Characterizations of Students' Metacognition in Solving Geometry Problems through Positioning Group Work

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***Abstract* – Group discussions can help students have a more effective problem-solving process because students exchange ideas and understand the provided problems. Accuracy in solving problems can be observed from students' knowledge and metacognition. This study used the descriptive qualitative approach to describe the characterization of students' metacognition during their interactions in positioning expert students, facilitators, and beginners. Problem-solving in group discussions is very helpful in bringing up students' metacognitive activities because they do not use only self-metacognitive but also social metacognitive.**

***Keywords* – metacognition, problem-solving, student positioning, geometry.**

1. Introduction

Discussions expand opportunities for students to come up with new ideas that are interconnected among group members. Group discussions aid students consider the process of working on a new task that requires them to exchange ideas with each group member [1].

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Group discussions expand students' learning opportunities because they do not only focus on finding the correct answer but also on exploring their knowledge [2]. Students showed improved learning outcomes after group discussions as they explicitly made different strategies and reasons. During group discussions, the students experience a process of interaction between group members and themselves in solving problems. Students can interact by thinking about their thoughts. Also, they can develop their perspectives before and during interactions with others [3]. This process of thinking about their thinking is a metacognitive process. Metacognition is essential for students because it facilitates them to express ideas and thoughts in solving mathematical problems both orally and in writing. Metacognition refers to individuals' awareness of their thoughts, as well as their evaluation and their regulation of those thoughts. By realizing their thinking skills, students can improve their metacognition through learning mathematics. The effectiveness of mathematical learning needs to be observed from the quality of students' mathematical concepts understanding through the characteristic process. Problem-solving is one of the main crucial skills in mathematics learning as it is included in the mathematical framework [4]. In group discussion, problem-solving represents the ability of individuals to engage effectively with two or more students in solving problems using their various understandings. Additionally, a mathematics subject matter that is difficult for students to do is geometry. Geometry is one of the most important branches of mathematics learning. Geometry is a field that carries excellent correlation with other areas and is extensively used in real life [5]. Geometry helps students improve various skills such as deductive reasoning, imaginary visuals, logical argument, and verification [6]. Basically, in studying geometry, students are not only asked to find answers but rather the process of finding these results. This study used a geometric problem because it can develop students' cognitive and metacognitive abilities. Metacognition is a guide to the course of cognitive processes through instructions to improve the ability to think critically, quickly, and logically. A good metacognitive in solving problems carries positive effects on the learning process and learning achievement [7], [8]. Differences in students' academic achievement in mathematics have spurred the investigation of its primary root. Metacognition is essential in solving mathematical problems [9]. Metacognition positively impacts students who learn through problem-solving because it provides an efficient way to obtain, store, and convey information [10]. If students own the skills to investigate their metacognition, they will attain more strategic thinking skills in solving problems. Research findings have shown that good problem-solving skills are associated with high metacognitive activity, while metacognitive activities can occur in various phases of the problem-solving process, individually or in groups [11].

2. Literature Review

Group discussion is a type of social interaction model that aims to build cooperative, interactive, and productive relationships among students. The practice of group discussion is supported by the NCTM, which suggests that students should learn to listen to and respond to their peers' ideas, build arguments, and solve complex problems. Group discussion is a learning model that aids students attain common goals in groups by helping each other [12]. Students' reasoning can be seen from the ongoing interaction in the discussion process [13]. By working together in groups and discussing solutions to a problem, the students will be encouraged to participate in social interaction [14].

Student interaction in small groups can help them gain more conceptual understanding [15]. Besides, students also negotiate with themselves through their dialogue in group discussions concerning mathematics problems [16], [17]. Unfortunately, dialogue between peers to learn mathematics tends to be a monologue. During the discussion, sometimes students make the regulation. They agree on appropriate ideas while mostly refraining from discussing differences in perspective and finally ending the dialogue [18]. Regulation, in this case, represents the metacognitive, so students need to have awareness and understanding of their thoughts during group discussions [19].

Metacognition occurs when students are aware of their thought processes in solving problems. Cognitive processes help students learn, while metacognitive strategies help them ensure that they have learned [20]. Students who use cognition and metacognition can understand information, make decisions and solve problems. Metacognition refers to a person's knowledge of their cognitive processes or relatedness. Metacognition is a process of thinking about thinking. In other words, it can be interpreted as a person's understanding of his thinking process [21]. Research related to metacognition conducted by Asik et al. showed a significant relationship between students' metacognitive knowledge and problem-solving performance [22]. Previous researchers have carried out various studies on metacognition and student positioning. Table 1. presents the position of this research against previous research.

Table 1. Research position

Study Object	Researcher (Year)	Research Focus
Meta-cognition	Magiera & Zawojewski (2011)	Characterizing socially-based and self-based contexts relates to student awareness, evaluation, and thought regulation [23].
	Iiskala et al. (2011)	Investigating how social metacognition emerges in students in collaborative problem solving [24].
	Abdel Rahman (2020)	Assessing metacognitive awareness and academic motivation, as well as their impact on academic achievement [25].
	Backer et al. (2022)	Assessing the function of social metacognitive regulation with collaborative student understanding of learning content [26].
	Haataja et al. (2022)	Investigating students in groups: Predictors of metacognitive learning [27].
Student Positioning	DeJarnette & González (2015)	Characterizing students' positioning during group work on problem-solving [28].
	Backer et al. (2015)	Exploring social metacognition in groups and identifying correlated metacognitions [29].
	Rasmussen & Schmidt (2022)	Investigating interactions between students during group discussions [30].
This research		Characterizing students' metacognition in solving geometry problems by positioning expert students, facilitators, and beginners.

Many studies on metacognitive processes focus on individuals' metacognition, and no research associates social metacognition with student positioning in group discussions [31], [32].

Meanwhile, research on metacognition in mathematical problem solving is mainly focused on identifying metacognitive awareness, regulation, and evaluation [23]. In contrast, this study characterized metacognition based on students' thinking processes in groups regarding student positioning. Therefore, knowledge about the students' metacognitive characterization in groups is vital in the process of skills improvement because students' metacognitive activities in group discussions facilitate dialogues between expert students, facilitator students, and beginner students. When experienced, facilitator, and beginner students communicate, it can lead to student interactions that produce metacognition based on their own and communal thinking. This study describes the characterization of students' metacognition in group discussions in solving geometry problems following student positioning.

3. Methods

This study used a descriptive qualitative method. The data were collected in the form of words presented in sentences or non-numeric [33]. The obtained illustrated the characterization of students' metacognition in solving geometry problems in group discussions following the student positioning. Descriptive research was selected as this study described or explained the research variables, namely students' metacognition and positioning. Specifically, this study described the characterization of students' metacognition in solving geometry problems in group discussions using students' positioning. The selection of research subjects began with setting a class where the students were asked to complete a geometry test. From the results of the geometry test, students were divided into several heterogeneous groups, with three students in each group. Students can learn to respect and assist one another very effectively in heterogeneous groups. After the teacher explained the materials, students were given geometry problems and asked to work in group discussions. Researchers observed metacognition in student interactions in the specific students positioning, consisting of expert students, facilitator students, and beginner students. Besides, in-depth interviews were also required to explore the metacognition process and characteristics of each student in the group [34].

During group discussions, students interact with themselves and others in solving mathematical problems [26]. The positioning of students in group discussions was carried out by ensuring that each group consisted of experts, facilitators, and beginner students. The student movement guidelines during group discussions are presented in Table 2., while the criteria for determining expert, facilitators, and beginners students are shown in Table 3.

Table 2. Student movement guidelines during group work

Information/ Code	Movement	Guidelines
Prior Knowledge (K1)	Information Provision	Students make statements to provide information
Delayed Primary Knowledge (dK1)	Giving Stimulus	Students need a moment to provide information and ask for confirmation of the submitted suggestions
Repeat K1 (rK1)	Repeat K1	Students restate information from K1
Secondary Knowledge (K2)	Information Request	Students ask questions to gain information
Response K1 (rK2)	Giving Response for K1	Students respond to K1 information as a follow-up to K2
Prior Actor (A1)	Take Action	Students take action without any request
Delayed Prior Actor (dA1)	Volunteered	Students recommend themselves to take action
Secondary Actor (A2)	Action Request	Students ask others to take action. Students ask questions related to whether or not they have done the task
Response A2 (rA2)	Giving Response for A2	Students give follow-up responses to A2
Question (Q1)	Confused Response	The student states that he does not understand the information
Activation Discuss (P)	Giving Control Activation Discussion	Students remind that the time is almost over and ask about the last part of the discussion
Inappropriate (X1)	Inappropriate response to the student movement	The student gives a response without request for confirmation, statement, or statement information

(adapted from DeJarnette & González, 2015)[28]

Table 3. Student positioning guidelines

Position	Guideline
Expert	Students often perform K1, K2, rK2, dK1, A1, and dA1
Facilitator	Students often do A1, A2, and P1
Beginner	Students often do K2, X1, and Q1

(adapted from DeJarnette & González, 2015)[28]

4. Results and Discussion

This study focuses on student interactions in the group. Each group consisted of students categorized as experts, facilitators, and beginners. During the group discussions, students' metacognition was observed. Students were assigned to form groups and start working on a given geometry problem. Subjects in this study were coded as SPF01, SFI02, and SAG03 in each discussion group. They were provided two questions, and the discussion lasted for 17 minutes on each question. Table 4. shows the frequency distribution of K1, dK1, K2, A1, Q1, and X1 during the student interaction.

Table 4. Frequency distribution of subject movement

Code	K1	dK1	K2	A1	Q1	X1
SPF01	1		4	1	4	1
SFI02	3		2	3	1	
SAG03	19	6				
SPF01	4,35%		66,67%	25%	80%	100%
SFI02	13,04%		33,33%	75%	20%	
SAG03	82,61%	100%				

Based on Table 4., it can be concluded that SPF01 often presented K2 (ask questions to ask for information) with a percentage of 66.67%, Q1 (reveal that they do not understand the data) with a rate of 80%, and X1 (give responses without requests for confirmation, statements or questions. Then, SFI02 often did the A1 (take actions without a request) with a percentage of 75%. At the same time, SAG03 often performed K1 (make statements to give information) with 82.61% and dK1 (delay providing information, students ask for confirmation of the suggestions) with a percentage of 100%. Additionally, Table 5. shows the frequency distribution of the number of students' exchanges and actions in working on geometric problems.

Table 5. Frequency distribution of the number of the subjects' exchanges and actions

Subject	Knowledge Exchange	Action Exchange
SPF01	5	6
SFI02	5	4
SAG03	19	6

From the acquired data, we concluded that SPF01 tended to be a beginner student, SFI02 tended to be a facilitator student, and SAG03 tended to be an expert student in working on geometry problems. The given geometry problem is illustrated in Figure 1.

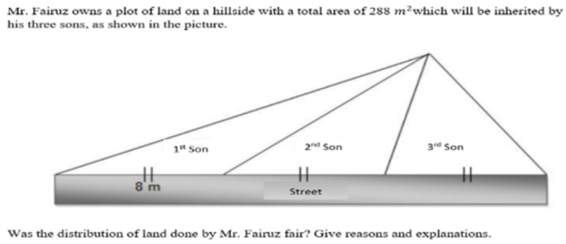

Figure 1. Geometry problems

In the discussion process, at first, the teacher gave instructions for solving the questions, and then students were asked to have discussions with their group members. Problem-solving in mathematics involves formulating concepts and constructing new ideas [4]. Initially, students were focused on their respective activities when the discussion process began. Students tended to have no interaction in the early minutes, but the teacher gave further direction for students to work together on the geometry problems. The students took 17 minutes to solve the geometry problems. At minute 01.09, SFI02 began to read the geometry problems aloud; SPF01 and SAG03 carefully observed the questions. At 02.01 minutes, SPF01 seemed to be daydreaming and was instructed to stay focused on doing group assignments.

At minute 03.03, SAG03 took the questions from SFI02 and started rereading the questions to understand the problems. At minute 03.42, SAG03 began working on the issue but did not write on paper. Instead, SAG03 started to think, as illustrated by the gestures of SAG03, such as scratching their heads and occasionally looking at SPF01 and interacting with SFI02. Further, SPF01 was appointed as note-takers by SAG03 without SFI02 approval. SAG03, at minute 4.00, seemed to be trying to rethink the alternative solutions for the geometry problem. At minute 04.07, SAG03 told SFI02 that the answer to number 1 point a was "fair enough" SFI02 confirmed again by asking SAG03, "yes?" SAG03 answered firmly, "yes yes". Then SFI02 interrupted the discussion, expressing disagreement with SAG03's answer. As long as students interact in group discussions, they are capable of expressing new ideas in solving problems, creating and generating new ideas or ways of vieweing things about mathematics [35]. The SFI02 disagreed because he had a different view on solving the problem, as shown in the following interview transcript:

SFI02: this is not fair, which is number 1 a [A1]. if it's clear from the picture that it's not fair [K1], this picture is longer [K1]

SAG03: what do you think? (ask SFI02 by trying to think again (aware of what he is thinking) [dK1], is it the same or not? [dK1]

SFI02: this is a different picture. This one has an extra length (pointing to the concept of child 1's land area in the problem). The size is more significant [K1]

SAG03: what do you think? (asking SPF01 opinion while rethinking (aware of what he's thinking)) [dK1]

SPF01: I don't know [Q1]. How come it's not fair (ask SFI02)? [K2]

In the interactions in the interview transcripts, there was a shift in the movement of each group member. The exchanges made the discussion process enjoyable because of the emergence of new understandings. Student interaction in mathematics learning can help them better understand the concepts [15]. In the process of exchange, there was also metacognition that appeared from SAG03 when he was initially sure of the answer, but because of a different opinion from SFI02, SAG03 tried to rethink what he was thinking consciously. Besides being aware of the regulation and evaluation of the interaction process, in attempting to re-examine the initial answer, SAG03 rechecked it by asking for approval from other colleagues. Mazancieux et al. (2019) argue that students must observe, relate, ask questions, find reasons, and draw conclusions to solve problems [36]. At the awareness stage, SAG03 made statements (thinking results) showing their thoughts about what is known, what needs to be done, and what has been done after doing the mathematical thinking assisted by the other members. At the regulation stage, SAG03 make statements (thinking results) showing planning strategies, setting goals and choosing problem-solving strategies after doing their mathematical thinking and are also assisted by the opinions of others. In addition, SAG03 also made evaluation (thinking results) on the results, considering problem difficulties, assessing progress, abilities, and understanding after the mathematical thinking carried out along with the other members, as shown in the following interview transcript:

SPF01: I don't know [?] what do you think 3? [K2]

SAG03: if we look at it at a glance in the picture, it's not fair [K1]. (while trying to think of a strategy that needs to be done). but the question is whether the distribution of children is fair or not? [dK1]. (trying to study by asking colleagues and also himself) This one (show the picture) should be fair because the sides here are the same (pointing to the side of the line with an equal sign, 8 meters) [K1]. This one (show the triangle) is a triangular shape with the same base [K1].

SPF01: maybe the 3rd one is correct, [K2], because the base is the same 8 meters, so it's fair. [K1]

SAG03: ok, the answer is fair. [K1] So 288 divided by 3 = 96 [K1]. so each child has a land area of 96. [K1] This has something to do with the second question, number 1, point B. [K1]

The way students think logically can be observed in the interaction process during the group discussion [37]. SAG03 was quite confident with the answers that he had previously made, although there were doubts when there were different opinions from other group members. Because SAG03 was able to use his metacognition with awareness, regulation, and evaluation, he was able to think in thinking about what he was doing. Besides, he also applied the principle of knowing, making planning strategies, setting goals, choosing a problem-solving approach, and evaluating the results and method. Students are expected to be able to use their abilities and skills in the problem-solving process [9].

Then, SPF01, who initially looked passive when provided with problems, tried to think about what they were thinking by making statements (thinking results) after doing their mathematical thinking, assisted by others. SAG03's report made SPF01 aware of his thoughts, but SPF01 had not been able to carry out regulations or evaluations, only to be mindful of his thoughts. SPF01 at the interaction time is more passive because he ignored the cognitive and metacognitive processes. Students experience difficulties in mathematics and problem-solving tasks because they miss various cognitive or metacognitive strategies [38].

SFI02, when using his metacognition, was aware that by making statements (thinking results) after doing systematic thinking himself or with others, he realized that his initial anwer was not correct because he realized that the pictures on the questions were not the same. Thus, he made a solution strategy by making statements (thinking results) after another mathematical thinking by focusing on visual images. It made other group members try to rethink the chosen answer. However, shortly after SAG03 presented his explanations, SFI02 finally agreed that his understanding of what he thought was a metacognitive failure, so he could not evaluate his answers. SFI02 was only able to determine when he got help from SAG03. Due of social connections, SFI02 first struggled to comprehend, but he used his metacognitive processes to work through the issue. The process of problem solving, decision making and metacognition affect the development of Higher Order Thinking Skills, while social interaction with other students affects their achievement in mathematics [17]. The metacognitive characterization of beginner students, facilitators and experts in group discussions in solving geometry problems can be seen in Table 6. below.

Table 6. Characterization of metacognition beginner students, facilitators and experts

Subject	Student Movement	Metacognitive Activity	Characterization
Beginner	• Students often ask questions to ask for information • Students often reveal that they do not understand the information • Students often give responses that without requests for confirmation or statements • Students rarely take action without a request	Awareness	Students make statements (thinking results) after doing mathematical thinking by themselves or others who show thoughts about: • What is known (task-specific knowledge, relevant mathematical knowledge, problem-solving strategies) • What needs to be done, what has been done or what could be done
Facilitator	• Students often take action without being asked • Students sometimes make statements to provide information • Students sometimes ask questions to ask for information	Awareness	Students make statements (thinking results) after doing mathematical thinking themselves or others who show thoughts about: • What is known (task-specific knowledge, relevant mathematical knowledge, problem-solving strategies) • What needs to be done, what has been done or what could be done
		Regulation	Students make statements (thinking results) after doing mathematical thinking by themselves or others, which show: • Planning strategy • Choose a problem-solving strategy
Expert	• Students often make statements to provide information • Students often delay providing information. • Students often ask for confirmation of the submitted suggestions	Awareness	Students make statements (thinking results) after mathematical thinking with themselves or others who show thoughts about: • What is known (task-specific knowledge, relevant mathematical knowledge, problem-solving strategies) • What needs to be done, what has been done or what could be done
		Regulation	Students make statements (thinking results) after doing mathematical thinking by themselves or others, which show: • Planning strategy • Setting goals • Choose a problem-solving strategy
		Evaluation	Students carry out evaluations (thinking results) after doing mathematical thinking themselves or others, showing thoughts about: • The effectiveness of the chosen strategy • Evaluation of results • Evaluation of the difficulty of the problem • Evaluation of progress, ability or understanding

5. Conclusion

In carrying out their metacognition, beginner students can make statements (thinking results) after mathematical thinking given by others so that they show the results of thinking about what is known and what has been done. Beginner students are not capable of carrying out regulations and evaluations. Besides, in student interactions, they are more passive and only give opinions after being given the opportunity by other members. By facilitating their metacognitive activities, students are aware of their metacognition by giving statements (outcomes of thinking) after their mathematical thinking. They are also assisted by other group members who show their thoughts about what they know and what needs to be done. Facilitating students are also able to regulate by making statements (thinking results) after mathematical thinking on their own and assisted by others in planning strategies and setting goals. In comparison, expert Students use the complete metacognitive activities, namely awareness, regulation and evaluation. During the interaction, the expert students take place more as group leaders who also help other students do metacognition. Expert students play an essential role in the discussion process as that talented students with high abilities have complete metacognition, consisting of awareness, regulation and evaluation. We think that metacognition is very important in problem-solving. Meanwhile, problem-solving in group discussions is very helpful in bringing up student metacognitive activities because students do not only perform self-metacognitive but also metacognitive through social interactions during group discussions.

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