecting learning outcomes, the suggested strategy applies fuzzy logic in conjunction with quantitative and qualitative analytic techniques. [5], [12] The goal is to offer a flexible evaluation methodology that considers the unique qualities of each student, the subject matter being taught, and the impact of outside variables. The study's specific objectives are to:

- identify important aspects impacting knowledge acquisition;
- define suitable language variables and membership functions;
- develop fuzzy evaluation criteria;
- construct an objective and flexible decision-making system.

The data sources, analytical techniques, and suggested design of the assessment model are presented in the next sections. They provide a thorough approach built on the examination of current data, the identification of important educational components, and the development of a decision-making framework focused on the long-term viability of the learnt material. In this regard, the model for sustainable assessment that is being given is founded on a careful examination of empirical data and the creation of a set of guidelines that enable a more accurate and equitable evaluation of educational results.

3.1. Description of the Data Source- Description of the approach

The proposed model is based on an analysis of the data source collected from [20], [21], [22], [23], [24], [25] with information related to the factors of assessment of the acquired knowledge and skills of the learners. The assessment rule is the various listed attributes, such as the criteria used for assessing a course project, plagiarism check, re-checking of the acquired knowledge, answers in a test, application of new knowledge for discovery, reconfirmation, and improvement in the specific scientific field [1], [3], [4], [5], [6]. Understanding the taught material is crucial while assessing the acquired and mastered knowledge and skills by the learners for their lasting deepening in the specific subject or in the studied discipline. The assessment models used give the distribution of potential models related to the impact of the discipline on the overall sustainable improvement of the learner.

Standard quantitative metrics (test scores, number of tasks completed successfully) and qualitative indications (proactivity in applying new information, depth of comprehension) are both included in the evaluation guidelines. The study examines the effects of these characteristics together on the application and long-term retention of learnt information.

Every text message utilised as empirical data is pre-anonymized by substituting distinct codes for individual identification. Only authorized members of the study team have access to the data, which is kept on secure servers and sent over end-to-end encrypted channels.

The General Data Protection Regulation's (GDPR) obligations, as well as the educational institution's ethical standards, are met by the storage and processing practices [26].

3.2. Fuzzy Logic with the Factor-Criteria Approach for Sustainable Learner Assessment

Innovative assessment methods attract learners who are more curious to learn or, more precisely, to apply what is been learned in practice to succeed in realizing the different challenges they face through the acquired material. To promote awareness and knowledge of various materials, learners try to apply and use them in preparing and using their projects or course assignments. The need for knowledge in a specific discipline increases over time, which requires improvements in the quality and innovation of the prepared project and the implementation of the learning material. Uncertain information presented or used in an innovative way of assessing the knowledge of learners is imperative to understand strategies for improving the way of learning and to ensure more sustainable knowledge in the long term. Fuzzy logic provides a more flexible and adaptive paradigm for decision-making in evaluation methods, which is essential for dealing with the inherent uncertainties and subjective nature of the quantitative assessment being made [10], [11], [18].

Stakeholders can make well-informed decisions that help assessment methods develop sustainably. Fuzzy sets are assigned values based on the input determined by the membership function of each input factor in the learner's learning in a given discipline.

Fuzzy theory defines it as a limit of innovative assessment methods related to information that is a challenge to process the sustainability of acquired knowledge, skills, and competencies. Determining whether this theory depends on a specific limit is the main goal of fuzzy logic to process the influencing factors affecting all dimensions. Its numerical value is obtained from the interval (0, 1) [27]. This limit defines the many boundary states in which the fuzzy set operates, where the value depends on either 0 or 1. Figure 2 depicts the schematic representation of the proposed approach for the development of assessment methods.

The primary assessment criteria (number of visits, teaching style, task performance, and economic and social considerations) are first defined by the scheme and then converted via fuzzy functions.

These functions are converted into fuzzy sets using the "fuzzy logic analysis" module. A fuzzy rule-based system (IF–THEN) then processes the fuzzy sets to provide outputs that have been partially defuzzied. The uncertainty and complexity of each rule are then measured by fuzzy entropy values, which are then combined into a "real quantitative assessment" in the final rule-based evaluation. Lastly, a factor correlation analysis is performed on the findings to determine the key factors influencing knowledge sustainability.

An objective qualitative evaluation approach that establishes a set of criteria (number of visits, teaching style, task performances, economic and social aspects) is the first step in the system shown in Figure 2. Following their transformation by fuzzy functions, these input variables are sent into a fuzzy analysis and rule basis, from which fuzzy sets are subsequently derived. Ultimately, a rule-based and intuitive evaluation is made using the fuzzy entropy values, which results in a true quantitative evaluation and correlation study of the variables.

One advantage of this method is its flexibility in modelling both quantitative and qualitative data, which makes it possible to deal with estimations that are ill-defined, ambiguous, or subjective. Experts can easily justify weight definitions and gain transparency through the usage of delimited rules and FAHP. Fuzzy entropy is also included to guarantee that uncertainty is considered while making decisions. However, the definition of triangular functions and rules is prone to human bias and necessitates extensive expert labour. Furthermore, real-time implementation may be hampered by the computational complexity and requirement for parameter calibration that arise with an increase in the number of input variables and rules.

Thematic grouping of rules (such as "visits-tasks" and "economic-social factors") can also be suggested for a more seamless transition between modules, which makes maintenance easier. Manual tuning can be minimized by using evolutionary algorithms or other optimization approaches to automate the calibration of triangle functions and FAHP weights. The inclusion of an interactive panel will assist in rapidly identifying "bottlenecks", boost user trust through text message encoding, and show in real time how changes in input data, rules, and entropy impact the outcome.

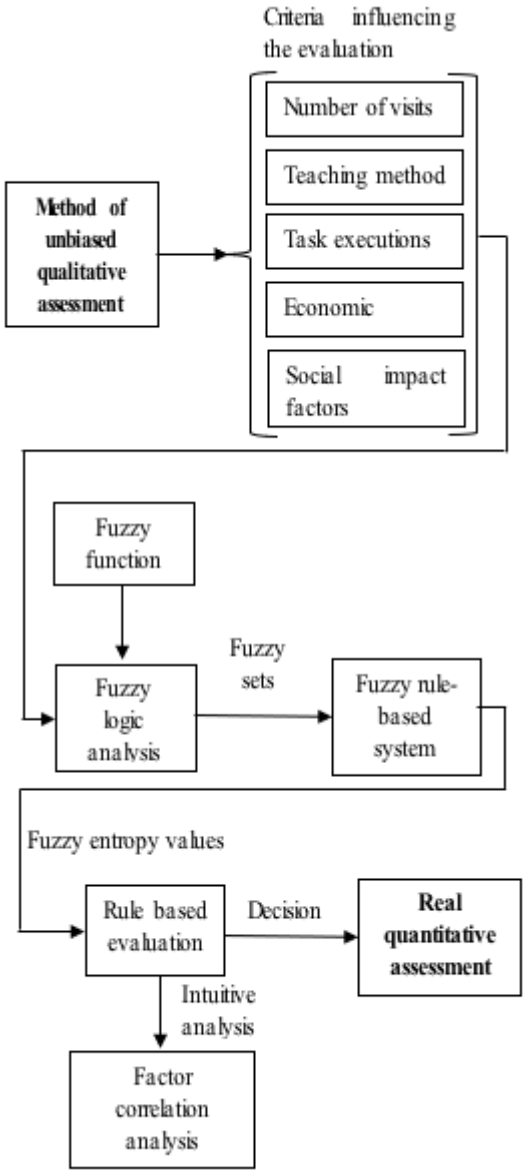

Figure 2. The overall architecture of the proposed FLFCA paradigm

3.3. Identification of the categories of the factor-educational approach

Fuzzy logic applies multidimensional factors to analyze the paradigm of assessing acquired knowledge, encompassing aspects of prior acquired knowledge and social and economic factors, to offer a numerical assessment of the sustainable acquisition of the taught material, as well as its implementation in different disciplines. Attributes or indicators with each dimension are evaluated with its multiple assessment factors.

The impact on the learners' previously acquired knowledge and its implementation in the newly studied discipline includes {weak impact, moderate impact, and strong impact}, which can capture the degree of assimilation and the correctness of a previous final grade.

Sustainable assimilation of the studied material includes {low, medium, high} net use of previous basic knowledge, use of the taught knowledge, skills, and competencies in the current discipline, and the implementation of the acquired knowledge in the implementation of course projects [3], [10]. The equations (10)-(12) given below represent the assimilation, where 100 represents the maximum allowable low impact.

$$\mu Low(x) = trai(0, 0.100), \quad (10)$$
$$\mu Moderate(x) = trai(0, 50,100), \quad (11)$$
$$\mu Hig\ h(x) = trai(50,100,100), \quad (12)$$

Fuzzy logic systems can use these membership functions to mimic the vague nature of assessments of the impact on prior background knowledge required to acquire new knowledge in a given subject area. Fuzzy logic can be useful in making decisions about the implementation of old knowledge. The meaning of prior knowledge includes background and non-background knowledge. Linguistic variables include "poorly acquired," "averagely acquired," and "well acquired," which can be used to assess the status of the factors of the CHI (Competence in Human Intelligence). The accessibility of the assessment method requirements helps learners navigate by providing precise and unambiguous information about their place on the assessment scale. The corresponding linguistic accessibility conditions are represented as {poor, average, good}. The application of fuzzy logic operators, such as AND and OR, is used to combine multiple fuzzy sets in rule-based systems.

3.4. Fuzzy Function Values

The values of the fuzzy functions for the different categories and linguistic terms for each linguistic variable are identical. Fuzzy AND (minimal operator) with A_1 and A_2 are two fuzzy sets together with Fuzzy OR (maximal operator); the fuzzy AND operator is denoted by $A_1 \cap A_2$, and the OR operator $A_1 \cup A_2$ is given as shown in Equations (13)-(14).

$$\mu A_1 \cap A_2\,(x(in\ 1)) = min\{\mu A_1\,(x), \mu A_2\,(x)\}, \quad (13)$$
$$\mu A_1 \cup A_2\,(x(in\ 2)) = max\{\mu A_1\,(x), \mu A_2\,(x)\}, \quad (14)$$

For the CHI metric, fuzzy rules can be generated as fuzzy as presented in Equations (15)-(16).

$$\mu poor \cap \text{good}\ (x) = min\ \{\mu\text{poor}(CHI), \mu\text{good}(accessibility)\}, \quad (15)$$
$$\mu well \cup \text{poor}(x) = max\{\mu\text{well}(CHI), \mu\text{poor}(accessibility)\}, \quad (16)$$

are then used in the fuzzy inference system to determine the final fuzzy output, which is subsequently "defuzzied" to obtain a clear value for the coefficient of sustainable knowledge.

The factor of knowledge considered for CHI is dependent on the number of visits C1 (0.4), the form of teaching C2 (0.3), the number of course assignments submitted C3 (0.2), and autonomy in the development of projects C4 (0.1) [8], [12]. Each factor score value should be evaluated on a scale from 0 to 1, with 1 being the highest level, and the weights defined above reflect the relative importance of each factor. The evaluation coefficient for these evaluation variables includes (0.8, 0.7, 0.9, and 0.6). CHI can be calculated as the weighted sum of the individual weighted score values of each factor considered out of the total number defined in equation (17):

$$CHI = \sum_{i=1}^{n} W_i * v_i, \quad (17)$$

The economic factors considered in the assessment method include {low, moderate, high} related to the form of attendance, having options such as {weak, moderate, strong}. Key factors of economic factors include household expenses. The social elements of education, including health, family, or language (communication) issues, are an important component of the social dimension of the education place visited.

The potential long-term effects of assessing newly acquired knowledge and skills can be assessed using fuzzy logic principles. The language variables of the long-term impact outcome metric are represented as "sustainable," "moderate," and "unsustainable" and can capture the degree of different long-term impacts. The sustainability dimension with language preferences is labeled as 'insufficient, balanced, stable'.

3.5. Fuzzy Rule-Based System

It considers a fuzzy rule base that associates various linguistic variables and their effects on input and output variables with the expected robust assessment. Each rule in the rule base defines a relationship between linguistic variables. The assessment method can benefit from a rule-based system because it offers a rational and organized framework for administration and decision-making. It combines aggregate fuzzy sets using fuzzy minimum and maximum operators to derive the final output of the influencing factor. Fuzzy entropy is a measure that is used to quantify the levels of complexity and uncertainty that are present in sustainability and acquired knowledge research. It supports decision-making in unbiased assessment by assessing the fuzzy qualities of language words, validating models, comparing scenarios, etc. Since the assessment of acquired knowledge, skills, and competencies has many dimensions, fuzzy entropy addresses uncertainties in the field, strengthening analytical robustness.

$$I = \frac{\sum_{i=1}^{n}(\mu A_1 \cap \mu A_2 \cap \ldots \cap \mu A_n) \times B_i}{\sum_{i=1}^{n} \mu A_i}, \quad (18)$$

In equation (18), the variables A_1, A_2, ... are fuzzy sets, and B_i is a fuzzy set representing the result of an influencing factor. A measure of uncertainty or instability of the learned knowledge within the fuzzy set is called fuzzy entropy. When balancing the exploration of different possibilities or linguistic concepts, maximizing fuzzy entropy can be useful. Based on fuzzy entropy, assign weights to the rules of fuzzy logic, give greater weight to rules with higher uncertainty.

Determining the fuzzy entropy of each rule for each of the input knowledge variables is a factor, which shows that rules with higher entropy have a greater impact on decision-making. Maximizing fuzzy entropy gives the ideal fuzzy values for thresholds described in terms of the fuzzy sets discussed above.

$$I = \frac{\sum_{i=1}^{n}(\mu A_1 \cap \mu A_2 \cap \ldots \cap \mu A_n) \times B_i}{\sum_{i=1}^{n} \mu A_i} \quad , \quad (19)$$

Here, equation (10) is used to maintain persistent knowledge; (19) is the membership function of a fuzzy set, and the lower as well as the upper extreme points of the influencing factors are represented in triangular membership functions with the specific linguistic terms defined in the interval [0,1] and are represented by equation (20).

$$\mu_{A_i} = \max\left(0, \min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right)\right) \quad , \quad (20)$$

The term gives the input variable associated with the available assessment method and parameters that define the lower and upper bounds of the triangular membership function. The evaluation of fuzzy models is supported by fuzzy entropy using linguistic ambiguity, which can assess the adaptability of the system to a set of settings for the unbiased assessment method of acquired knowledge. This helps to ensure the reliability of the decision support model. The robustness performance score (S) of the unbiased assessment can be given as the decision factors collected from all input factors with the appropriate linguistic preferences of the knowledge assessment method used, which is given by $\Delta k = 1$, where $\Delta k < 1$ indicates that the factor belongs to all individual decision factors of the unbiased assessment, to determine the overall robustness score of all factors related to acquired knowledge, skills, and competencies in the given learning domain, as given in equation (21). [18], [27].

$$\Delta k(S) = \frac{\sum f \in F \Delta_k^f}{F}, \forall k \in K, \quad (21)$$

3.6. Result of Fuzzy Rule-Based Evaluation

Applying the generated fuzzy rules and following the associated impact factors of each input variable, then taking into account the linguistic values assigned to each variable and the estimated corresponding degree of membership of each identified factor to which each rule is activated based on the input values. For the evaluation result, the changes in adaptability using FL (Fuzzy Logic) include the inclusion of linguistic variables, fuzzy sets, membership functions, and fuzzy rules.

Rule 1: If the number of visits is well maintained AND the number of submitted works is high AND the teaching format is present, i.e., strong, then the sustainability of the acquired knowledge is stable.

Rule 2: If the number of visits is moderate AND the number of submitted works is high AND the teaching format is online, i.e., weak, then the sustainability of the acquired knowledge is balanced.

Rule 3: If the number of visits is low AND the number of submitted works is low AND the teaching format is online, i.e., weak, then the sustainability of the acquired knowledge is insufficient.

Rule 4: If the number of visits is well maintained AND the number of submitted works is high AND the form of teaching is face-to-face, i.e., high, then the sustainability of the acquired knowledge is stable.

Rule 5: If the number of visits is well-maintained AND the number of submitted works is high AND the teaching format is online, i.e., weak, then the sustainability of the acquired knowledge is relatively stable.

Rule 6: If the impact on the assessment method is medium AND the number of visits is moderately maintained, the number of submitted works is low AND the teaching format is online, i.e., moderate, then the sustainability of the acquired knowledge is balanced.

Rule 7: If the sustainability of the acquired knowledge is balanced, AND the number of submitted works is low AND the teaching format is online, i.e., weak, then the assessment method is unsustainable.

The influence factor (IF) number of visits is considered an input factor, and the corresponding output variable is the level of adaptability of the proposed paradigm. The membership function for both variables is demonstrated as low, medium, and high as linguistic variables with rules like $\mu IF - L(x)$ representing the membership function for a low influence factor $\mu IF - M(x)$, medium as $\mu IF - H(x)$ as a high factor. Similarly, the variable is denoted as $\mu AL - L(y)$, $\mu AL - M(y)$, and $\mu AL - H(y)$.

The submitted work is analyzed by the attributes assignments (D) and coursework/projects submitted (R). The degree to which each linguistic phrase is appropriate is determined using fuzzy rules, taking into account what is seen in the values of D and R.

The fuzzy minimum and maximum operator with the implication method for determining the fuzzy output for sustainable knowledge is achieved by applying fuzzy rules.

If it is L, then it is H.
If it is M, then it is M.
If it is H, then it is L.
If D is high (1) AND R is low, then AL (22) is low.
Aggregate the output:

According to these fuzzy rule-based systems, the variable represents the number of linguistic terms or fuzzy sets considered in the adaptability assessment. Denoting the degree of membership for the itch linguistic term, represented in equation (22).

$$AL = \frac{\sum_{i=1}^{n}(\mu_i \times crisp-output_i)}{\sum_{i=1}^{n}(\mu_i)}, \qquad (22)$$

The weighted average value of the clear values, taking into account the degree of membership in fuzzy sets for adaptation in the sustainability of the acquired knowledge, skills, and competencies, is reflected in the equation AL (22). By introducing variations in the input X_1, X_2, ..., X_n and observing the changes in the output. The partial derivative of the output Y – $\frac{\partial Y}{\partial X_i}$represents each input X_i. This can be calculated using the rules and the membership function Y of the fuzzy logic system X_i. In general, the relationship between the variables can be identified using a sensitivity analysis that uses linguistic variables with the most significant impact on adaptability, calculated using equations (23) and (24), depicted in Fig. 3 as a visualization of the sensitivity analysis.

$$Sensitivity\ Index\ for\ X_i = \frac{\left|\frac{\partial Y}{\partial X_i}\right|}{max_{allX_i}\left|\frac{\partial Y}{\partial X_i}\right|}\ , \qquad (23)$$

$$Overall\ Sensitivity = \frac{\sum SI}{n}, \qquad (24)$$

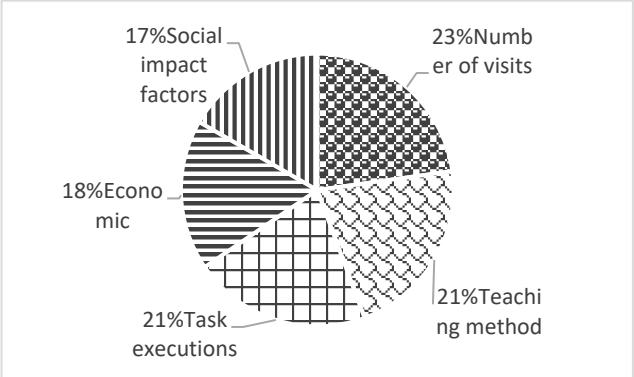

Figure 3. Global sensitivity analysis: Identifying influential factors in the valuation method

The respective proportions of the five major determinants of the evaluation (taking into account the CHI coefficient or persistence of knowledge) are shown in Figure 3:

23 % The quantity of visitors:

The frequency of visits receives the highest portion (approx. one quarter of the total). This indicates that the model's strongest signal is the quantity of in-person or online instruction.

21% Assignments:

This element demonstrates that active participation through practical tasks is equally crucial for the outcome and almost equals the significance of the instructional methodology.

21% Teaching method:

The assignments are given identical weight regardless of the instructor's method (online, hybrid, or in-person), which highlights the importance of the learning material's presentation.

18% Economic factors:

Although it accounts for a lesser percentage of the total score, the economic component (such as accessibility and educational expenses) shows that the resource component is also quite important.

17% Social impact factors:

The fact that social factors – such as family dynamics, communication problems, and health challenges – are given the least weight while remaining relatively close to economic factors indicates that the model also considers the socio-emotional backdrop.

The following are some of the main conclusions:

1. **Priority on engagement**: The evaluation is dominated by attendance and assignment completion (44% combined), indicating a substantial correlation between sustained learning and active involvement.
2. **Equivalence of method and assignments**: The teaching format and practical tasks have equal percentages (21% each), emphasizing the need for a well-rounded "theory + practice" approach.
3. **Complex model**: Although social and economic elements (35% combined) have less of an impact, they should not be disregarded because they provide the foundation for learning opportunities and incentives.

The notion of a thorough, multi-criteria evaluation is supported by this diagram, which integrates quantitative (attendance, tasks) and qualitative (methodology, economics, sociality) elements to provide a more objective picture of the learning process.

Sensitivity analysis is used to assess how changes in input parameters affect the model output, revealing information about the robustness of the model and the relative importance of different aspects.

When using global sensitivity analysis, the entire range of input uncertain parameter values is taken into account while assessing the impact of different parameter values and their relationship to each other on the model output indicators, such as adaptability level, CHI, resilience index coefficient, and long-term impact plan of the assessed result, providing decision-makers with information about an important factor, stable knowledge, influencing the implementation of the assigned course projects related to the acquisition of the taught material and the assignment of a final unbiased assessment. For each academic discipline, the specified ranges or distributions are in force, which best covers all possible variations in the choice of method for assessing the acquired knowledge, skills, and competencies. The findings of the global sensitivity analysis shed light on the relative importance of the optimal assessment method in determining the results of the acquired knowledge.

The methodology described here provides a methodical and transparent way to create and apply a fuzzy logic-based model for sustainable knowledge evaluation. By establishing pertinent variables, developing a set of linguistic variables, and developing fuzzy rules, a system that enables a more adaptable and flexible evaluation of educational outcomes was developed. Its foundation is the need for a more equitable and impartial assessment of students' level of knowledge acquisition and application. The results of the application of the model are presented in the article, along with an assessment of its efficiency, the extent to which it corresponds to the actual data, and possible directions for further development and improvement.

4. Conclusion

The proposed FLFCA paradigm applies fuzzy logic concepts to create a comprehensive paradigm for assessing the optimal method for assessing the acquired knowledge, skills, and competencies of learners. It analyzes the effects of multiple elements, such as stable and long-lasting knowledge, participation independently or in a team in the development of various projects, the implementation of which requires the use, and more precisely the application, of the acquired knowledge through the use of vague rules and linguistic variables.

Taking into account elements including teaching location, number of visits, learner status, and participation in mandatory or recommended projects, CHI is presented as a measure. The study applies fuzzy entropy to decision-making and uncertainty assessment and uses fuzzy rule-based systems to measure improvements in the adaptation of resilience indicators.

Sensitivity analysis is used to determine which factors have the greatest effect on the level of adaptability.

The paradigm consists of a methodological flow that integrates sensitivity analysis, fuzzy logic estimation, and data analysis using entropy to identify the relationship between variables. Sustainability is essential in guiding choices to expand long-term acquired knowledge related to the curriculum and assessment method. In the complex and ever-changing world of learning and assessment development, the proposed paradigm seeks to support sustainable practices by supporting well-informed decision-making with the improved outcome of developing sustainable knowledge compared to existing schemes.

The current task of designing a fuzzy interpretation of the results of student assessment is a component of artificial intelligence that is widely used today. The presented formulation of fuzzy logic can significantly simplify the work of specialists in the field of pedagogical diagnostics.

A wide range of professionals, both in the educational process and in practice, advocate the fuzzy logic technique. The objectification of assessment and the exclusion of randomness in the interpretation of the results of the assessment of student knowledge would be facilitated by the implementation of such systems in the practice of educational institutions for checking the quality of student training.

However, the ability to integrate modern fuzzy control methods is becoming increasingly feasible with the advancement of programmable controllers and their processing power.

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