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Neurotic Fuzzy-Data-Envelopment Analysis to Forecast Efficiency of Bank Branches

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***Abstract* – In this study the prediction of efficiency of four different Bank Branches have been done by using Neurotic Fuzzy Data Envelopment Analysis approach. In the first stage of the study, Artificial Neural Network (ANN) model has been modelled and trained using the last five years data. The data belonging any year has been taken as input of ANN, next year data has been defined as output of ANN. Fuzzyfication process has been applied to obtained predictions based on asking managers of bank branches, after Fuzzy Data Envelopment Analysis process has been applied to fuzzy values. As a result, the bank branches parameters belonging to 2021 year have been obtained. The efficiency of 2021 for bank branches have been calculated based on Fuzzy Data Envelopment Analysis (FDEA).**

***Keywords* – Artificial Neural Networks, Forecasting, Fuzzy Data Envelopment Analysis.**

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1. Introduction

Contributions of banks to economy is very important role for the economy. They protect people's savings and financially support local and global business and trade.

Moreover, many studies have shown that effective financial activities have a positive effect on economic growth, while bankrupt companies have very negative effects on the economy.

Therefore, the performance of banks attracted the attention of customers, investors and the public. Previously, the performance of banks was traditionally evaluated on the basis of financial ratios, now it is evaluated by techniques such as operations research and artificial intelligence.

DEA is an operations research model used to improve production quantities and measure efficiency accordingly. In data envelopment, a score between 0 and 1 is calculated for each bank. Efficiency with other banks is calculated with reference to this score. The most important feature of the DEA we know is that it works relatively well with small samples. DEA is also subject to several limitations. Some of these deficiencies assume that the DEA's measurements of data are accurate and are sensitive to inappropriate values.

Measurement of bank productivity is a subject that has been widely studied in recent years. The liberalization of the banking sector, financial crises and low interest rates had adversely affected the profitability of banks. In the area of bank productivity measures, the analysis of bank branches is particularly important to facilitate management control. Bank branches are one of the main cost sources for banks and contribute to a large portion of the value provided to the end customer, despite the digitalization of many banking transactions and the many transactions that can be made before arriving at the branches [1].

Methods of representing and measuring internally its multidimensionality remain a problem. Because of their complex and variable structure, the outputs and inputs of banking activities can be measured in different ways and productivity itself can be defined in different approaches. Various techniques have been used to address such issues: (a) income / total factor productivity or productivity ratios; (b) standard iterations that analyse the effect of a series of inputs on a given output; and (c) production or cost limit functions obtained by parametric and nonparametric methods. However, each approach has its own strengths and weaknesses [1].

If you have price data for inputs and/or outputs, you should have no problem estimating cost and/or profit efficiency. Cost effectiveness is conceptually a product of technical efficiency and allocation efficiency. As a result, cost efficiency demonstrates a bank's ability to provide services without wasting resources as a result of technical or mixed inefficiencies. The financial performance (FP) is very significant for the survivorship of a bank that is observed by various stakeholders: depositors, potential investors, management teams of banks and the central bank of nation creditors. Additionally, there are practical requirements for the prediction and exploration of the changes of performances of banks. The improvements of the FP can be supplied for supporting investment decisions, and the disruptions can be thought as warning signs for the prevention of financial crises. Because of its importance, various studies have been published based on the examination or prediction of the performances' changes of the banks. The analysis of the performance to branch-level was also widely examined by the researchers in the recent studies. While most of the researchers appear agreeing that the FP may be estimated based on the analyse of the historical data operational indicators and key financial ratios [2], [3].

In modeling nonlinear datasets and generating computational activity to find the optimal solution, computational intelligence approach has attracted considerable attention of researchers recently due to its solving capability. Data envelopment analysis (DEA) is an analysis technique used by banks to analyze performance and measure changes. [4]. In the DEA method, variables do not have probability distribution, but researchers need to choose input and output variables according to individual consideration. Shyu and Chiang [23] proposed a three stage DEA approach to measure bank branches efficiency in Taiwan. Paradi et al. [24] used two stage evaluation in DEA.

ANN has the ability to minimize errors in a sample set to be trained. However, it is very difficult to obtain understandable rules by looking at the weights and structure of a trained ANN. Neuro-fuzzy inference mechanism is a technique based on the working together of ANN and fuzzy inference system (FIS). In addition to this, ANN is able to learn complex nonlinear data set; but, the taken results from the ANN cannot assist for the explanation of the causal relationship between the target output and the considered variable. Besides, the FIS may bestow interpretability to reason imprecisely. With a high degree of accuracy and intelligibility if-then rules, fuzzy reasoning can be modeled by a combination of two complementary techniques. A neuro-fuzzy inference system often starts with an ANN and continues with a set of if-then rules and vice versa. Neuro-fuzzy inference systems have been widely applied in different fields, and multi-specific studies have been carried out to analyze banks' loans and identify business failures. Previous research has mainly focused on obtaining effective rules for identifying bad business or risky credit, and relatively little attention has been paid based on banks' FP analysis [5].

LaPlante and Paradi [1] evaluated bank branch growth potential using the DEA. Aghavi [6] developed a cost efficiency measurement using fuzzy DEA. Wu et al. [7] applied fuzzy data envelopment analysis method to measure cross region bank branches efficiency analysis. Puri and Yadav [8] applied Intuitionistic fuzzy data envelopment analysis for Indian banking sector. Kodogiannis and Lolis [9] proposed a fuzzy system-based technique to forecast financial time series. The literature based on fuzzy membership functions developed for fuzzy logic applications is examined in two categories as direct and indirect approaches. Polling, direct rating, reverse rating, interval estimation, pairwise comparisons and membership exemplification may be given as examples for the direct approaches. Direct approaches involve an expert, usage of subjective information and are expensive. The indirect approaches including neural-fuzzy techniques and fuzzy clustering methods, form objective membership functions from data. Further refinement of membership functions has been explored by several researchers. Several probabilistic programming approaches have been developed that take expert opinion into account. Human/expert participation is still required by most of membership elicitation approaches and a few of them are inclined to errors and failures, while they are useful. For example, expert intervention is needed for both neural-fuzzy techniques and fuzzy clustering to identify certain forms of fuzzy functions (Trapezoidal or Gaussian).

Various research studies have been presented by the researchers in different areas based on using a DEA and ANN. Athanassopoulos and Curram [10] presented a study comparing the benefits and drawbacks of the DEA and the ANN. In the case of a combination of continuous and discrete data, ANNs were allowed to take them into account without any additional changes necessary to the DEA. These advantages get ANN a convenient element in a two-step hybrid methodology composed of the DEA and ANNs since the advantages of the DEA may be utilized for processing the data and the ANN may implement the predictions because of the advantages of both. Wu at al. [25] used a combination of Data Envelopment and Artificial Neural Network systems to evaluate the efficiency of branches of a bank located in Canada. Hatami-Marbini et al. [12] published a paper about the reviews of fuzzy DEA methods and the presentation of the classification schemes evaluating many papers published in last 20 years. These kinds of approaches using DEA would be compulsory for the future of a hybrid DEA approach evaluating various dataset as an organ dataset used in Hatami-Marbini et al. study. Furthermore, these approaches are very important for the future research studies in this area. Liu et al. [13] presented a study of a system that provides a non-hybrid approach, using a very efficient DEA model to rank observations. The super-efficient DEA model does not yield results for multiple efficiencies. A study on the energy efficiency of an industrial sector organization was presented by Olanrewaju et al. [14]. DEA has been utilized for ranking the predictions of the ANN. In the study, they used real energy consumption as input and estimates as output. This will permit the efficiency of the predictions to be seen. An ANN has been trained by Bolat et al [21] for the prediction of the efficiency of the input-oriented DEA. Narin [22] used neuro-fuzy-DEA method to forecast bank branches efficiency.

In this study, data belonging to four bank branches were used. Unlike other studies, for each bank branch, the data for 2021 are predicted by using the artificial neural network for Total Loans Net of Provision Loans, investment, Deposits, Operational Expenses, Net Personnel Expenses, Personnel Expenses and Interest Rates Expenses. For the fuzzyfication process, the expertise of the branch managers has been applied. These estimated data have been fuzzyficated by the manager and assistant manager of each bank branch. Managers, who are experts in their fields, have determined that estimated data by ANN how much less and how much more can be. Fuzzy DEA has been applied to defuzzyficated data to measure efficiency of bank branches. In this study, real data of a Turkish bank has been used. But we do not say about which city of bank branches. The data is converted from Turkish money to any money unit due to secretion. This paper is organized as follows; in section 2, training ANN and forecasting process explained with data and training parameters. In section 3, Fuzzy Data Envelopment Models. In section 4, Fuzzy DEA is applied to the result of neuro-fuzzy forecasting system and calculated efficiency of bank branches for the next year.

2. Training of Artificial Neural Networks and Forecasting Process

We have total loans net of provision loans, Investment, Deposits, Operational Expenses Net of Personnel Expenses, Personnel Expenses, Interest Rates Expenses data for four bank branches. The data belongs to years 2016, 2017, 2018, 2019 and 2020. If the data for any year is input of the ANN, the data for the next year is output of the ANN. For example, if the data of 2016 are input, the data of 2017 are output. It means that a neuro-autoregressive model has been obtained. The ANN has been trained using data between 2016 and 2020 years. Four different ANN has been trained for four bank branches. To forecast the data of 2021, the data of 2020 has been given to ANN as an input. Designed ANN can be seen in Fig.2. Proposed solution system can be seen in Fig.1. Training set of the four ANNs for each bank branch can be seen in Table 1, Table 2, Table 3 and Table 4. The ANN training process for Bank Branch-1 can be seen in Figure 2. The ANN training parameters are given in Figure 3. Error plot and training stages of the ANN can be seen in Figure 4. Forecasted Data of 2021 for Bank Branches can be seen in Table 5.

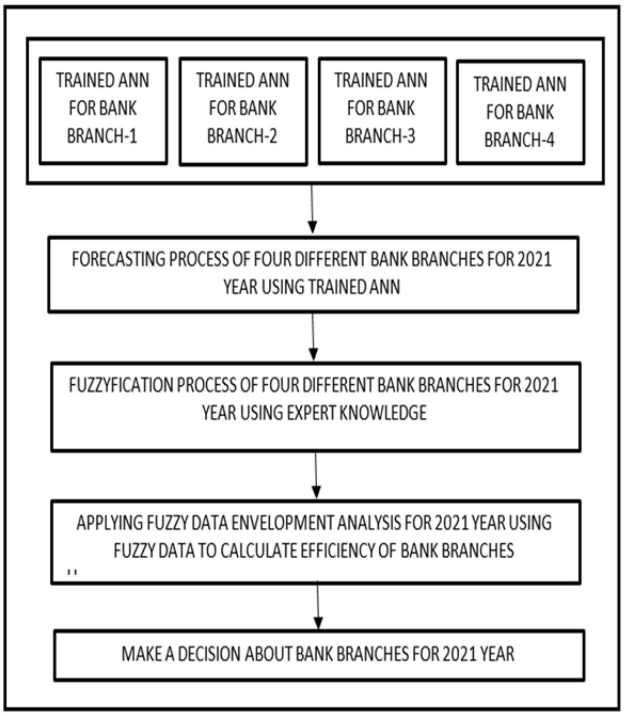

Figure 1. Flow diagram of the proposed solution model

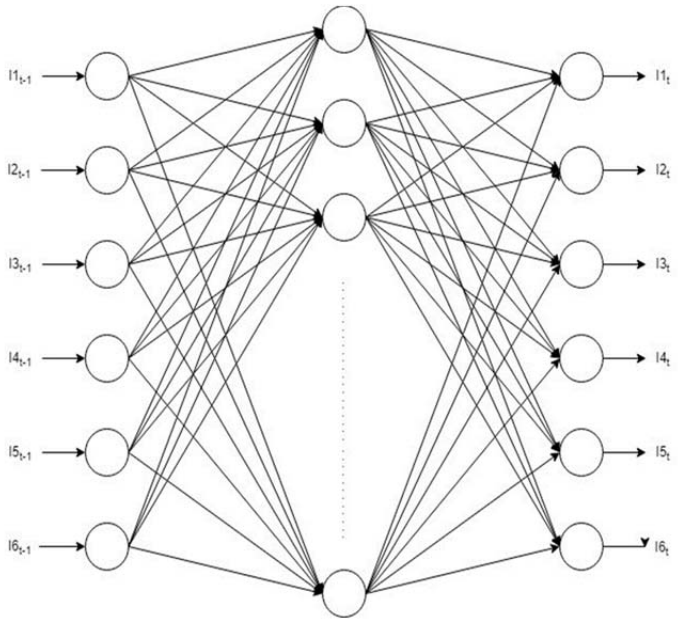

Figure 2. Structure of Designed Artificial Neural Network Model

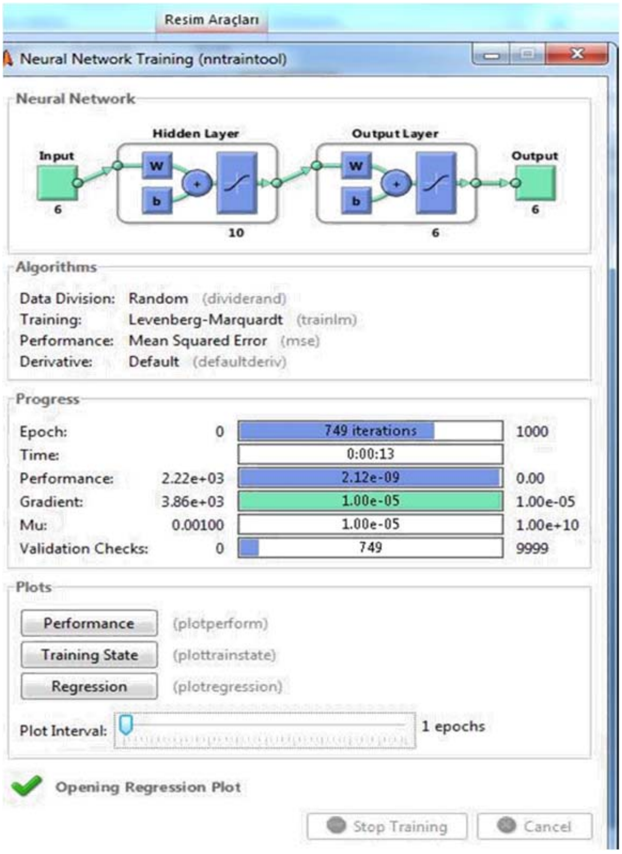

Figure 3. Training parameters of ANN

Table 1. Data of Bank Branch-1 which is used to train ANN

Year	Operational Expenses Net of Personal Expenses	Personnel Expenses	Interest Rate expenses	Total Loans Net of Provision Loans	Investments	Deposits
2016	471	4003	56	137	40	83
2017	470	4010	58	138	42	85
2018	465	4000	57	137	41	84
2019	450	3980	56	135	40	82
2020	484	4139	60	140	43	88

Table 2. Data of Bank Branch-2 which is used to train ANN

Year	Operational Expenses Net of Personal Expenses	Personnel Expenses	Interest Rate expenses	Total Loans Net of Provision Loans	Investments	Deposits
2016	377	1698	139	49	19	49
2017	380	1695	137	48	18	39
2018	375	1690	135	52	20	40
2019	370	1700	120	50	22	41
2020	384	1686	140	49	17	38

Table 3. Data of Bank Branch-3 which is used to train ANN

Year	Total Loans Net of Provision Loans	Investments	Deposits	Operational Expenses Net of Personal Expenses	Personnel Expenses	Interest Rate expenses
2016	154	855	29	46	9	25
2017	150	869	28	45	7	24
2018	156	874	26	48	9	28
2019	155	870	28	46	8	25
2020	157	879	27	47	9	27

Table 4. Data of Bank Branch-4 which is used to train ANN

Year	Total Loans Net of Provision Loans	Investments	Deposits	Operational Expenses Net of Personal Expenses	Personnel Expenses	Interest Rate expenses
2016	45	372	18	36	6	21
2017	47	370	20	35	6	22
2018	44	365	20	35	7	23
2019	42	360	18	34	7	21
2020	46	371	19	33	5	20

Table 5. Forecasted Data of 2021 year for Bank Branches

	FORECASTED INPUTS – 2021			FORECASTED OUTPUTS - 2021		
Bank Branch	Total Loans Net of Provision Loans	Investments	Deposits	Operational Expenses Net of Personal Expenses	Personnel Expenses	Interest Rate expenses
Bank Branch-1	482.99	4137.697	56.399	136.295	41.599	82.629
Bank Branch-2	349.666	1690.044	120.853	48.444	20.764	39.879
Bank Branch-3	155.305	872.153	28.66	47.493	6.78	24.316
Bank Branch-4	43.907	366.527	19.666	30.999	7.333	22.666

3. Fuzzy -Data-Envelopment Analysis

Performance measures and indicators mean to evaluate or measure the performance of a service, program or business. The DEA, which is frequently used in performance measurement, is very sensitive to data, so possible errors and uncertainties in the data affect the efficiency limits. The success of the DEA method is that inputs and outputs are exact values; It is due to the correct measurement. However, it is often difficult to measure precisely because a production process or service structure has complex inputs and outputs.

As an example of the change in input and output values in the DEA, it is possible to give the operational efficiency of airlines. Fuel and labor as inputs; As a printout, flight length can be given for each passenger in km. Weather, season, working situation etc. For reasons, inputs and outputs can be changed easily. The DEA, because it is a boundary method sensitive to extreme values, is quite complicated to evaluate efficiency with variable inputs and outputs with traditional DEA models.

Fuzzy Data Envelopment Analysis (FDEA) does not force the decision maker to make a precise formulation for mathematical reasons. The decision maker can take into account minor violations of restrictions and give different importance to violations of different restrictions [15].

3.1. Kao-Liu Model

The basic principle of the Kao Liu model is to transform fuzzy DEA models into traditional DEA models using different α-segments and Zadeh's extension principle. This model can only be applied to data with limited and precise values. The exact ranges of fuzzy input and output whose α-segments are taken are expressed as;

$$(\tilde{X}_{ij})_\alpha = \left[\min_{x_{ij}}\left\{x_{ij} \in S(\tilde{X}_{ij}) \middle| \mu_{\tilde{X}_{ij}}(x_{ij}) \geq \alpha\right\}, \max_{x_{ij}}\left\{x_{ij} \in S(\tilde{X}_{ij}) \middle| \mu_{\tilde{X}_{ij}}(x_{ij}) \geq \alpha\right\}\right] \quad (1)$$

$$(\tilde{Y}_{ik})_\alpha = \left[\min_{Y_{ik}}\left\{y_{ik} \in S(\tilde{Y}_{ik}) \middle| \mu_{\tilde{Y}_{ik}}(y_{ik}) \geq \alpha\right\}, \max_{y_{ik}}\left\{y_{ik} \in S(\tilde{Y}_{ik}) \middle| \mu_{\tilde{Y}_{ik}}(y_{ik}) \geq \alpha\right\}\right] \quad (2)$$

$$\mu_{\tilde{E}_r}(z) = \sup_{x_{ij}} \min\left\{\mu_{\tilde{X}_{ij}}(x_{ij}), \mu_{\tilde{Y}_{ik}}(y_{ik}), \forall_{i,j,k} \middle| z = E_r(x,y)\right\} \quad (3)$$

According to Zadeh's expansion principle, the activity of the r decision-making unit is defined as the membership function.

The proposed approach to construct the $\mu_{\tilde{E}_r}$ membership function is derived from α-segments. Accordingly, the r'th decision-making unit can create a membership function $\mu_{\tilde{E}_r}(z) = \alpha$ only if it meets the condition.

$min\left(\mu_{\tilde{X}_{ij}}(x_{ij}), \mu_{\tilde{Y}_{ik}}(y_{ik})\right) = \alpha$. Here, the lower and upper limits according to any part of the membership function of the r decision-making unit;

$$\left(\sum_{k=1}^{t} \mu_k (Y_{ik})_\alpha^L \Big/ \sum_{j=1}^{s} v_j (X_{ij})_\alpha^U\right) \leq 1 \quad (4)$$

The two-level mathematical model given as lower and upper bound can be made conventional one-level as follows. Accordingly, the lower and upper limit of the relative efficiency value in a certain α segment of the DMU to be compared are calculated as follows. The lower limit of the related DMU is created by using cluster values that minimize the output level and maximize the input level. The upper limit of the related DMU is created by using cluster values that maximize the output level and minimize the input level.

Based on this, the lower limit of the relative efficiency value of the r th DMU;
Constraints;

$$(E_r)_\alpha^L = max\left(\sum_{k=1}^{t} \mu_k (Y_{rk})_\alpha^L \Big/ \sum_{j=1}^{s} v_j (X_{rj})_\alpha^U\right) \quad (5)$$

$$\left(\sum_{k=1}^{t} \mu_k (Y_{rk})_\alpha^L \Big/ \sum_{j=1}^{s} v_j (X_{rj})_\alpha^U\right) \leq 1 \quad (6)$$

$$\left(\sum_{k=1}^{t} \mu_k (Y_{ik})_\alpha^U \Big/ \sum_{j=1}^{s} v_j (X_{ij})_\alpha^L\right) \leq 1 \qquad \boldsymbol{i \neq r}, \ i=1,\ldots,n \quad (7)$$

$$\mu_k, v_j \geq \varepsilon > 0$$

while expressing the relative efficiency upper limit of the same DMU:

$$(E_r)_\alpha^U = max\left(\sum_{k=1}^{t} \mu_k (Y_{rk})_\alpha^U \Big/ \sum_{j=1}^{s} v_j (X_{rj})_\alpha^L\right) \quad (8)$$

$$\left(\sum_{k=1}^{t} \mu_k (Y_{rk})_\alpha^U \Big/ \sum_{j=1}^{s} v_j (X_{rj})_\alpha^L\right) \leq 1 \quad (9)$$

$i \neq r$, i=1,…,n

$$(E_r)_\alpha^L = \min_{\substack{(X_{ij})_\alpha^L \leq x_{ij} \leq (X_{ij})_\alpha^U \\ (Y_{ik})_\alpha^L \leq y_{ik} \leq (Y_{ik})_\alpha^U \\ \forall_{i,j,k}}} \begin{cases} E_r = max\left(\sum_{k=1}^{t}\mu_k y_{rk} \Big/ \sum_{j=1}^{s} v_j x_{rj}\right) \\ Kısıtlar: \left(\sum_{k=1}^{t}\mu_k y_{ik} \Big/ \sum_{j=1}^{s} v_j x_{ij}\right) \leq 1 \quad i=1,\ldots,n \\ \mu_k, v_j \geq \varepsilon > 0 \end{cases} \quad (10)$$

defined as Kao and Liu, 2000) [11].

3.2. Sengupta Model

It is Sengupta (1992) who first used the fuzzy set theory in the DEA [16]. Sengupta (1992) [16] used fuzzy data envelopment analysis to measure the effectiveness of decision-making units in the case of lack of information and uncertain data. It has blurred the constraints and objective function of the CCR model of data envelopment analysis with fuzzy linear programming under uncertain data conditions. In this way, the relations of the DEA model were relaxed and flexibility in the model was provided [17]. In his study, he used three different fuzzy approaches: fuzzy mathematical programming, fuzzy regression and fuzzy entropy; For the fuzzy mathematical programming model, two different membership functions are discussed, namely "linear and nonlinear membership functions".

3.3. Cook-Kress - Seiford Model

Cook, Kress and Seiford [18] proposed a standard CCR model consisting of input variables containing only ordered data. The authors developed this model in 1996 and proposed an approach that includes precise and sequential data.

3.4. Saati-Memariani - Jahanshahloo Model

The CCR model created by using triangular fuzzy numbers was used in the model. In order to transform the fuzzy CCR model into a linear programming model consisting of exact numbers, α-cut sets were used. The model also suggests a ranking method for decision-making units using the fuzzy DEA approach [19].

4. Application of Fuzzy Data Envelopment Analysis for Bank Branches

The criteria and data to be used in practice are given in Table 5.1. Since the data is fuzzy, first how to convert it to a normal number, then what is fuzzy data and how it can be used. The fuzzy lower A membership function in the set X is defined by (μ_A)˜: x → [0,1]; where 〖μ〗_A˜:(x) value in x indicates the membership degree of x in A. The triangle can be described by a fuzzy number (a, b, c), where c≥b≥a.

The membership function is given by μ_A˜:(x) :

$$\mu_{\tilde{A}}(x): \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-C}{b-C} & b \le x \le C \\ 0 & others \end{cases} \qquad (11)$$

M is the center value, a is the left width and b is the right width. The membership function is as follows:

$$\mu_{\tilde{A}}(x): \begin{cases} \frac{x-M+a}{a} & M-a \le x \le M \\ \frac{M-x+\beta}{\beta} & M \le x \le M+\beta \\ 0 & diğer \end{cases} \qquad (12)$$

In order to do more calculations, it is necessary to convert the integer to fuzzy number. Kang et al. [20] proposed an efficient and easy method to convert an integer to a fuzzy predictive fuzzy number. The method of Kang et al. [20] is as follows:

Suppose an integer is Z = (A˜, B˜). Convert the second part (B˜) to a net number. Blur converts a blurry number to a net value. The most commonly used blur technique is the central blur method. This calculation is done using the equation.

$$a = \frac{\int x\mu_{\tilde{B}}(x)\,dx}{\int \mu_{\tilde{B}}(x)\,dx} \qquad (13)$$

B = (b1, b2, b3) if the equation is: α = (b1 + b2 + b3)/3. Definition of a weighted integer: by multiplying the z-value, the weighted integer is converted to the classic fuzzy number below:

$$\tilde{Z}' = \sqrt{a} * \tilde{A}^{a} = \left(\sqrt{a} * a, \sqrt{a} * b, \sqrt{a} * C, \sqrt{a} * d\right) \qquad (14)$$

In this way, the integer z is converted to a conventional fuzzy number. The data to be used in this study are in fuzzy number format. Therefore, it is multiplied by weights and the data is converted into a normal number (defuzzification). In order to get a better result, calculations will be made for a = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 weights and the results will be compared. Weights are processed. The forecasted input and output values can be seen in Table 6 and Table 7.

Table 6. Forecasted Fuzzy Inputs Values of bank branches

	FUZZY INPUTS								
	I1			**I2**			**I3**		
B1	480	483,697	485	4135	4138.036	4140	55	56.169	58
B2	280	283.999	385	1688	1690.132	1692	119	120.56	123
B3	153	155.915	157	870	872.46	874	27	28.98	30
B4	42	44.722	45	365	366.583	368	18	20	21

Table 7. Forecasted Fuzzy Outputs Values of bank branches

	FUZZY OUTPUTS								
	O1			**O2**			**O3**		
B1	135	136.885	137	40	41.798	43	79	83.886	85
B2	46	48.333	51	18	20.292	24	38	40.639	41
B3	46	47.481	49	5	7.34	8	22	24.948	26
B4	32	34.999	26	6	7	9	21	22.999	24

For Branch 1:

For α = 0.1, the clarified outputs and inputs are calculated;

Branch 1 in Table 5.1 data, for quality output

$\tilde{A}$ = (135,136,885,137) α = 0.1;
O1=((135x√0.1)+(136.885x√0.1)+(137x√0.1))/3 =43.1

Example of x2 calculation for α = 0.1:

Branch 1 in the data in Table 5.1, for its environmental cost output $\tilde{A}$=(40,41.798,43) α= 0.1;

O2=((40x√0.1)+(41.798x√0.1)+(43x√0.1))/3=13,155

Example of calculation for x3 for α = 0.1: Branch 1 in Table 5 data shows $\tilde{A}$ = (79,83,886.85) α = 0.1 for green supply chain management output;

O3=((79x√0.1)+(83.886x√0.1)+(85x√0.1))/3=26.159

Example of y1 calculation for α = 0.1:
Branch 1 in the data oin Table 5.1, for the cost input

$\tilde{A}$ = (480,483,697,485) α = 0.1;
I1=((480x√0.1)+(483.697x√0.1)+(485x√0.1))/3 =152.706

Example of y2 calculation for α = 0.1:
Branch 1 in the data in Table 5.1 shows that $\tilde{A}$ = (4135,4138,036,4140) α = 0.1 for green production input;

I2=((4135x√0,1) + (4138,036x√0,1) + (4140x√0,1)) / 3 = 1308,448

Example of y3 calculation for α = 0.1:
Branch 1 in Table 5.1 data, for occupational safety and health input $\tilde{A}$ = (55,56,169,58) α = 0.1;

I3=((55x√0.1)+(56.169x√0.1)+(58x√0.1))/3 = 17.832

Similar calculations are made for other branchs and other α values and other tables are created. Then, using inputs and outputs for each branch, mathematical models are built according to the CCR model in data envelopment analysis.

In the mathematical expression of this model,
M: number of entries
P: number of outputs
N: the number of DMU
It is expressed as. The mathematical expression of the weighted CCR model is as follows (Cooper et al., 2007) [26] :

Appendix: Relative efficiency measure

ur: k. r by the decision-making unit, weight given to output,
vi: k. by the decision-making unit I, weight given to input,
yrk: k. produced by the decision-making unit r. output,
xik: k. used by the decision-making unit i. entry,
yrj: j. produced by the decision-making unit r. output,
xij: j. used by the decision-making unit i. entry,
ε: A sufficiently small positive number (e.g. 0.00001),

$$E_k = max\left(\sum_{r=1}^{p} y_{rk} u_r\right) \quad (15)$$

This function is under the following constraints.

$$\sum_{i=1}^{m} x_{ik} v_i = 1 \quad (16)$$

$$\sum_{r=1}^{p} v_r y_{rj} - \sum_{i=1}^{m} x_{il} v_i \leq 0 \text{ j=1,2,...,N} \quad (17)$$

ur ≥ ε r=1,2,…..,p
vi ≥ ε i=1,2,…..,m

Defuzzyficated Input and Output values for α=0.1, 0.2, 0.3 can be seen in Table 8., 9. and 10.

Table 8. Defuzzyficated Input and Output values for α=0.1

	Outputs			Inputs		
	O1	**O2**	**O3**	**I1**	**I2**	**I3**
Şube 1	43.100	13.155	26.129	152.706	1308.448	17.832
Şube 2	15.320	6.566	12.611	100.033	534.439	38.217
Şube 3	15.019	2.144	7.689	49.112	275.799	9.063
Şube 4	9.803	2.319	7.168	13.885	115.906	6.219

***α* =0,1 mathematical models for α=0.1**

Branch 1 :

Maks. 43.100 a1 +13.155 a2 + 26.129 a3

Constraints:

43.100 a1 +13.155 a2 + 26.129 a3 - 152.706 b1- 1308.448 b2 -17.832 b3 ≤0
15.320 a1 +6.566 a2 +12.611 a3 -100.033 b1 - 534.439 b2 -38.217 b3 ≤0
15.019 a1 +2.144 a2 +7.689 a3 -49.112 b1 - 275.799 b2 -9.063 b3 ≤0
9.803 a1 +2.319 a2 +7.168 a3 -13.885 b1 - 115.906 b2 -6.219 b3 ≤0
152.706 b1 +1308.448 b2 +17.832 b3=1
a1 , a2 , a3 , b1 , b2 , b3≥0

Branch 2 :

Maks. 15.320 a1 +6.566 a2 + 12.611 a3

Constraints:

43.100 a1 +13.155 a2 + 26.129 a3 - 152.706 b1-1308.448 b2 -17.832 b3 ≤0

15.320 a1 +6.566 a2 +12.611 a3 -100.033 b1 -534.439 b2 -38.217 b3 ≤0
15.019 a1 +2.144 a2 +7.689 a3 -49.112 b1 -275.799 b2 -9.063 b3 ≤0
9.803 a1 +2.319 a2 +7.168 a3 -13.885 b1 -115.906 b2 -6.219 b3 ≤0
100.033 b1 +534.439 b2 +38.217 b3=1
a1 , a2 , a3 , b1 , b2 , b3≥0

Branch 3 :

Maks. 15.019 a1 +2.144 a2 + 7.689 a3

Constraints:

43.100 a1 +13.155 a2 + 26.129 a3 - 152.706 b1-1308.448 b2 -17.832 b3 ≤0
15.320 a1 +6.566 a2 +12.611 a3 -100.033 b1 -534.439 b2 -38.217 b3 ≤0
15.019 a1 +2.144 a2 +7.689 a3 -49.112 b1 -275.799 b2 -9.063 b3 ≤0
9.803 a1 +2.319 a2 +7.168 a3 -13.885 b1 -115.906 b2 -6.219 b3 ≤0
49.112 b1 +275.799 b2 +9.063 b3=1
a1 , a2 , a3 , b1 , b2 , b3≥0

Branch 4 :

Maks. 9.803 a1 +2.319 a2 + 7.168 a3

Constraints:

43.100 a1 +13.155 a2 + 26.129 a3 - 152.706 b1-1308.448 b2 -17.832 b3 ≤0
15.320 a1 +6.566 a2 +12.611 a3 -100.033 b1 -534.439 b2 -38.217 b3 ≤0
15.019 a1 +2.144 a2 +7.689 a3 -49.112 b1 -275.799 b2 -9.063 b3 ≤0
9.803 a1 +2.319 a2 +7.168 a3 -13.885 b1 -115.906 b2 -6.219 b3 ≤0
13.885 b1 +115.906 b2 +6.219 b3=1
a1 , a2 , a3 , b1 , b2 , b3≥0

Table 9. Defuzzyficated Input and Output values for α=0.2

	Outputs			Inputs		
	O1	**O2**	**O3**	**I1**	**I2**	**I3**
Şube 1	60.953	18.604	36.953	215.959	1850.426	25.218
Şube 2	21.665	9.286	17.835	141.468	755.811	54.047
Şube 3	21.240	3.032	10.874	69.455	390.039	12.817
Şube 4	13.863	3.280	10.137	19.636	163.916	8.795

Table 10. Defuzzyficated Input and Output values for α=0.3

	Outputs			Inputs		
	O1	**O2**	**O3**	**I1**	**I2**	**I3**
Şube 1	74.652	22.785	45.258	264.449	2266.300	30.886
Şube 2	26.534	11.373	21.843	173.263	925.675	66.194
Şube 3	26.013	3.713	13.318	85.064	477.698	15.698
Şube 4	16.979	4.017	12.415	24.049	200.755	10.772

Table 11. Comparison of efficiency according to different α level

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
B-1	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**
B-2	0,646	0,648	0,595	0,703	0,607	0,692	0,614	0,649	0,685
B-3	0,979	0,972	0,971	0,946	0,926	0,920	0,963	0,949	0,935
B-4	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**

It can be seen Bank Branch efficiency values according to different α level in Table 11. Branch-1 and Branch-4 efficiency for each α level efficient. Branch-2 and Branch-3 inefficiency for each α level efficient. Because efficiency level is less than 1. We can propose the Bank Branc-2 and Bank Branch-3 to be closed. Bank Branch-2 maximum efficiency level at α=0.4 and 0,703, Bank Branch-3 maximum efficiency value at α=0.1 and 0,979.

5. Conclusion

In this study, the efficiency of four different bank branches in a Turkish city have been analyzed based on fuzzy data envelopment technique. Fuzzy data envelopment analysis is most widely used technique for the measurement of the performances of the decision-making units. In this study, the data of the next year has been predicted by using the data of the previous year such as total loans net of Provision Loans, Investment, Deposits, Operational Expenses Net of Personnel Expenses, Personnel Expenses and

Interest Rates Expenses. As a prediction tool, an artificial neural network has been designed and used. For example, the data of 2017 has been used as input and the data of 2018 has been used as output. After the prediction of the data for 2021, the bank managers fuzzificated these predictions. Thus, the predictions obtained from artificial neural network have been gotten more realistic by using the contributions of the bank managers. After that, the Fuzzy Data Envelopment Analysis method has been applied to fuzzy data for 2021. The decision has been taken which bank branches will stay open or will be closed based on the measurement of the efficiencies of each bank branch.

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