

# Analysis of the Evolution of Mathematical Models for Estimating Life Cycle of Power Substations

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**Abstract** – The article analyses the evolution of mathematical models used in life cycle estimation. A comparison between the different mathematical models and their areas of applicability is proposed. This analysis is generally done in terms of costs. The importance of the article lies in the proposal of a new mathematical model for estimating life cycle of power substations, compatible with current and future requirements. The study findings reveal that an optimization model can be established using the life cycle assessment method to obtain a longer lifespan, but also to minimize costs (maintenance, repairs, replacements, etc.). In order to establish a mathematical model of the life cycle of electrical substations, it is necessary to study several important design parameters and decision factors for each stage of the life cycle. Even if the paper does not provide empirical results, it must prove that it is necessary to develop a multi-algorithm optimization framework to find an adequate solution. The life cycle must be modelled according to: equipment availability technological evolution, maintenance activities, risk management, environmental protection, human error, etc. The study helps further research to build, implement and validate a robust model with development possibilities depending on the needs of the researchers.

**Keywords** – Life cycle, mathematical models, power assets, reliability analysis, substations.

## 1. Introduction

The increased development of electrical networks, customer requirements, and technological innovations also require the transformation of electrical substations. The argument for the elaboration of this article is closely related to the place and role that power substations have in the evolution of power systems and their potential to support change. For electricity companies, the need to use existing electrical equipment, at optimal parameters, with a high safety factor associated with the assimilation of new technologies (green substation, standard modularized, easy to deploy, with inert gases instead of SF<sub>6</sub> for insulation, the integration of renewable sources from long distances, digitization and advanced solutions for equipment testing, self-diagnosis of defects and self-healing, managed by artificial intelligence, reactive power compensation).

The evaluation of the life of power substations considers many aspects, from the economic ones to the environmental ones. Hence, when contemplating a mathematical model for estimating the life cycle of electrical equipment, it is imperative to encompass all stages in which each piece of equipment is engaged. This includes design, operation, maintenance, repair, support, and disposal phases [1], [2], [3]. Since the twentieth century, researchers have been concerned with estimating the remaining life of power substations. The main accent was on the reliability of the system, on the performance of the system depending on the condition of the equipment, on a better management of costs throughout the life cycle of the studied element [5], [8], [9]. Since 2000, the focus has been on the deterioration of electrical equipment in substations and their associated costs. According to studies [4], [5] it was considered that 80% of the decisions made regarding the extension of the life cycle are in the first 20% of the life of substation.

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The extension of the life of substations is influenced by the choice of long-term strategies [2]. It is mainly affected by maintenance costs but also by environmental issues. Therefore, most studies use the cost of life cycle as a mathematical model to increase the life of power substations.

Most studies use the mathematical model of life cycle cost for [1]: the identification of some typical substation projects; the comparative analysis of the types of substation projects and the evaluation of the advantages and disadvantages of each one; the evaluation of the cost elements that influence the total cost; identifying, evaluating and comparing different approaches for replacement, maintenance or disposal of old components; optimal allocation of investments for equipment development/improvement activities.

In order to categorize key publications, databases were explored; a significant number of journal articles and conference papers were identified and analyzed. Queries, metadata, titles, abstracts, and keywords were used. With an emphasis on a better understanding of the most effective LCC modeling strategies, different notions, benefits, and drawbacks are investigated along with an analysis of the most used applications and areas.

This article follows several main steps, and they are grouped into the following categories: (1) an updated presentation of the LCC model (taking into account the evolution over time for the main application areas identified); (2) an overview of LCC model approaches used by transport and distribution operators in the USA, Canada, the UK, the Netherlands; (3) a presentation of the key challenges resulting from the application of LCC in the field of power substations in Romania and of some aspects that must be properly addressed: environment, human errors, etc.; (4) an overview of some future trends and applications found in mathematical models.

## 2. Mathematical Modelling of the Life Cycle of Power Substations

Mathematical modeling finds applications in various fields, serving the purpose of describing phenomena, gaining insights into system operations, or making predictions regarding asset behavior [5], [6], [7]. This article follows the mathematical modeling of the life cycle of a substation to make predictions about the behavior of some assets [8]. The main aspects that must be taken into account in the creation of a mathematical model are summarized in Figure 1. In order to build a mathematical model of the life cycle of a power substation, certain factors that influence the safe operation of the electrical installation must be taken into account [10].

The aim of the paper is to increase the life span of electrical substations. Figures 1 and 2 show the evolution over time of publications depending on the field of analysis and the type of analysis used. Figure 3 represents the LCC publications depending on the applied fields.

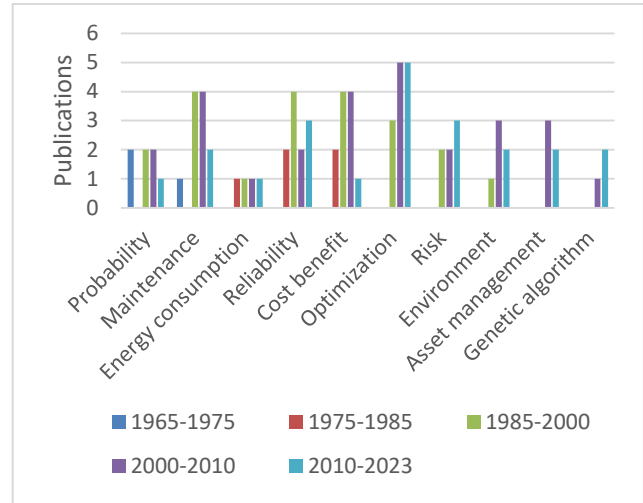


Figure 1. Evolution of publications depending on the field of analysis

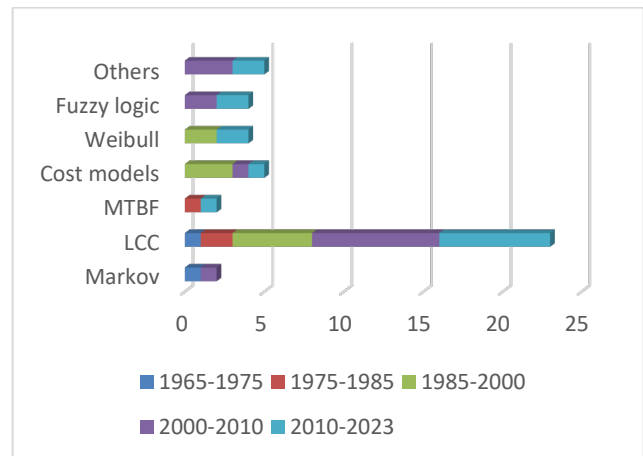


Figure 2. Evolution of publications depending on the type of analysis used

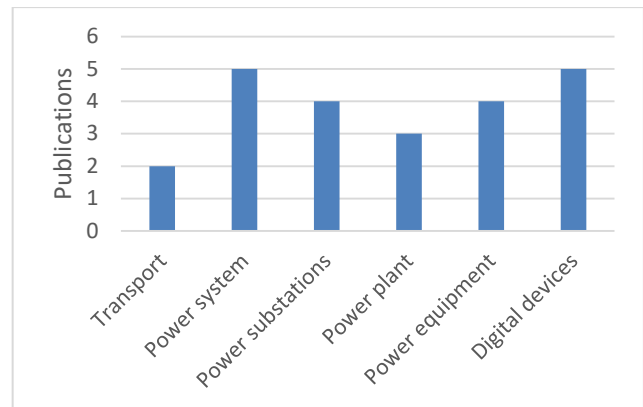


Figure 3. LCC publications depending on the applied fields

According to studies [8], [9], and [22], mathematical models are used in several forms depending on the purpose pursued. Regression models can be categorized as either linear or non-linear, and this choice is made to simplify complex static theories. They can be deterministic or probabilistic depending on the parameters used in the model and their variable states. They can be static or dynamic if differential equations are used. Also, they can be discrete or continuous if the introduction of the time variable in the mathematical model is desired or not.

### 3. State of the Art of Mathematical Models

Table 1 shows mathematical models used over time for life cycle estimation. The most representative models from various fields of activity were considered.

- Between 1965-1975 (3 papers), the mathematical models for equipment life cycle analysis were based on Markov techniques [10] and the life cycle cost model [11]. The first mathematical models used for this purpose were those from the marine and aviation fields.

- Between 1975-1985 (5 papers), reliability continued to be the main factor tracked in the life cycle analysis process [12], [13], but it was observed that as more investments were made in improving reliability, some cost elements increased. Therefore, the second factor tracked is the cost of the life cycle of the studied element [14]. During this period, a mathematical model of the life cycle cost is built that considers all the stages in which the studied element is during its life, from its design to its replacement.

- New approaches are considered in the period 1985-2000 (21 papers) in order to obtain as low a cost [14], [19], [22] as possible of the life cycle of the electrical substations and also to obtain a life span as high as possible. Among these are: the replacement of AIS substations with the GIS alternative, the use of a screening of all the equipment of an electrical substation to detect, reduce and even eliminate the occurrence of early failures; photovoltaic and offshore wind projects are also studied; several studies are carried out regarding environmental issues, both from the point of view of the development of new equipment and from the point of view of resource conservation and sustainability - based on these, cost-benefit analyzes are carried out. Reliability [20] continues to remain a main factor pursued in life cycle analyzes of power substations. Also in this period, the types of maintenance [17], [18], [23] are defined that strongly influence both the reliability of the electric power system and the cost of the life cycle.

- Since the 2000s, various mathematical models (27 papers) have been proposed: genetic algorithms [32], Markov chain [28], fuzzy Monte Carlo models [25], analytical analysis process method. Emphasis is also placed on the adequate evaluation of the state of the equipment to correctly determine the efficient operation, maintenance, and replacement strategies. There are many studies based on the optimal operation of transformers [30], [31], because as its average age increases, asset management becomes more and more important. Also during this period, an attempt is made to separate the fixed costs from the variable ones, in order to have a better coordination of the maintenance activities. Some studies focus on the impact of reducing SF6 emissions from high-voltage circuit breakers on the lifetime of power substations. They are trying to include the environmental cost in the total cost of a substation, which since 2000 has played an important role in the production of electricity [34], [35]. Some studies consider the life cycle cost (LCC) of renewable energy generation substations as the optimization objective to establish a complete optimization program for equipment maintenance.

- From 2010 until now (22 papers) the introduction of automation in substations is being tried to obtain a more flexible control and monitor the state of the electrical equipment, ensuring the reliability, efficiency, and quality of the delivered energy [40], [43], [47], [48], [49], [50]. Researchers are also trying to build some life cycle cost estimation software tools used to make comparisons between AIS and GIS variants, between conventional and smart substations [39]. The LCC continues to be a main tool in the process of increasing the lifetime of power substations [36], [38], [42], [45], [46]. During this period, efforts are made to optimize the life cycle cost by employing various mathematical models, including models for evaluating uncertainty. [33], [36] (the number of defects, the cost of operation, the economic losses caused by electricity interruptions; the Monte Carlo simulation method is used to solve this model), cost effectiveness evaluation model (gray correlation analysis for the selection of power transformers), genetic algorithms, Weibull distribution model [44] (used to estimate the equipment reliability function), dynamic programming [44] (the algorithm gives the minimum LCC; the numerical results demonstrate the effectiveness of the dynamic programming approach, compared to those using the genetic algorithm).

Table 1. A brief history of the most representative mathematical models used over time for life cycle estimation

Nr crt.	Model / Published	Mathematical expression	Study Purpose	Applied field
1	Markov Chain Techniques [10] – 1966	$\bar{F} [F_1(N), F_2(N), \dots, F_X(N)] = \bar{F} [F_1(0), F_2(0), \dots, F_X(0)] \times [F]^N$	A method of estimating the probabilities of availability and reliability for a missile system, to which the logistics part is added.	Naval - missile system.
2	Mcleod [11] – 1973	$\Delta I = f(A) - \Delta C_R - \Delta C_M$	The application of economic elements in the maintenance activities with the aim of minimizing the cost of the life cycle during the development period of the system/asset.	Engineering, social sciences, automatic data processing.
3	Forsythe [12] – 1980	$C_T = IC + EC \times C_E + OF$	Comparison model of the energy consumption of different traction systems, evaluated based on the life cycle cost concept.	Railway transport system.
4	MTBF (Mean Time Between Failures) [13] – 1983	$\theta_A = \theta_Z [1 + (\frac{\theta_G}{\theta_Z} - 1) (\frac{T}{T_G})^b]$	Reliability investment optimization (RIO) model based on the Duane model. This model is used to calculate the LCC.	Aviation.
5	Unit cost [14] – 1984	UNIT COST = DC + IC + FC FC = (RC + SPC) × t / MTBF	The study of the reliability of the design of an electric power system taking into account the failures that may occur in its components using MTBF.	Power energy.
6	Cost effectiveness model [14] – 1990	$E_K = \frac{\sum_{k=d+1}^K (P_j(k+1) \cdot \alpha_j(k+1) + C_{k+1})}{\sum_{k=d+1}^K C_k}$	Global system for evaluating the effectiveness of research and development costs that can be used in any field of activity.	Global.
7	Weibull distribution [16] – 1995	$R_{ci}(t t_{ci}) = p_{cssi}(t_{ci}) \cdot \left( \frac{\exp\left(-\left(\frac{t+t_{ci}}{\eta_{cssi}}\right)^{\beta_{cssi}}\right)}{\exp\left(-\left(\frac{t}{\eta_{cssi}}\right)^{\beta_{cssi}}\right)} \right) + p_{cghi}(t_{ci}) \cdot \left( \frac{\exp\left(-\left(\frac{t+t_{ci}}{\eta_{cgi}}\right)^{\beta_{cgi}}\right)}{\exp\left(-\left(\frac{t}{\eta_{cgi}}\right)^{\beta_{cgi}}\right)} \right)$	The environmental stress screening model is used to detect in time the occurrence of early defects and implicitly to eliminate them.	Electronic switching systems
8	True cost of cycling [17] – 1995	True cost of cycling $= C_M + C_R + C_P + C_C + C_G + C_I + C_D$	Evaluating the life cycle costs of the power plant by using appropriate maintenance strategies, dispatchable strategies with the aim of optimizing expenses.	Power plant.
9	Average annual cost [18] – 1995	$AAC_i = \frac{\alpha_j AC_j (1 + m_j) + IC + \sum_{j=1}^n [MC_j (1 + m_j)] (i + 1) + \sum_{j=1}^n C_{Mji}}{t_i}$	Reliability Based Design (RBD) system model for minimizing life cycle costs, increasing reliability and maintenance engineering. Genetic algorithms are used for cost optimization.	Power system.
10	Life Cycle Cost [19] – 1996	$LCC = CI + CP + CR + CO + CG + CF + CD$	The GIS reliability of the equipment is monitored during 20 years of operation. The advantages of GIS technology are presented through comparative analyses.	GIS power substations.
11	Weibull distribution [20] – 1996	$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right]$ $0 \leq x \leq \infty$	Probability-based reliability model for life cycle design. The model is used to calculate remanufacturing costs.	Power system.
12	LCC [21] – 1997	$LCC = CI + CO + CM + CF$	Applying the model for 2 different electrical schemes and comparing them from the point of	Power substation.

Nr crt.	Model / Published	Mathematical expression	Study Purpose	Applied field
			view of the extinguishing medium used.	
13	LCC [22] – 1999	$CT = C_{prod} + RC_{prod} + C_{reass} + C_{rp} + C_{disp} + B$	Effective model for the cost-benefit analysis of an equipment to which the recycling condition is added. Defining some indicators that contribute to determining the duration of operation.	Global.
14	Cost of operation and support [23] – 1999	$C_{O\&S} = C_{OP} + C_{LOSS} + C_{CM} + C_{PM} + C_{DIAG}$	Reliability centered maintenance program using stochastic Petri nets. The effectiveness, reliability and costs of the system are monitored.	Transportation systems.
15	Net present value [24] – 2000	$NPV_i = C_{0i} + ARC_i^d + NRC_i^d - SAV_i^d$	Algorithm based on life cycle cost evaluation using fuzzy set theory. Three major problems are proposed for the development of this algorithm.	Global.
16	Objective function – total cost [25] – 2003	Total cost = $IER_{fc}A_p + (IER_{dc} + IER_{du})A_e + LCC$	The model takes into account the needs of customers using SAIDI and SAIFI reliability indices, as well as the introduction of life cycle cost and investment costs for protection devices. The purpose of the model is to optimize costs to minimize the costs of non-delivery energy to customers.	Power system distribution.
17	Annual outage costs [26] – 2005	$C = \sum_{j \in J} \sum_{i \in I} \lambda_{zone} (a_i + b_i t_j) P_{ij} n_{ij}$	Model for reliability analysis in the network planning process. Alternative network solutions to which total life cycle costs apply. The maximum interruption load, network component failures, cost parameters and the power of each customer connected to the network are taken into account.	Electrical distribution network.
18	Fuzzy logic [27] – 2006	%Loss of life = $(F_{EQA} \times t \times 100) / \text{NormalInsul. Life}$ $F_{EQA} = \frac{\sum_{n=1}^N F_{AAn} \Delta t_n}{\sum_{n=1}^N \Delta t_n}$	It studies the remaining lifetime of the power transformer by fuzzy modeling. The introduction of appropriate asset management strategies is pursued, ageing factors are used.	Power transformers.
19	LCC [28] – 2006	$LCC = CI + (CF + CV) \left[ \frac{(1+i)^n - 1}{i \times (1+i)^n} \right]$ $PA = \frac{(1+i)^n - 1}{i \times (1+i)^n}$	Comparison of life cycle costs for different technological alternatives of two power substations. Technical-economic study for choosing the most optimal solution for replacing electrical equipment.	Power substations.
20	Deterministic model [29] – 2006	$n_{opt} = (k \frac{t_r}{t_i})^{1/2} - 1$	Model that takes into account the inspections that should be done within a power substation to minimize outage time.	Power substation equipments.
21	LCC Markov Chain [30] – 2006	$LCC = \int_0^t \sum_{i=1}^n (c_i(\tau) + \sum_{j=1}^n c_{ij}(\tau) \cdot p_{ij}) \cdot P_j(\tau) \cdot e^{-\alpha\tau} d\tau$	The life cycle is modeled using the Markov chain. The costs related to the process of damage to the electrical equipment are taken. The controlled energization of a power transformer is introduced.	Power transformers

Nr crt.	Model / Published	Mathematical expression	Study Purpose	Applied field
22	Total Owing Cost [31] – 2007	$TOC=BP+A \times NLL+B \times LL$	The purpose of the model is to highlight the importance of environmental aspects in making decisions regarding the replacement of a transformation unit. Factors are introduced that depend on the degree of loading, the specific characteristics of the network.	Power transformers.
23	Genetic algorithm LCC [32] – 2008	$C_{LC}(t) = C_A(t) + C_O(t) + C_R(t)$ $C_O(t) = C_M(t) + C_F(t)$ $C_F(t) = C_{CR}(t) + C_P(t)$ $C_{CR}(t) = \sum_{i=1}^n C_{Rep,i} * F_i(t)$ $C_P(t) = k_P * F_S(t) * t_{S,Rep}$	The use of a genetic algorithm to optimize the life cycle costs of the power substations. The aim is to determine the cost-sensitive components that could influence the decisions regarding the design of the substation.	Power plants
24	LCC [33] – 2009	$C = c_0 + \sum_{i=1}^m [c_1 p_0(t_i) + c_2]$	Model for optimizing the design of maintenance activities. The objective function is the life cycle cost, and the constraints are the system reliability.	Electromechanical systems.
25	Environmental cost [34] – 2009	$C_e = \sum_{i=1}^n (V_{ei} \cdot Q_i + C_{pi}) + C_{fe} \cdot d$	Environmental cost model for LCC of four types of power generation. The analytic hierarchy process (AHP) is used, which takes into account the amount of emissions of the studied pollutants and the charge for each.	Power plant.
26	Decision matrix LCC [36] - 2010	$TotalValue = \sum_{i=1}^c W_i V_i$	Determination of the optimal IEC61850 process architecture for a transformer bay using LCC. Decision matrix used to compare the proposed architectures.	Power transformer.
27	Fuzzy Optimal Power Flow Algorithm [37] – 2010	$\min f = \sum c_k \cdot P_{gk} + G \cdot \sum PNS_k$ $\sum P_{gk} + \sum PNS_k = \sum Pl_k^{ctr}$ $P_{gk}^{min} \leq P_{gk} \leq P_{gk}^{max}$ $PNS_k \leq Pl_k^{ctr}$ $P_b^{min} \leq \sum a_{bk} \cdot (P_{gk} + PNS_k - Pl_k^{ctr}) \leq P_b^{max}$	Modeling the uncertainties of transmission systems through the fuzzy algorithm, as well as the calculation of the risk indices used in the identification of investment solutions to improve the system.	Transmission systems.
28	LCC [38] – 2010	$LCC=FC+N(MC+SC+WC)$	Optimizing the cost of the life cycle of power generation equipment with the aim of optimizing asset management.	Power generation equipment
29	Life cycle input-output ratio [39] – 2012	$R = \frac{C_c + C_e + C_p + \sum_{lifecycle} C_o + C_r}{C_i}$	Comparison between the conventional substation and the smart substation through the life cycle cost. The application of sensitivity analysis to determine the components that affect the LCC of the smart substation. In the mathematical model, the increment of intelligent investment is also taken into account.	Smart substations.
30	Unavailability variable parcel [40] – 2014	$PVI = \frac{BP}{1440D} K_p \left( \sum_{i=1}^{NP} DVDP_i \right) + \frac{BP}{1440D} K_p \left( \sum_{i=1}^{NO} K_{O_i} DVOP_i \right)$	The optimal choice of a substation automation system for power substations, taking into account the minimization of costs and the increase of reliability by quantifying the number of interruptions and their duration.	Power substation automation.

Nr crt.	Model / Published	Mathematical expression	Study Purpose	Applied field
31	Security Efficiency Cost (SEC) [41] – 2016	$SEC = \sum_j (k_j \times SEC_j) \times f_s \times f_e$ $SEC_j = C_j / (E_4 \times \alpha)$	Uncertainty evaluation model that solves problems related to failures, operating costs. Entropy-weighted fuzzy comprehensive evaluation method is used.	Power substations.
32	LCC [42] – 2016	$LCC = CI + (CO + CM + CF) \times PA + \sum_{i \in M} C_i \times PF_{i-1} + \sum_{j \in M} CD_j \times PF_{j-1}$ $PF = \frac{1}{(1+\alpha)^n}; PF = \frac{(1+\alpha)^n - 1}{\alpha(1+\alpha)^n};$	Model used to select reliability-aware smart grid planning schemes. Case study for two planning schemes of a 110 kV substation.	Power systems
33	Reliability [43] – 2017	$E[n(t)] = N_0 - N(t) = \lambda \cdot t^\beta$	Development of different models to increase system reliability, as well as software development and incremental testing phase.	Integrated Defense Systems
34	Weibull distribution [44] – 2018	$\lambda(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1}$	Model used for scheduling maintenance with the aim of obtaining a minimum cost of the life cycle of the equipment. Dynamic programming is the one that ensures a minimum LCC and is more effective compared to the genetic algorithm.	Power substation equipments.
35	LCC [45] – 2019	$LCC = C1 + C2$ $C1 = CI + CJ + CL + CV$ $C2 = CO + CM + CF + CE + CD$	Model used to demonstrate the effectiveness of cost management in order to promote the modular power substation.	Modular substation.
36	LCC [46] – 2019	$LCC = W_1 + W_2 + W_3 + W_4$ $t_a = t_n - at_i$ $0 < a < 1$	The importance of secondary equipment within a substation is highlighted by the LCC model and the cuckoo algorithm for optimizing costs, especially maintenance costs. The model also offers the optimal revision time.	Secondary Equipment.
37	Building information modeling (BIM) [47] – 2019	$\sum_m \sum_c F U_c^m \times x_c^m + E U_c^m \times x_c^m$ $\sum_{\substack{c, \bar{c} \\ c \neq \bar{c} \\ m, \bar{m}}} I_{m,c,\bar{m},\bar{c}} \times x_c^m \times x_{\bar{c}}^{\bar{m}}$ $\sum_m \sum_c Q_{Heat,c}^m x_c^m + Q_{Cool,c}^m x_c^m + Q_{DHW,c}^m x_c^m$	Efficiency of energy consumption by modeling information about buildings and making decisions for durable and sustainable design. The quantity of heat in the heating and cooling modes caused by material used for each component of the building is taken into account.	Energy consumption in buildings.
38	Equipment state [48] – 2020	$Q(a, X^l) = \frac{1}{l} \sum_{i=1}^l  a(x) \neq y^*(x)  \cdot 100\%$	The model provides an adequate life cycle management for electrical equipment by using technical estimates regarding their condition.	High-Voltage Equipment
39	Quantitative benefit-cost analysis [49] – 2020	$C_T(t) = \sum_{i=1}^n (inv_i) + \left[ (1 + TCI)(1 + PCI) \sum_{i=1}^m \frac{CPM_i}{(1+r)^{iT_0}} \right] \left\{ [C_{REP}(t) + C_{PNL}(t)] \frac{(1+r)^{T_1} - 1}{r(1+r)^{T_1}} \right\} + \sum_{i=1}^n \frac{C_{R_i}}{(1+r)^{T_2}}$	Increasing the reliability of automation systems in power substations through cost-benefit analyzes and LCC evaluations according to IEC61850. It takes into account the number of elements of the process bus, the number of scheduled maintenance activities in the period established to reach the optimal solution.	Automation system.

#### 4. Using LCC Experience

Depending on the goal pursued, it was found that when studying the life span of an asset, one tries to narrow the research area to the most important

factors that contribute to the decrease of the remaining life span. In the paper [50], the main concerns of researchers from various countries of the world are presented.

Thus, the identification of failure modes, their causes and the application of diagnostic factors lead to the estimation of the time interval until failure.

Distribution network operators:

- In the UK, they start from an estimated lifetime of assets, and after identifying the current age, apply certain diagnostic indicators to find out the actual remaining lifetime of that asset [36], [15].
- In Romania, the development of optimization methodologies is being followed to establish strategies for replacing equipment, reducing power losses in the system. Also, the real-time monitoring of the parameters allows the improvement of the energy quality by identifying the necessary indicators related to the continuity of the user's supply.

Transmission operators:

- USA TSO [10], [11], [12], [13], [14], emphasizes the methodology and the choice of input data, finding an explanation for the use of a logarithmic or exponential method.
- Canada TSO [18], [27] have a strategic approach and provide a basis for prioritizing and justifying their investment strategies. They state that the deterioration of a component of an equipment should not be associated with age.
- Dutch TSO [15] expresses the condition in relation to the decommissioning rate or the remaining expected lifetime, makes a comparative analysis of reliability and uses availability analyses, makes a prediction regarding the failure modes and the development of maintenance strategies, the final goal being the elaboration appropriate investment plans.
- German TSO evaluates both the condition and the important factors for the realization of the maintenance program. The evaluation criteria are based on both the technical condition and the strategic condition that leads to an artificial aging of the installation. Through this method, it is possible to check the investments regarding which elements can be replaced or require maintenance.
- Romanian TSO use efficient analysis systems and applications for the continuous improvement of the system's operation, cost reduction, optimization of maintenance programs, increasing the lifetime of equipment, reducing the probability of incidents that lead to interruption of energy transfer. In recent years, their focus has been on the implementation of appropriate IT systems to ensure both the management of actions and maintenance costs as well as the determination of reliability indicators according to various criteria (age, specific local conditions).

## 5. Discussions

Analyzing the previously presented mathematical models, a significant number of approaches to mathematical modeling results, most of them being developed and improved for the optimization of the study regarding the life cycle of power substations. The majority is based on the economic component when it is desired to establish the lifetime of equipment, without taking into account technical aspects such as system stability, topology, and energy losses. They are also adapted to environmental and political requirements.

To develop a mathematical model, it is crucial to conduct a comparative analysis among different mathematical models. This analysis should consider the variables to be utilized and the availability of relevant data and parameters. In addition to these aspects, researchers must also take into account the simulation stages of the model, to have a broader vision of the current problems and needs.

To establish asset management strategies, it is necessary that the competing requirements of performance, cost and risk be balanced. This requires an accurate process and adequate experience to obtain a cost-effective and effective mathematical model for the lifetime of the assets. Thus, asset management systems must be built according to standards (PAS55 and ISO 55000). At the same time, maintenance, repair, or replacement costs must enter into a systematic risk and reliability assessment process.

Mathematical modeling optimization techniques to improve lifetime must use stochastic and probabilistic methods to satisfy the uncertainties related to the safety in operation of the system at the time of the occurrence of a fault.

Cost components are another issue that should be addressed. It is necessary to take into account the energy demand, ensuring the continuity of the supply of the users in relation to the investment cost requirements. And in this case, the use of optimization methods is the best solution.

Environmental requirements put pressure on most transmission and distribution companies to meet environmental policies. That's why, from the design process, carbon emission reduction studies must be carried out.

## 6. Conclusions

The mathematical models approached offer different perspectives on how to evaluate the life cycle of power substations.



With adequate maintenance and asset management according to the requirements, results can be obtained to increase the lifespan of a power substation. In the article, the studies highlighted the fact that in the analysis of the condition of the electrical equipment, in addition to the technical evaluation, there is also the economic component that has an essential role in decisions regarding repairs or even replacements. That is why the economic side must also be considered when the power substation planning project is carried out.

The work follows the evolution of mathematical models and their adaptability to changes over time. At the beginning, the focus was on ensuring the safe operation of the system by operating the electrical equipment using reliability models, Markov techniques, etc. Subsequently, efforts were made to seek solutions related to equipment technical condition assessment and environmental issues. Mathematical models were then developed to facilitate cost-benefit analyses. After that, genetic algorithms were used to obtain a clearer view of life cycle costs.

Approaching various methods for increasing the life of a power substation, from reducing SF6 emissions, the advantages of using GIS to improving maintenance programs, introducing automation systems, introducing more advanced monitoring systems, uncertainty assessment methods, programming dynamics, etc., have led to advantageous results on this subject. Throughout this period, reliability continued to be one of the important factors followed.

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#### References:

- [1]. International Electrotechnical Commission. (2017). IEC 60300-3-3: 2017 Dependability management—Part 3-3: Application guide—Life cycle costing. *International Standard*. Geneva, Switzerland.
- [2]. Balzer, G., & Neumann, C. (2011). Asset simulation and life cycle assessment for gas insulated substation. *CIGRE, Germany*.
- [3]. Balzer, G., Gaul, A., Neumann, C., & Schorn, C. (2007). The general asset management process of power systems. In *Konferenzbeitrag CIGRE-Symposium, Osaka, I(4)*.
- [4]. Costinas S., Opris I., Gogoase Nistoran D.E., Triștiu I., Ionescu C.S. (2018). Hybrid Socio-Technical & Economic Interaction Networks Application: the Theoretical Cost of Penalties for Non-Delivery of Power Energy. *TEM Journal*, 7(1).
- [5]. Kawauchi, Y., & Rausand, M. (1999). Life Cycle Cost (LCC) analysis in oil and chemical process industries. *Toyo Engineering Corp, Chiba*.
- [6]. Opris, I., Cenușă, V. E., Norișor, M., Darie, G., Alexe, F. N., & Costinaș, S. (2020). Parametric optimization of the thermodynamic cycle design for supercritical steam power plants. *Energy Conversion and Management*, 208, 112587.
- [7]. Gogoase Nistoran, D. E., Ionescu, C. S., Brașoveanu, L., Armaș, I., Opris, I., & Costinaș, S. (2018). Modeling hydrodynamic changes induced by run-of-river hydropower plants along the Prahova River in Romania. *Journal of energy engineering*, 144(2), 04017078.
- [8]. Xiaoyuan, L., & Jun, L. (2013). Review of Mathematical Modeling in Applied Mathematics Education. In *2013 Fourth International Conference on Intelligent Systems Design and Engineering Applications*, 549-552. IEEE.
- [9]. Sanz-Bobi, M. A., Palacios, R., Vieira, R. J., Nicolau, G., & Ferrarons, P. (2010). Assisting tools for a new maintenance planning in a power distribution system. In *IEEE PES General Meeting*, 1-6. IEEE.
- [10]. Eliot, C. C., & Sellers, W. H. (1966). Automated Mission Analysis by Markov Chain Techniques. *IEEE Transactions on Aerospace and Electronic Systems*, (4), 204-209.
- [11]. Mcleod, R. B. (1973). Requirements of an economic approach to maintenance. *IEEE Transactions on Engineering Management*, (3), 75-80.
- [12]. Forsythe, J. B. (1980). Light Rail/Rapid Transit: New Approaches for the Evaluation of Energy Savings, Part I-Life-Cycle Cost from Synthetic Routes/Operational Models. *IEEE Transactions on Industry Applications*, (5), 655-665.
- [13]. Seger, J. K. (1983). Reliability investment and life-cycle cost. *IEEE Transactions on Reliability*, 32(3), 259-263.
- [14]. Monteith, D. O. (1984). Reliability considerations for power system design. In *INTELEC'84-International Telecommunications Energy Conference*, 491-494. IEEE.
- [15]. CIGRE Working Group B3.48. (2021). Asset health indices for equipment in existing substations. Technical Brochure no 858. Cigre.
- [16]. Pohl, E. A., & Dietrich, D. L. (1995). Environmental stress screening strategies for multi-component systems with Weibull failure-times and imperfect failure detection. In *Annual Reliability and Maintainability Symposium 1995 Proceedings*, 223-232. IEEE.
- [17]. Lefton, S. A., Besuner, P. M., & Grimsrud, G. P. (1995). Managing utility power plant assets to economically optimize power plant cycling costs, life, and reliability. In *Proceedings of International Conference on Control Applications*, 195-208. IEEE.
- [18]. Monga, A., Zuo, M. J., & Toogood, R. (1995). System design with deteriorative components for minimal life cycle costs. In *1995 IEEE International Conference on Systems, Man and Cybernetics. Intelligent Systems for the 21st Century*, 2, 1843-1848. IEEE.

- [19]. Eriksson, A., Pettersson, K. G., Krenicky, A., Baker, R., Ochoa, J. R., & Leibold, A. (1995). Experience with gas insulated substations in the USA. *IEEE transactions on power delivery*, 10(1), 210-218.
- [20]. Shu, L. H., & Wallace, D. R. (1996). Probabilistic methods in life-cycle design. In *Proceedings of the 1996 IEEE International Symposium on Electronics and the Environment. ISEE-1996*, 7-12. IEEE.
- [21]. Karlsson, D., Wallin, L., Olovsson, H. E., & Solver, C. E. (1997). Reliability and life cycle cost estimates of 400 kV substation layouts. *IEEE transactions on power delivery*, 12(4), 1486-1492.
- [22]. Giudice, F., La Rosa, G., & Risitano, A. (1999). Models and indicators for cost-benefit analysis of a green product. In *Proceedings First International Symposium on Environmentally Conscious Design and Inverse Manufacturing* (pp. 77-82). IEEE.
- [23]. Savio, S. (1999). Modelling of diagnostics aided RCM procedure for transportation systems. Dependability and cost evaluation. In *ISIE'99. Proceedings of the IEEE International Symposium on Industrial Electronics (Cat. No. 99TH8465)*, 2, 877-882. IEEE.
- [24]. Kishk, M., & Al-Hajj, A. (2000). A fuzzy approach to model subjectivity in life cycle costing. In *Proceedings of the BF2000 national conference of postgraduate research in the built and human environment*, 9-10. University of Salford.
- [25]. Bupasiri, R., Wattanapongsakorn, N., Hokierti, J., & Coit, D. W. (2003). Optimal electric power distribution system reliability indices using binary programming. In *Annual Reliability and Maintainability Symposium, 2003.*, 556-561. IEEE.
- [26]. Verho, P., Järventausta, P., Kivikko, K., Pylvänäinen, J., Partanen, J., Lassila, J., ... & Kaipia, T. (2006). Applying reliability analysis in evaluation of life-cycle costs of alternative network solutions. *European Transactions on electrical power*, 16(5), 523-531.
- [27]. Arshad, M., & Islam, S. M. (2006). A novel fuzzy logic technique for power transformer asset management. In *Conference Record of the 2006 IEEE Industry Applications Conference Forty-First IAS Annual Meeting, 1*, 276-286. IEEE.
- [28]. Villa, G. (2006). Feasibility of the Modernization and Upgrade of the Electrical High Voltage Substations. In *2006 IEEE/PES Transmission & Distribution Conference and Exposition: Latin America*, 1-5. IEEE.
- [29]. Endrenyi, J., & Anders, G. J. (2006). Aging, maintenance, and reliability-approaches to preserving equipment health and extending equipment life. *IEEE Power and Energy Magazine*, 4(3), 59-67.
- [30]. Politano, D., & Frohlich, K. (2006). Calculation of stress-dependent life cycle costs of a substation subsystem-demonstrated for controlled energization of unloaded power transformers. *IEEE Transactions on Power Delivery*, 21(4), 2032-2038.
- [31]. Amoiralis, E. I., Tsili, M. A., Georgilakis, P. S., & Kladas, A. G. (2007). Energy efficient transformer selection implementing life cycle costs and environmental externalities. In *2007 9th International Conference on Electrical Power Quality and Utilisation*, 1-6. IEEE.
- [32]. Hinow, M. (2008). Genetic algorithm based methodology for optimisation of innovative switchgear design. In *Conference Record of the 2008 IEEE International Symposium on Electrical Insulation*, 437-440. IEEE.
- [33]. Huang, L., & Yue, W. (2009). Reliability modeling and design optimization for mechanical equipment undergoing maintenance. In *2009 8th International Conference on Reliability, Maintainability and Safety*, 1029-1034. IEEE.
- [34]. Yi, H., & Wen-jie, H. (2009). Analysis of the life cycle cost of the power plants based on analytic hierarchy process method. In *2009 International Conference on Information Management, Innovation Management and Industrial Engineering, 4*, 639-642. IEEE.
- [35]. Du, L., Wang, Z., Huang, H. Z., Lu, C., & Miao, Q. (2009). Life cycle cost analysis for design optimization under uncertainty. In *2009 8th International Conference on Reliability, Maintainability and Safety*, 54-57. IEEE.
- [36]. Anombem, U. B., Li, H. Y., Crossley, P., Zhang, R., & McTaggart, C. (2010). Process bus architectures for substation automation with life cycle cost consideration. *10th IET International Conference on Developments in Power System Protection (DPSP 2010). Managing the Change*.
- [37]. Gomes, B. A., & Saraiva, J. T. (2010). Evaluation of transmission reinforcement strategies using risk indices in the presence of uncertainties. In *2010 7th International Conference on the European Energy Market*, 1-6. IEEE.
- [38]. Huang, Y., & Luo, Y. (2010). An Optimization Research of Power Generation Equipment Management Based on Life Cycle Cost. In *2010 International Conference on Logistics Engineering and Intelligent Transportation Systems*, 1-4. IEEE.
- [39]. Song, Y., & Li, J. (2012). Analysis of the life cycle cost and intelligent investment benefit of smart substation. In *IEEE PES Innovative Smart Grid Technologies*, 1-5. IEEE.
- [40]. Farias, V. S., & Fernandes, N. C. (2014). Life cycle cost analysis of substation automation implementation. In *2014 International Telecommunications Symposium (ITS)*, 1-5. IEEE.
- [41]. Zhang, J., Du, Z., Xie, D., Jiang, S., Liu, Y., Ma, J., & Chen, Y. (2016). Improved SEC model based evaluation approach for design scheme of the new generation smart substation. In *2016 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, 1169-1173. IEEE.
- [42]. Liao, Z., Chen, C., & Zang, X. (2016). Research on comparison of power grid planning program based on life cycle cost. In *2016 IEEE International Conference on High Voltage Engineering and Application (ICHVE)*, 1-5. IEEE.
- [43]. Krasich, M. (2017). Mathematical models and software reliability can different mathematics fit all phases of SW lifecycle?. In *2017 Annual Reliability and Maintainability Symposium (RAMS)*, 1-7. IEEE.

- [44]. Laohanan, S., & Banjerdpongchai, D. (2018). Dynamic programming approach to optimal maintenance scheduling of substation equipment considering life cycle cost and reliability. In *2018 15th International Conference on Electrical Engineering/ Electronics, Computer, Telecommunications and Information Technology (ECTI-CON)*, 388-391. IEEE.
- [45]. Jun, L., Fan, Z., Yinyuan, L., & Beibei, Z. (2019). Life Cycle Cost Analysis and Optimization of Modular Substation. In *E3S Web of Conferences*, 136, 01049. EDP Sciences.
- [46]. Zhang, Z., Yu, Z., Li, W., Ji, J., Li, B., & Han, X. (2019). Research on the whole life cycle cost management and maintenance decision of secondary equipment based on physical ID. In *2019 IEEE 3rd Conference on Energy Internet and Energy System Integration (EI2)*, 2691-2696. IEEE.
- [47]. Najjar, M., Figueiredo, K., Hammad, A. W., & Haddad, A. (2019). Integrated optimization with building information modeling and life cycle assessment for generating energy efficient buildings. *Applied Energy*, 250, 1366-1382.
- [48]. Khalyasmaa, A., Eroshenko, S., Elaev, M., & Chung, T. D. (2020). Machine learning application for the high-voltage equipment lifecycle forecasting. In *2020 21st International Symposium on Electrical Apparatus & Technologies (SIELA)*, 1-4. IEEE.
- [49]. Diaz, F. F., Guerrero, F. G., & Barandica, A. (2020). Technical-economic evaluation model for a process bus based on IEC 61850. *Sustainable Energy, Grids and Networks*, 21, 100288.
- [50]. Stan, A.; Costinas, S.; Ion, G. (2022). Overview and Assessment of HVDC Current Applications and Future Trends. *Energies*, 15, 1193.