Reflection of Problem Solving Task and Importance of A-priori Analysis in Preparing Teacher for the Teaching Process

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***Abstract* – In this article, we focus on a priori analysis as a part of teacher's lesson planning. We show a concrete use of this analysis during a posteriori analysis of a lesson (analysis after lessons) and describes its importance in teachers' work. We chose a problem from geometry, which was solved by secondary school students. The goal of the article is to point out the importance of a priori and also a posteriori analysis for teachers from the point of view of the theory of didactic situations.**

***Keywords* – theory of didactic situation, problem solving task, geometry, students, teacher´s lesson planning**

1. Introduction

Mathematics teacher has learned in his higher education how to solve mathematical and various problems, how to analyze their solutions from a didactic point of view, he can also judge the suitability of selecting specific tasks for the teaching process. In addition, the teacher should conduct the teaching in such a way as to appropriately stimulate the student's thought processes, and thus all didactic activities should be prepared and planned by the teacher.

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The theory of didactic situations is suitable for this, in our opinion, while the related terminology is used in the sense introduced by [1], [2], [3] and his school of mathematics didactics, but also in the sense of the work by [4], [5] and also a colleague of a university in the Czech Republic in [6], [7], [8].

2. Theory of Didactic Situations in the Teaching Process

This theory will allow us to analyze the didactic problem in the teaching process from several aspects. It is a theoretical tool that examines the nature of the problem. And it is the analysis of the problem on individual levels of didactic situations that forms the basis of didactic research of the theory of didactic situations, but also of the practical activity of a mathematics teacher. It is advisable to analyze the problem before the teacher solves it for the students. In this case, he tries to answer how the given problem will allow him to follow the set goal. However, the analysis may show that the current problem formulation is not suitable for pursuing the set goal, or is not very suitable for pursuing another goal. Likewise, the analysis can show how the problem needs to be changed for the students if we want to achieve the learning goal [9].

According to the authors, a priori analysis is therefore a very suitable tool for teachers in preparing their preparation for the teaching process at their disposal. In mathematics, the theory of didactic situations is based on the assumption that mathematical knowledge can always form a specific corresponding mathematical situation.

If the teacher uses a priori analysis for the teaching process, it will give him a more detailed idea of how the lesson can take place, or what he can expect, how the students will react, what questions or problems they may have. Thus, the teacher prepares for an adidactic situation, a situation where students acquire their knowledge without the help of a teacher. It is an analysis that, according to [10], is helpful in planning the lesson and selecting activities. It is then necessary to compare the a priori analysis and the a posteriori analysis, which will be helpful for the teacher with regard to the interpretation of unexpected student strategies, their argumentation, student errors, ... [11].

Author [12] also characterizes an a posteriori analysis, which understands the teacher's analysis of a specific lesson taught. The teacher can compare his assumptions, which he proposed in the a priori analysis with the results obtained in the a posteriori analysis.

In addition, [13] argue that if a teacher uses a priori analysis in his or her school practice, he or she is also able to discuss with colleagues' problems and issues related to teaching mathematics and learning mathematics.

3. A Priori Analysis of the Problem Task and Preparation of the Teacher for the Teaching Process

Due to the fact that the basis of a correctly solved stereometrical problem is a suitable sketch of the geometric solid, which we then use to solve the problem, we gave the task of constructing projections of various solids to secondary school students. Images of three-dimensional geometric shapes need to be distinguished, named and sketched correctly within the framework of mathematics education. This above-mentioned issue is also followed up in mathematics education at higher secondary school. Before assigning the task to the students, we prepared an a priori analysis of the given problem task, which can be part of the preparation of the mathematics teacher for the teaching process.

The problem task had several variants for students, in parallel projection they had to draw:

- cone with the given height, length and a radius,
- cylinder with the given height, length and a radius,
- pyramid with the given side length,
- triangular, square, hexagonal based pyramid with the given side length and height length,
- triangular, hexagonal prism with the given side length and height length.

Characteristics of Problem Task Assignment

The teacher assigns the task to the students in writing, while the assignment is in accordance with the curriculum for the lower secondary school. Each student has to draw one geometric solid with specific data, it is up to the teacher to determine the values. We assume that the above wording of the assignment is understandable for students.

Thematic Unit Where the Task Can be Classified

We recommend the teacher to include the task (polyhedrons) in the 7th grade of secondary school as part of the thematic unit Parallel Projection (thematic area Geometry and Measurement). The task can also be included in the 8th and 9th grade of secondary school, where the knowledge and properties of various bodies (polyhedrons and non-polyhedrons) are fixed, including the calculation of their volumes and surfaces.

Problem Task Goal

Based on the properties of parallel projection, the student should be able to display various spatial geometric shapes into a plane.

Problem Solving Time Range

The teacher repeats the characteristics of parallel projection and subsequently the students will work on a problem task during one lesson.

Course of Work

We recommend students to work independently, respectively at the teacher's discretion, the task can be solved in pairs. Students should not have a problem understanding the assigned task, or they can discuss ambiguities with the teacher individually. We do not recommend working in larger groups, as weaker students may not be involved in solving the problem.

Tools

Students have the following tools available to solve the task: drawing aids (ruler, compass), paper, pencil.

Didactic Variables for a Priori Analysis

The formulation didactic variables are: draw, projection. Cognitive didactic variables are: regular/rotational. When we are omitting these concepts in the task assignment, students may have ambiguities with the type of geometric solids as pyramid/prism/cone/cylinder. Teacher can enter another spatial geometric shape (not often used in the teaching process) and thus change the construction adopted by the students (given radius/given height/given side length). Teacher can also enter a specific value or other data related to the construction of the base shape, which will change the solution of the task (triangular/square/hexagonal). If the teacher replaces the given number of walls with another, larger number, the task will be more difficult for the students, because such selected bodies are often not used in the teaching process.

Students' initial knowledge and skills

Student distinguishes and can name planar and spatial geometric shapes, he can also sketch them. They also know and can apply the properties of parallel projection on specific demonstrations, they must have mastered the basic rules of drawing and be able to use drawing aids.

Students´ Reactions

We expect students to ask questions about the naming of individual geometric solids, as well as how to draw correctly and accurately these solids.

Teachers´ Reactions

The teacher can modify the task according to the age and abilities of the students.

Problem Solving Strategies

We assume that students will be able to use in their solutions:

- use the property of parallel projection: parallelism is maintained,
- use the property of parallel projection: the arrangement of points on the line is unchanged,
- use the property of parallel projection: the dividing ratio is unchanged,
- use the property of parallel projection: projections of parallel and identical segments are parallel and identical segments,correctly display of a triangle / square / hexagon in parallel projection,
- correctly display of the circle in parallel projection, which will be entered to the projection of the square described by the given circle,
- construct projections of identical faces in a prism/cylinder in parallel projection,
- construct a projection the center of base of solids in parallel projection,
- display the height of a given solids in parallel projection,
- display the edges of the displayed solids,
- mark the visibility of the edges.

Incorrect Problem Solving Strategies

We expect the students to construct the incorrect projection of the triangle as the base of the solids, or the projection of the hexagon when the parallelism of the respective sides will not be observed. Some students display the base of the solids in actual lengths and shapes. We also expect that projections of given parallel segments will not have identical lengths. In problem solving, students incorrectly use the property of right angle projection in parallel projection when they will by display square shapes. When displaying different projections of geometric solids in parallel projection, students display these solids without marking the visibility of the edges.

Assessment of the Problem Task

The assessment will be formative, which can take place in the next lesson too.

4. Reflection of Problem Solving Task and Importance of a Priori Analysis of Task

The problem task fits into the thematic unit Parallel Projection (7th grade of secondary school). In the opinion of teachers, it is appropriate to include it in the higher grades of secondary school, where students work over again with individual geometric solids. The given task was solved by the students of the 8th year of secondary school with regard to the students´ abilities of our chosen class. Of course, the teacher does not have to include this task in a specific thematic unit, but when he decides to include it in the teaching process, he has to clarify what knowledge and knowledge the students have.

The goal was formulated in the a priori analysis clearly. There wasn´t a need to explain to students in more detail what their role was, so they didn´t ask the teacher questions such as "Why are we learning this?" Thus, the assignment of the problem was related to the repetition and extension of the discussed issue. Of course, it is possible to solve the task within the explanation of the described issues. Then assign to solve similar tasks to students in the next mathematics lessons, or as a homework.

The course of students' work in the classroom and the aids were appropriately chosen, students can solve tasks of this type independently. Didactic variables were also all included in our a priori analysis, there was no need for the teachers to modify the assignment. Due to the age of the pupils, the teacher entered the specific data needed to identify the individual geometric solids (the given average/given height/given edge were replaced by a specific integer). Therefore, the same problem assignment did not occur among several students. However, there were student questions as to whether we could continue the number of vertices indefinitely and how these solids would appear in parallel projection. Some students said that not solving such a task would be difficult for them to imagine. Therefore, we think that all the geometric solids listed in the assignment of the problem task were appropriately chosen for the students of the 8th year of secondary school.

In the a priori analysis, we determined the necessary initial knowledge and skills of the students, based on the thematic units in which the problem task could be included. In our case, we chose the 8th grade students because the lower grade students did not have the necessary knowledge and skills from the thematic unit Parallel projection. We think that they would not be able to successfully solve the task with respect to the set goal. However, 7th grade secondary school students routinely work with projections of solids such as a cube, a block, a square based pyramid or pyramid, but this knowledge has not been sufficient for us due to the goal, and thus specifically using the properties of parallel projection to draw the projection of other bodies than we have stated.

The above-mentioned reactions of our students and teachers have not been confirmed to us. We thought of the correct and incorrect student solutions to the problem sufficiently, because in these solutions there was no other correct, resp. incorrect strategy used by students in solving the task. We can therefore say that we would respond appropriately to any student questions during the solution of the given task. Therefore, we state that if the teacher has prepared an a priori analysis of any activity, the course of this activity will be easier for him to manage from the point of view of students' questions. The following examples in Figure 1. are in line with our proposed correct or incorrect solution strategies, which the students used in their solutions to the problem task.

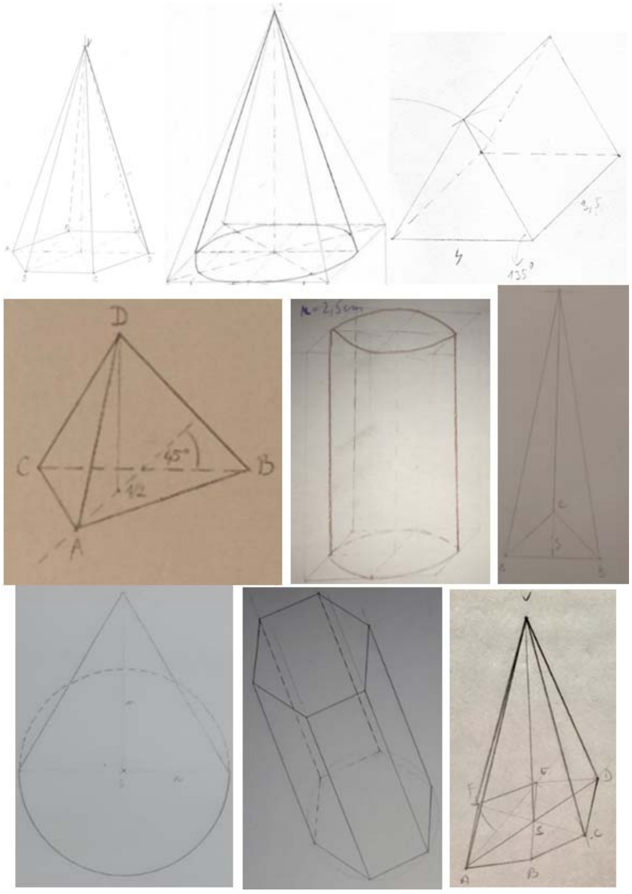

Figure 1. Samples of student solutions to a problem task

We suggested that students evaluate the solved problem only formatively, which was also appropriately chosen by us. We think that due to this fact, the students were not afraid to ask the teacher to the ambiguities that arose in their solutions, and also responded to any questions from classmates.

5. Conclusion

Our findings suggest that a priori analysis is a useful tool for teachers and researchers. Teacher of mathematics should not only master the solution of mathematical problems, their selection and analysis, but he should also introduce them in the teaching process so that they stimulate the students' thought processes well. Therefore, each selected problem should be well thought out and then also planned. In the theory of didactic situations in mathematics is a priori analysis described as a professional tool that can help teachers in planning the teaching.

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