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Devising VIMATE Framework to Assess the Impact of Visualization Tools on Geometric Reasoning and Problem-Solving Skills

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Abstract **– Traditional instructional approaches often fail to engage students and solidify conceptual understanding. This study proposes the VIMATE (Visualization Impact on Mathematical Transformations Assessment in Education) framework and introduces a novel empirical approach for evaluating the role of visualization tools in enhancing student comprehension of geometric transformations. In order to conduct this analysis, an original dataset was obtained from 300 high school students. The VIMATE framework involves several components of the research and is complemented by a literature review and meta-analysis of selected mathematical software platforms, such as MATLAB, Mathematica and GeoGebra, and quasi-experimental student assessments, as well as, questionnaires capturing the teachers' and learners' opinions. The overall aim of this approach is to fill major gaps in the prior research by offering an integrated methodology to objectively evaluate these visualization instruments on student achievement, reasoning abilities, and problem-solving strategies. The research makes four pivotal contributions by presenting the VIMATE framework as the methodological approach for assessing the use of educational technology in teaching practice.**

Keywords **– Transformational geometry, visualization in mathematics education, visualization tools, educational technology, problem-solving skills.**

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1. Introduction

This field of mathematics education is almost always developing to address the evolving need of creating a student populace that can understand the abstract and cognitively intense subject being taught. The field of geometric transformations which has been given basic definitions such as rotation, dilation and reflection, and translation has been found to be very difficult. Thus, the current study introduces the novel VIMATE (Visualization Impact on Mathematical Transformations Assessment in Education) framework to fill these essential gaps. The VIMATE paradigm is multidimensional, theoretical, and research-based by integrating diverse lines of enquiry to provide a rich account about how the instructional functions of visualizations can enhance learners' appreciation and learning of geometric transformations.

The VIMATE framework's objectives are multifaceted and are the following: (1) Construct a replication-based method for evaluating the impact of visualization on students' performance in problem-solving and achievement in geometry related contexts; (2) Define the strengths and weaknesses of the visualization tools under consideration in responding to different educational needs and, (3) Provide practical recommendations for enhancing the use of visualization methods for mathematics instruction.

2. Literature Review

Data visualization literacy has become an essential competency in the 21st century affecting different disciplines such as mathematics education.

This review considers how visualization can be used to enhance learners' mathematics comprehension and discusses tools and strategies for its efficient employment.

2.1. Conceptual Frameworks and Assessment

Discussion on identifying data visualization literacy and a conceptual framework was elaborated by [1]. Their work is concerned with the capacity to understand, analyze and design visualizations proficiently [1]. It describes exercises and tests as a vehicle to assess visualization literacy suggesting that the framework will be useful to teachers [1]. However, it must be said that while the study is presented as relevant to both business and mathematical visualizations, the emphasis of the study is made on general data visualization competencies, so the study may not encompass certain specifics of mathematical visualizations in their totality.

2.2. Mathematical Software and Visualization Integration

Some papers deal with incorporating visualization tools into the environment provided by mathematical software systems. In addition, [2] indicate the advantages of visualization functionalities of these programs in improving students' understanding. Authors [3] employed visualization in understanding of the concept of congruence as a facet of mathematics. From their studies, they postulate that visualization is the cross between concrete as well as the abstract context in mathematics [3]. However, both works fail to describe the contributions of certain features of software to learning results in more detail [2], [3].

2.3. Optimizing Data Visualization for Learning

In a comprehensive approach, [4] reviewed the literature on effectiveness of data visualization. They specify guidelines for making effective and meaningful graphics, focusing on simplicity, relevance and interactivity [4]. These principles can be useful when developing the diagrams in mathematics through planning for student learning. However, this study was conducted to establish general principles of data visualization and may not address some of the peculiarities of graphing mathematical concepts [4].

2.4. Case Studies in Visualization Tools

Some examples of how the particular instruments of visualizations are used in learning mathematics are described in the following studies.

[5] present a case study of using computer visualization software in a general mathematics course. [6] focus on GeoGebra, a dynamic geometry software program, emphasizing its ability to combine modeling, visualization, and programming for effective STEM education [6]. In addition, [7] investigate the impact of integrating GeoGebra into pre-service mathematics teacher training, highlighting its potential to enhance visualization skill. Further, [8] demonstrate the use of GeoGebra's visualization environment for teaching linear algebra. These case studies provide valuable insights into the practical application of visualization tools in specific mathematical domains [5], [6], [7], [8]. However, a comparative analysis of different tools and their strengths and weaknesses is lacking across these studies.

2.5. Visualization in Advanced Mathematical Computing

A final group of studies explores visualization in advanced mathematical computing environments. In [9] authors introduce MatSurv, a MATLAB toolbox for survival analysis visualization. Around the same time [10] provides a general overview of the mathematical modeling system MATLAB, highlighting its visualization capabilities [10]. The authors [11] showcase the use of SolidWorks, MATLAB/Simulink, and Unity for simulating and visualizing dynamic systems in virtual reality. These above mentioned studies demonstrate the power of visualization tools in advanced mathematical computing for research and potentially for upper-level mathematics courses. However, the focus on specialized software may limit the generalizability of these findings to broader mathematics education contexts [9], [10], [11].

3. Research Methodology

This study utilizes a multifaceted research methodology to comprehensively assess the influence of visualization aids on the instruction and acquisition of geometric transformations. The VIMATE framework was established using this methodology and includes the following essential components:

1. A comprehensive literature review was carried out to examine current research on the function of visualization in mathematics education and assessment techniques, software platforms, and case studies pertaining to the integration of visualization tools. This review identified gaps in current knowledge and emphasized the need for a comprehensive framework to evaluate the effects of visualization tools on the achievement and reasoning of students.

2. Evaluation of Mathematical Software Comparatively: A thorough comparison of three well-known mathematical software platforms—MATLAB, Mathematica, and GeoGebra—has been conducted. Their visualization capabilities were evaluated, especially in terms of demonstrating geometric alterations. A usability examination looked at each platform's visual representation of changes, interactivity, and ease of use.

3. Quasi experimental Student Assessments: The quasi-experimental student assessments were conducted with a sample size of 300 students from three secondary schools in North Macedonia: A - High School Gymnasium Skopje, B - High School Cvetan Dimov Skopje, and C - Technical High School Gostivar. Studies that are almost entirely experiential and include student participants in geometric changes have been planned and carried out. Students with a range of socioeconomic and cultural backgrounds were chosen, and they were then randomly assigned to experimental and control groups. The experimental group received training enhanced by the use of visualization tools, such as GeoGebra, while the control group received conventional instruction. The students were randomly assigned to the experimental and control groups, guaranteeing that the pertinent factors (e.g., prior knowledge, mathematical ability, demographics) were distributed equally. Qualitative data sources were also incorporated, such as student interviews and classroom observations that provided richer insights into the learning experience with visualization tools. Pre and post assessments tests measured students understanding, reasoning abilities and problem solving skills related to geometric transformations.

4. Surveys of Teachers and Students: In order to gather teacher and student perspectives on the use of visualization aids in teaching and learning geometric transformations, comprehensive surveys have been developed. Data on the precise and overall effectiveness of the visualization tools, their consistent ease of use and clarity of explanations, potential challenges, and characteristics that facilitate successful integration have all been gathered.

5. Synthesis and Analysis of Data: With the use of suitable statistical techniques, both quantitative and qualitative data from student assessments and teacher/student surveys have been analyzed. The differences in student achievement and reasoning skills between the experimental and control groups have been determined to be significant. The results obtained from the literature review, comparative analysis, student assessments, and surveys have been combined to create a comprehensive understanding of visualization.

6. Hypothesis H1: The use of electronic visualization tools such as GeoGebra in teaching geometric transformations will significantly improve students' conceptual understanding, reasoning abilities and problem-solving skills when compared to traditional instructional methods. Furthermore, the incorporation of these tools improves the standard of instruction and makes it easier to explain and demonstrate abstract geometric concepts. This research attempts to establish a strong methodology for assessing the impact of visualization tools on student achievement and reasoning proficiency through the use of the VIMATE framework comprehensive approach. The results offer practical guidance on how to best utilize visualization technology in mathematics education, thereby promoting higher order reasoning and problem-solving skills in students.

4. Results and Insights

The results of the comparison of Matlab, Mathematica and GeoGebra platforms focusing on the case study – geometric transformations are provided below. In general, in terms of usability, the tools offer the possibility to perform geometric transformations such as: translation, rotation and scaling. In the illustrated examples for Matlab, the original figure and the transformed figure are always presented through the full function. In Mathematica such thing could be accomplished if the Show function is used, as illustrated in Figure 1. The following commands to test such layout can be used:

```
1  x = Graphics[{Green,Disk[]}]
2  y = Graphics[{Pink, Scale[Disk[], {1, 2}]}]
3  show[x,y]
```

Line 1 creates the unit circle x in green, while line 2 creates the scaled figure 2 times y in pink. Line 3 performs the graphical display on the screen.

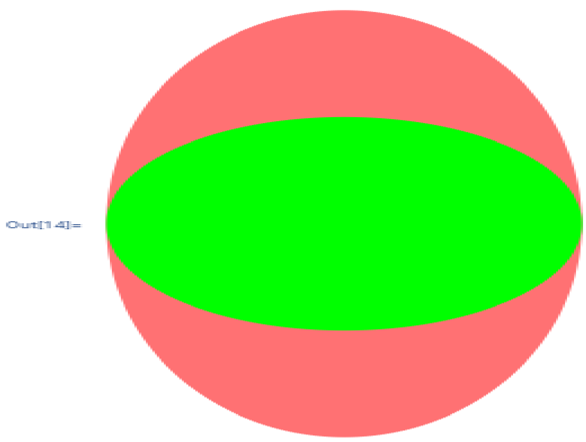

Figure 1. Placing the original and transformed figure in Mathematica

On the other hand, in the examples of Mathematica, the transformations in 3D figures are ilustrated through the Graphic3D command. In Matlab, to realize this functionality, a specific command affine3d [26] is used, which is more difficult to implement than the Graphic3D command of Mathematica. As for translation in Mathematica, it is possible to perform multiple translations with the same command. In Example 2, if the circle for 1 in the x direction does not need to be translocated, but also the same circle for 3 in x and 2 in y, then the following command can be used:

Graphics[Translate[Circle[], {{1,0},{3,2}}], Axes -> True]

In this case, another translocated figure as presented below is obtained. To achieve this effect in Matlab more than one command is needed. Specifically, one command to translate the image and another to display the graph.

In this study, the techniques that enable GeoGebra to solve tasks related to geometric transformations were also demonstrated.

Several examples of solutions to tasks from translations, reflections and rotations were demonstrated. It can be concluded that:

1. GeoGebra provides a powerful platform for solving tasks from geometric transformations with the tools it provides.
2. Interactive animations allow students to visually imagine the transformation of figures, be they translations, reflections or rotations.
3. Using the tools is quite intuitive and easy for students to use.

Finally, it can be concluded that any of beforehand offered - Matlab, Mathematica or GeoGebra can be used for teaching geometric transformations, depending on the teacher's preference and familiarity.

4.1. Discussions about Performance Results

Inferential Statistics was realized with an objective to:

- Report the results of paired t-tests to show within-group changes.
- Compare groups using independent t-tests and report significance.
- Present ANOVA results to highlight differences among visualization tools.
- Use ANCOVA to control for initial differences and report adjusted means.

Based on the results obtained and analyzed, from the results of the students' testing, it can be deducedthat the students managed to correctly answer over 60% of the questions from the topics of translation and axial symmetry, while they achieved lower skills (over 44 %) on the topic of rotation. This result is expected, based on the finding described in [10] where it is proven that students have low performance in the topic of transformations and especially in the topic of rotation. Also, looking at the results of the students' survey, based on the values in the quartiles, it cane be noticed that the students mostly answered with "agree" and "completely agree", while from the teachers' survey, in addition to these options, their answers also preferred the "neutral" option.

Further, from the students' test performance, it can be noted that the experimental group, i.e. the one to whom the transformations were clarified through GeoGebra, reached an average of 18.05 points (out of a maximum of 30), while the control one 13.06. To make this difference clearer, the correlation between the points obtained and the clarification with/without GeoGebra can be used. To prove whether this difference is visible i.e. significant, *the* test for Equality of Means was used in Table 2, and gives the values obtained from the Independent Sample Test. According to the rules for *the* independent t-test, the t-value is greater than the critical value. From the t-Table of Distribution, for df=139 there is($t.975 = 1.962$) < ($t = 2.897$). Also, according to the second rule P or Sig (2-tailed)=0.004 < 0.05 and the confidence interval does not include the value 0. As all three conditions of the test rules are met, it follows that the difference is obvious or significant. This also confirms the second research question: "Does the transfer of competences to students become faster in this way even in distance learning conditions?" Therefore, students who learned the transformations with GeoGebra achieved a significantly better result on the test than those who did not use it. From this result, as well as from the results of the students' survey, in which the questions about the use of GeoGebra were answered by a majority that completely agree or agree, it can be marked that the hypothesis is proven.

Table 1. Independent Sample Test for variables of research groups

		Levene's Test for Equality of Variance		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
points	a	0.45	0.50	2,897	139	0.004	4.981	1.719	1.581	8.38
	b			2,978	50.22	0.004	4.981	1.672	1.622	8.340

Legend: a-Equal variances assumed
b- Equal variances not assumed

Table 2. Independent tamples test for research group variables (backward learning)

		Levene's Test for Equality of Variance		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
points	a	3.146	.081	.369	63	.713	.637	1.726	-2.811	4.085
	b			.453	44.340	.652	.637	1.406	-2.195	3.469

More specifically, the use of electronic tools during the teaching of the geometric transformations unit increases the quality of learning in the field of geometric transformations.

It is worth mentioning an intresting detail,that a significant difference was not reached in the case of reverse learning, where the control groups reached an average score of 14.17 (experimental group) versus 13.53 (control group), as evidenced in Table 2.

To refer to the first research question "Does the use of electronic tools facilitate, clarify and better clarify the concepts of geometric transformations?", there are some indicators that confirm this claim.

According to Table 3, a list of arguments that support each of the clarification properties through GeoGebra is given. The results from surveys and testing, including quantitative and qualitative data, are listed across the rows.

Table 3. Analysis of ease, clarity and precision of explanations with GeoGebra

76% of teachers think that software applications for teaching mathematics help a lot in increasing students' skills.
88% agree that the learning objectives of geometric transformations are clearly expressed through GeoGebra.
70% think that the use of GeoGebra affects the integration of geometric transformations in school curricula
43% avoid teaching geometric transformations
41% of teachers think that GeoGebra does not need an instructor.
Significantly higher mean in the test of the experimental group than the control group
55% of students fully agree that clarifications are facilitated through the use of electronic tools and 53% fully agree that they increase the quality of teaching and learning
The visual aspect means the movements of the figures (animations) during the transformations
Carrying out or solving the task with demonstration,
Quick learning of the topic of transformations as well as clarity and precision of calculations

Since GeoGebra facilitates, clarifies and specifies the explanations of geometric transformations, then this means that the remaining half of the hypothesis of this study is confirmed, which was "the use of electronic tools also increases the quality of teaching geometric transformations".

4.2. Important Correlations

From the analysis of the significant correlations calculated from the teachers' survey variables, a correlation between work experience and teachers' avoidance of explaining geometric transformations has emerged. Specifically, as can be observed, the following statement resulted:

- Statement 1. Teachers with less work experience fully agree or agree with the claim that teachers avoid teaching the topic of transformations. Also, an interesting correlation that has been detected is the one between the increase of teachers' knowledge on the topic of transformations through the use of GeoGebra and the IT requirement of students' prior skills. Accordingly, the following statement can be derived:

- Statement 2. Teachers who agree that GeoGebra has increased their knowledge of geometric transformations also agree that students' prior IT skills are required to work with GeoGebra.

Table 4. Results from ANOVA and ANCOVA

Analysis	Source	df	F	p	η^2p
One-Way ANOVA	Group	1	32.28	<0.001	0.18
	Error	148			
ANCOVA	Pre-test Score	1	46.57	<0.001	0.24
	Group	1	27.65	<0.001	0.16
	Error	147			

- **ANOVA** compared post-intervention scores between experimental and control groups.
- **ANCOVA** controlled for baseline differences using pre-intervention scores.

Interpretation:

- **ANOVA:** Significant difference in post-intervention scores between groups, $F(1, 148) = 32.28$, $p < 0.001$, with a large effect size ($\eta^2p = 0.18$).
- **ANCOVA:** Significant group effect on post-intervention scores after controlling for pre-intervention scores, $F(1, 147) = 27.65$, $p < 0.001$, with a medium to large effect size ($\eta^2p = 0.16$). Pre-intervention scores were a significant covariate, $F(1, 147) = 46.57$, $p < 0.001$.

[Legend: df = degrees of freedom, F = F-statistic, p = p-value, η^2^p = partial eta squared (effect size)]

These results support the hypothesis that visualization tools positively impacted students' performance, even after accounting for initial knowledge and abilities. ANCOVA analysis strengthens the findings by controlling for potential confounding factors and baseline differences.

5. Conclusion

A novel empirical approach to assessing the role of visualization tools is provided by the VIMATE framework, which offers a strong methodological framework for assessing the effectiveness of visualization tools in improving students' comprehension of geometric transformations as well as their overall mathematical reasoning and problem-solving abilities. The findings from the original dataset, collected from 300 high school participants, indicate that visualization tools significantly improve students' performance compared to traditional instructional methods. GeoGebra, in particular, demonstrated the highest efficacy among the tools analyzed. The results of the comparative analysis and student assessments support the hypothesis, showing that the experimental group using visualization tools performed significantly better than the control group. Several important contributions were introduced by the VIMATE framework. These contributions advance theoretical understandings in addition to illuminating the practical ramifications. Moreover, there is a more thorough explanation of these small contributions:

5.1. Theoretical Contributions

Framework creation: The VIMATE framework bridges the divide between mathematical pedagogy and visualization technologies. By incorporating a systematic method to evaluate and compare the effectiveness of different visualization tools, the framework adds to the body of knowledge by providing an organized model that can be replicated and modified in a variety of educational contexts. The frameworks' applicability across many mathematical disciplines are expanded.

Improved comprehension of geometric transformations: The study advances a deeper understanding of how dynamic visualization tools can help with the comprehension of abstract geometric concepts through its empirical findings. The present findings are consistent with earlier theoretical frameworks that highlight the cognitive advantages of interactive learning environments in mathematics education.

5.2. Practical Contributions

Evaluation of Educational Tools: A noteworthy pragmatic contribution of this research is the comparative evaluation of widely used mathematical software tools, including GeoGebra, MATLAB, and Mathematica. The research offers significant perspectives that might assist educators in choosing appropriate technologies that complement their pedagogical goals and recommendations for Educational Practice.

5.3. Empirical Contributions

Evidence-Based Validation: The rigorous validation of the effectiveness of visualization tools in enhancing mathematical reasoning and problem-solving abilities is provided by the VIMATE framework's empirical approach. This supports the benefits of educational technology in mathematics and serves as a benchmark for future research.

Qualitative and Quantitative Understanding: Through the use of both qualitative and quantitative research methods, the study adds to the body of knowledge highlighting the subtle effects of visualization tools on student learning.

5.4. Policy and Educational Strategy Contributions

The results and the developed framework provide educational policymakers who are tasked with making decisions on the integration of technology into curricula, with a useful resource. The VIMATE framework's evidence-based design ensures that decisions are based on research findings.With its numerous contributions, the VIMATE framework can deeply influence theoretical constructs, educational strategies, and policy formulation.

5.5. Future Work

Expanding the application of the VIMATE framework beyond mathematics to other STEM (Science, Technology, Engineering, and Mathematics) disciplines could uncover the broader utility of visualization tools in enhancing problem-solving and reasoning skills across various fields. This cross-disciplinary approach could lead to the development of integrated educational technologies that support holistic STEM education.

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