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Teaching Short - Circuit Calculation with Off - Nominal Turns Ratio Transformers

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Abstract **–Teaching short-circuit analysis is conducted primarily through case studies; however, there are not many validated short-circuit studies available on the subject, especially when considering off-nominal turns ratio transformers. In order to improve the teaching of short-circuit analysis, a three phase short-circuit study in an industrial system according to ANSI/IEEE standards by means of Z-matrix method is presented; two case studies are considered: the industrial system with nominal and off-nominal turns ratio transformers, in both cases the step by step solution is given in an explicit manner and the analytical results are validated through software simulation.**

Keywords **– Short-circuit, Z-Matrix, off-nominal turns ratio transformer.**

1. Introduction

Power system analysis is a compulsory course of the electrical engineering career, its main goal is to provide students with a complete overview of interconnected power system operation. At the completion of the course students should be able to develop appropriate models for an interconnected power system, and know how to perform power flow, economic dispatch, and short-circuit analysis [1], [2].

Load flow and short-circuit analysis are necessary during the designing and planning of industrial power systems to select the equipment and improve the performance of an existing or proposed power system [3], [4], [5].

Nowadays, research applied to the teaching of power systems analysis is scarce, a large percentage of this research is based on creating software and problem base learning (PBL) [2], [6], [7]. However, it is not only necessary for the student to use software to solve the exercises, it is also important that they understand the fundamental concepts to achieve a good electrical engineer curriculum.

The fundamental concepts of short-circuit analysis can be learned through short-circuit calculation of industrial power systems using analytical methods. Ohmic, Point-to-Point, MVA and P.U. are the most commonly analytical methods used for calculating 3-phase fault current magnitude. Due to the size and complexity of many modern industrial systems, fault calculations can be impractically time consuming, this is the main problem with MVA and Ohmic methods because a new equivalent circuit need to be made for every faulted node [8]. The major obstacle with Point-to-Point method is the inaccuracy of the calculation methodology. On the other hand, the mathematical model of the electric power system is completely described by Z-Matrix. Due to the impedance matrix nature, the diagonal elements (driving-point impedance) correspond to the Thevenin impedance between nodes and ground; and the off-diagonal elements (transfer impedance) are denoted as the Thevenin impedance between nodes, consequently is convenient to use Z-matrix to calculate the current fault magnitude [9], [10], [11].

The main aim of the article is to present a detailed three-phase short-circuit current calculation in an industrial system according to ANSI/IEEE standards [12], [13], [14] by means of Z-matrix method. Case studies considering transformers with nominal and off-nominal turns ratio will be analyzed, the step by step procedure for the proper short-circuit calculation will be presented and finally the results will be validated through commercial software.

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2. Industrial System Data

The IEEE 13 bus industrial system selected for conducting the short-circuit analysis is depicted in Figure 1 and its data is given in Tables 1, 2 and 3. This system has been used in load flow and harmonics analyses [13], [14], [15].

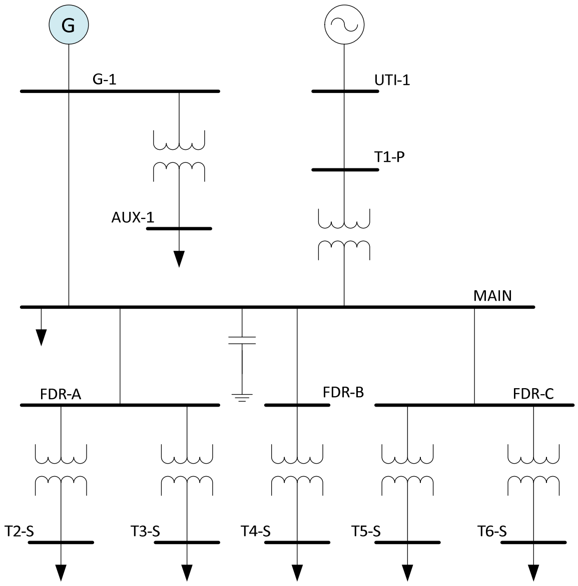

Figure 1. IEEE Industrial System

The system is fed from two sources: a utility supply at 69 kV and a local plant at 13.8 kV. Due to the nature of the analysis (balanced three phase fault); only positive sequence data is provided. Utility data is shown in Table 1.

Table 1. Utility supply data

Tag	Node	kV	MVA$_{SC}$	X/R
UTILITY 1	UTI-1	69	1000	22.2

The local generator connected at bus G-1 is represented as a Thevenin equivalent, sub-transient impedance (.0366 + 1.3651j Ω) is used as the equivalent impedance.

Table 2. Per unit line impedance data (Base values: 13.8 kV, 10 MVA)

Node A	Node B	R	X
UTI-1	T1-P	0.00139	0.00296
MAIN	G-1	0.00122	0.00243
MAIN	FDR-A	0.00075	0.00063
MAIN	FDR-B	0.00157	0.00131
MAIN	FDR-C	0.00109	0.00091

Table 3. Transformers data

Node A	Node B	kV	Tap	MVA	R %	X %
T1-P	MAIN	69:13.8	69	15	0.4698	7.9862
G-1	AUX-1	13.8:0.48	13.45	1.5	0.9593	5.6694
FDR-A	T2-S	13.8:0.48	13.45	1.25	0.7398	4.4388
FDR-A	T3-S	13.8:4.16	13.11	1.725	0.7442	5.9537
FDR-B	T4-S	13.8:0.48	13.45	1.5	0.8743	5.6831
FDR-C	T5-S	13.8:0.48	13.8	1.5	0.8363	5.4360
FDR-C	T6-S	13.8:2.4	13.11	3.75	0.4568	5.4810

It is important to notice that static load and capacitor data is not provided because they are not short-circuit current sources, and therefore can be omitted for short-circuit current analysis purposes [13], [14].

3. Short-circuit Current Calculation

In this section, the calculation of a three phase short-circuit current of an industrial system by means of Z-Matrix method is presented, considering: a) transformers with nominal turns ratio and b) transformers with off-nominal turns ratio. A step by step procedure for the calculation of short-circuit currents with nominal transformer tap settings is fully detailed in subsection nominal taps. Subsection off-nominal taps deals with the Y-Matrix modifications to take into account the off-nominal transformers tap settings.

Nominal Taps

The process for the calculation of a three phase short-circuit current by means of Z-matrix method is illustrated with a flowchart in Figure 2 [9], [10], [11].

The first step for the calculation of short-circuit current is to convert system values to per unit values, in this case the arbitrary base power selected was 10 MVA and the base voltages were chosen as the nominal voltage of the transformers. According to Tables 1, 2 and 3 not all quantities are expressed in per unit, so it is necessary to convert them. Utility supply impedance in p.u can be calculated using eqn (1) and data provided in Table 1:

$$Z_{pu} = \frac{V_{LL}^{2}}{S_{SC}} \cdot \frac{1}{Z_{base}}, Z_{base} = \frac{V_{base}^{2}}{S_{base}} \tag{1}$$

Where V_{LL} is the line voltage at node UTI-1 and S_{SC} is the fault MVA given in Table 1.

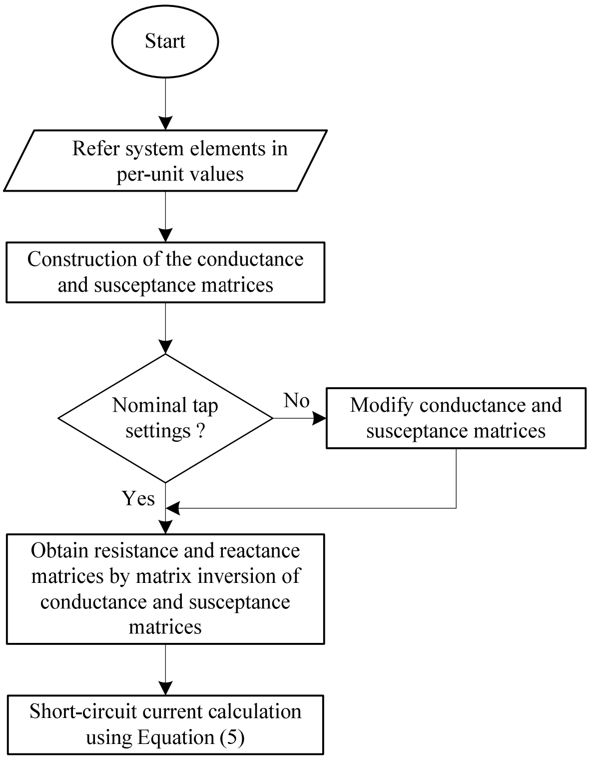

Figure 2. Algorithm for the calculation of a three phase short-circuit current by means of Z-matrix method

Table 2 includes line impedances in p.u for a base power of 10 MVA and base voltage of 13.8 kV, however the line from UTI-1 to T1-P is connected at a 69 kV node, then a change of base is necessary for this line (all other lines data remain unchanged). The change of base is calculated using eqn (2) where base power new and old are equal and then they cancel each other out

$$Z'_{pu} = \frac{(V_{LL,old})^2}{(V_{LL,new})^2} \cdot \frac{(S_{3\phi,new})^2}{(S_{3\phi,old})^2} \cdot Z_{pu} \qquad (2)$$

substituting values we obtain:

$$Z'_{pu} = \frac{(13.8\ \text{kV})^2}{(69\ \text{kV})^2} \cdot (0.00139 + j0.00296) = \\ = 0.0000556 + j0.0001184$$

Table 3 shows transformers impedance in p.u, however base powers are different from 10 MVA, then a change of base is necessary for these transformers. As an example, let us change the base of the transformer connected between buses T1-P and MAIN, the change is calculated using eqn (2) where base voltages new and old are equal and then they cancel each other out.

$$Z'_{pu} = \frac{10\ \text{MVA}}{15\ \text{MVA}} \cdot (\frac{0.4698 + j7.9862}{100}) = \\ = 0.003132 + j.053241$$

Once all the system elements impedance have been referred to the selected base quantities, the next step is the construction of the susceptance and conductance matrices. These matrices are constructed separately, the following steps have to be performed: first, the single line diagram is converted to conductance and susceptance diagrams, in each diagram only one equivalent element has to be connected directly between the buses, then the matrices are created in a straightforward manner: off-diagonal element Y_{ij} is the negative of the conductance or susceptance between nodes i and j and diagonal element Y_{ii} is the sum of all the conductance or susceptance terminating at the node (including the conductance or susceptance to ground at bus i) identified by the repeated subscripts as in [10], [11].

As an example, let us build a part of the susceptance matrix considering only the elements connected to the nodes G-1 (node 1) and AUX-1 (node 2), in matrix form which we get:

$$B_{bus} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \qquad (3)$$

and following the process previously described, matrix elements can be written as:

$$B_{11} = B_{gen} + B_{LT} + B_{tra} \\ B_{12} = B_{21} = B_{tra} \quad B_{22} = B_{tra} \qquad (4)$$

where generator, line and transformer were abbreviated as gen, LT and tra respectively.

Figures A.1 and A.2 (see appendix) show the complete conductance and susceptance matrices for the industrial system with nominal taps respectively. It is important to notice that nominal taps were considered for the construction of these matrices.

The next step is to obtain resistance and reactance matrices, they are computed by matrix inversion of conductance and susceptance matrices respectively. Diagonal elements of resistance and reactances matrices are important in short-circuit calculations because they represent Thevenin resistance and reactance.

Finally, short-circuit current is calculated by Thevenin equivalent method, in eqn (5), I_k represents short-circuit current at node k, V_k is the Thevenin voltage at node k, R_{kk} and X_{kk} represent Thevenin resistance and reactance at node k respectively [11].

$$I_k = \frac{V_k}{\sqrt{R_{kk}^{\ 2} + X_{kk}^{\ 2}}} \qquad (5)$$

As an example, let us calculate the short-circuit current at node AUX-1, according to Figure A.3 and A.4 (see appendix), which are the resistance and reactance matrices for the nominal case and considering pre-fault voltage as 1 pu [12], [13], [14], after substitution in (5) we get:

$$I_{AUX-1} = \frac{1}{\sqrt{0.065330^2 + 0.412262^2}} = 2.39575\ pu$$

Finally the conversion from pu to amperes is done by multiplying the current in pu by the base current of node AUX-1

$$I_{AUX-1} = 2.39575\ pu\left(\frac{10\,MVA}{\sqrt{3}\cdot 480\,V}\right) = 28.816\,kA$$

Off - Nominal Turns Ratio

Tap changers are indispensable in power transformers used in electrical energy networks and industrial applications. Tap changers enable voltage regulation by varying the transformer turns ratio, adding or subtracting turns from either the primary or the secondary winding. An off-nominal turns ratio transformer is required to compensate the varying voltage drops in the system and control reactive power flow. In industrial applications it often has 5 positions and the voltage between the taps lies around 2.5% of the rated voltage of the transformer; this tap changing modifies the value of the transformer impedance [16], [17], [18].

Calculation of short-circuit currents with nominal transformer tap settings following the guidelines previously presented is a straightforward task. However, in the real-life many electrical power systems have transformers operating at different rated voltages. Such transformer operating conditions introduce a mathematical problem. If the transformer has off-nominal turns ratio, its equivalent circuit is different from the standard transformer model. Figure 3 shows an equivalent model of an off-nominal turns ratio transformer [19], [20]. The transformer equivalent circuit comprises an admittance connected in series with an ideal transformer with turns ratio t. Transformer admittance is defined as the reciprocal of nominal impedance. The off-nominal position of the transformer's tap changer is located at node m.

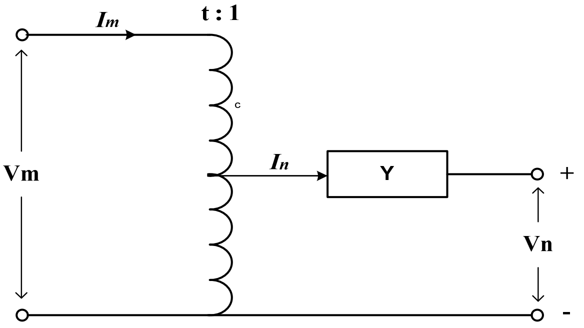

Figure 3. Off-nominal turns ratio transformer equivalent circuit

From Figure 3, V_m and I_m can be expressed as a function of V_n and I_n as:

$$V_m = t\left(V_n + \frac{I_n}{Y}\right)$$

$$I_m = \frac{I_n}{t} \qquad (6)$$

and eqns (6) are represented in matrix form as:

$$\begin{bmatrix} V_m \\ I_m \end{bmatrix} = \begin{bmatrix} t & t/Y \\ 0 & 1/t \end{bmatrix}\begin{bmatrix} V_n \\ I_n \end{bmatrix} \qquad (7)$$

In order to include the transformer with off-nominal taps within the Ybus of the system, it is necessary to represent it as an equivalent pi circuit and subsequently obtain the node admittances. The equivalent pi circuit of the off-nominal turns ratio transformer equivalent circuit is depicted in Figure 4 [19], [20].

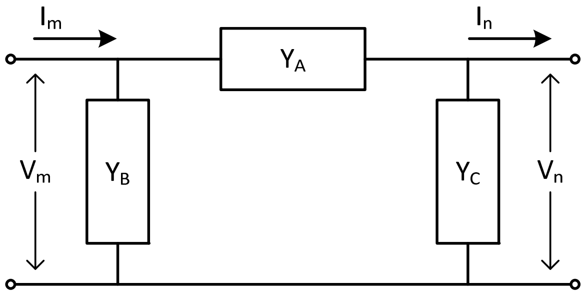

Figure 4. Equivalent pi circuit

From Figure 4, V_m and I_m can be expressed as a function of V_n and I_n as:

$$V_m = V_n + \frac{1}{Y_A}\left(I_n + Y_C V_n\right)$$

$$I_m = \left(V_n + \frac{I_n + V_n Y_C}{Y_A}\right)Y_B + V_n Y_C + I_n \qquad (8)$$

and eqns (8) are represented in matrix form as:

$$\begin{bmatrix} V_m \\ I_m \end{bmatrix} = \begin{bmatrix} 1+\frac{Y_C}{Y_A} & \frac{1}{Y_A} \\ Y_B + Y_C + \frac{Y_B Y_C}{Y_A} & 1+\frac{Y_B}{Y_A} \end{bmatrix}\begin{bmatrix} V_n \\ I_n \end{bmatrix} \qquad (9)$$

comparing term to term (7) and (9) we obtain:

$$\frac{1}{Y_A} = \frac{t}{Y} \quad 1+\frac{Y_B}{Y_A} = \frac{1}{t} \quad 1+\frac{Y_C}{Y_A} = t \qquad (10)$$

where the following identities are easily verifiable

$$Y_A = \frac{1}{t}Y \quad Y_B = \frac{1-t}{t^2}Y \quad Y_C = \frac{t-1}{t}Y \tag{11}$$

Finally, the nodal admittance matrix can be created in a straightforward manner: off-diagonal element Y_{mn} is the negative of the admittance between nodes m and n and diagonal element Y_{mm} is the sum of all the admittances terminating on the node:

$$Y_{mm} = Yt^{-2} \quad Y_{nn} = Y \quad Y_{mn} = Y_{nm} = Yt^{-1} \tag{12}$$

in matrix form which we get:

$$Y_{bus} = \begin{bmatrix} Yt^{-2} & Yt^{-1} \\ Yt^{-1} & Y \end{bmatrix} \tag{13}$$

In order to take into account the mathematical model of the off-nominal turns ratio transformer, the conductance and susceptance matrices (G-Matrix and B-Matrix respectively) are modified using eqn (13); where the transformer is represented between nodes m and n, and the tap changer is on the m side. Consequently, the matrix elements to be modified are the diagonal and off-diagonal elements associated with the nodes where the transformer is connected.

As an example, let us modify susceptance matrix elements linked with the off-nominal transformer connected between nodes G-1 (node 1) and AUX-1 (node 2)

$$B'_{bus} = \begin{bmatrix} B'_{11} & B'_{12} \\ B'_{21} & B'_{22} \end{bmatrix} \tag{14}$$

Equation (14) shows the modified susceptance matrix of the industrial system segment. This section comprises the local plant, a line transmission and an off-nominal turns ratio transformer connected between nodes G-1 and AUX-1. According to (13) and considering the transformer tap changer located at node G-1, matrices elements can be written as:

$$\begin{gathered} B'_{11} = B_{gen} + B_{LT} + t^{-2}B_{tra} \\ B'_{12} = B'_{21} = t^{-1}B_{tra} \quad B'_{22} = B_{tra} \end{gathered} \tag{15}$$

where:

$$t = \frac{V_{tap}}{V_{nominal}} \tag{16}$$

Since the tap changer is at node G-1, $t = 13.45\text{ kV}/13.8\text{ kV} = 0.9746$ and from eqns (14)-(16), modified B-Matrix becomes:

$$B'_{bus} = \begin{bmatrix} -j428.26 & j2.71 \\ j2.71 & -j2.65 \end{bmatrix}$$

Figures A.5 and A.6 (see appendix) show the complete modified conductance and susceptance matrices, which are calculated repeating the previous procedure for all off-nominal turns ratio transformers in the system.

Once modified matrices are calculated, resistance and reactance matrices are computed by inversion of modified conductance and susceptance matrices respectively, these matrices are shown in Figures A.7 and A.8 (see appendix). Finally, short-circuit currents are calculated using eqn (5) as described in nominal taps section.

4. Results

This section presents the analytical results of the short-circuit calculation and the simulations performed in ETAP for both the nominal and off-nominal cases.

Analytical calculation of three phase short-circuit currents of the system with nominal and off-nominal transformers tap settings is achieved using the algorithm presented in Figure 2 and all matrices used for these calculations can be found in the appendix.

Both analytical calculation and simulation are conducted according to ANSI/IEEE standards [12], [13], [14]. Usual assumptions are used: pre-fault voltage is 1 pu, shunt capacitor and static loads are neglected.

Results from analytical calculation and simulation for the three phase short-circuit study with nominal turns ratio are presented in Table 4.

Table 4. Comparison of analytical short-circuit calculations and ETAP results (nominal turns ratio)

Node	Isc Zbus (kA)	Isc Etap (kA)	Error %
G-1	12.187	12.174	0.11
AUX-1	28.816	28.811	0.02
MAIN	12.234	12.233	0.01
UTI-1	9.023	9.023	0.00
T1-P	8.924	8.924	0.00
FDR-A	11.998	11.997	0.01
T2-S	30.472	30.472	0.00
T3-S	3.627	3.627	0.00
FDR-B	11.747	11.746	0.01
T4-S	28.715	28.714	0.00
FDR-C	11.894	11.893	0.01
T5-S	29.939	29.938	0.00
T6-S	13.229	13.229	0.00

Table 4 shows close agreement between analytical results and simulations, maximum difference occurred at bus G-1, this error is due to software generator modelling rounding, however maximum error is only of the order .11%. The formula used to calculate the error is:

$$error\ \% = \left| \frac{I_{Etap} - I_{Zbus}}{I_{Etap}} \right| x100\% \qquad (17)$$

where I_{Etap} and I_{Zbus} represent the results obtained by software simulation and the analytical method respectively.

Analytical results of the short-circuit study match almost exactly with the simulation results, so it can be concluded that the methodology used is adequate for the short-circuit calculation with nominal taps.

Table 5. Comparison of analytical short-circuit calculations and ETAP results (off-nominal turns ratio)

Node	Isc Zbus (kA)	Isc Etap (kA)	Error %
G-1	12.187	12.174	0.11
AUX-1	**28.693**	**28.687**	**0.02**
MAIN	12.234	12.233	0.01
UTI-1	9.023	9.023	0.00
T1-P	8.924	8.924	0.00
FDR-A	11.998	11.997	0.01
T2-S	**30.331**	**30.331**	**0.00**
T3-S	**3.592**	**3.592**	**0.00**
FDR-B	11.747	11.746	0.01
T4-S	**28.586**	**28.586**	**0.00**
FDR-C	11.894	11.893	0.01
T5-S	29.939	29.938	0.00
T6-S	**12.958**	**12.958**	**0.00**

Once the methodology for the nominal case has been validated, we proceed to show the results for the off-nominal case. From Table 5 we can observe close agreement between analytical results and simulations. It is important to note that short-circuit current magnitude has changed at the nodes where off-nominal taps are considered (marked in bold), in all cases this current decreased as expected.

Figure 5 depicts the error between the simulations and the analytical method for the off-nominal case. It can be observed that for all the nodes where the tap position has changed, the percentage error has not varied between the nominal and the off-nominal case, remaining less than .02 %.

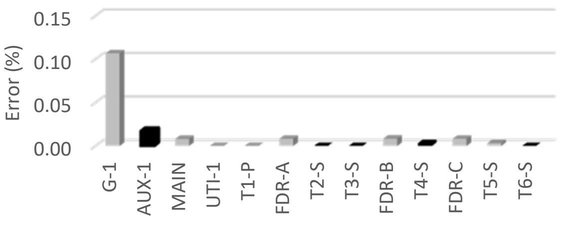

Figure 5. Short-circuit current percentual error for off-nominal case

Therefore, the results obtained through software simulation validate our analytical results and can be stated that the analytical procedure is adequate to model transformers with off-nominal turns ratio for three phase short-circuit studies by means of Z-matrix method.

5. Conclusion

A step by step procedure for three phase short-circuit current calculation for systems with nominal and off-nominal turns ratio transformers by means of Z-Matrix method was presented.

In order to validate the method, short-circuit studies were performed on an IEEE 13 bus system for nominal and off-nominal case. Analytical results were compared with software simulation and maximum difference due to rounding between analytical and simulation results was less than .02% for the nodes with off-nominal turn ratio transformers. These results validate the models and procedure used for three phase short-circuit studies by means of Z-matrix method.

This work provides a detailed short-circuit study case according to ANSI/IEEE standards, which can be useful for teachers, students or researchers in order to improve the teaching of this subject. The paper also contains all the matrices developed in the study (see appendix) and a clear exposition of how to model off-nominal transformers, therefore it can be used to teaching topics such as: admittance matrix construction, off-nominal transformer modeling, among others.

Future work may involve short-circuit calculation of asymmetrical faults with off-nominal transformers according to ANSI/IEEE standards.

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Appendix

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	1355.64	-15.64	-819.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AUX-1	-15.64	15.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MAIN	-819.67	0.00	4026.66	0.00	-319.28	-1333.33	0.00	0.00	-636.94	0.00	-917.43	0.00	0.00
UTI-1	0.00	0.00	0.00	20207.86	-17985.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T1-P	0.00	0.00	-319.28	-17985.6	18304.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FDR-A	0.00	0.00	-1333.33	0.00	0.00	1373.41	-16.90	-23.18	0.00	0.00	0.00	0.00	0.00
T2-S	0.00	0.00	0.00	0.00	0.00	-16.90	16.90	0.00	0.00	0.00	0.00	0.00	0.00
T3-S	0.00	0.00	0.00	0.00	0.00	-23.18	0.00	23.18	0.00	0.00	0.00	0.00	0.00
FDR-B	0.00	0.00	-636.94	0.00	0.00	0.00	0.00	0.00	654.10	-17.16	0.00	0.00	0.00
T4-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-17.16	17.16	0.00	0.00	0.00
FDR-C	0.00	0.00	-917.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1017.46	-17.94	-82.09
T5-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-17.94	17.94	0.00
T6-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-82.09	0.00	82.09

Figure A.1 Conductance matrix (nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	-428.12	2.65	411.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AUX-1	2.65	-2.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MAIN	411.52	0.00	-3879.87	0.00	18.78	1587.30	0.00	0.00	763.36	0.00	1098.90	0.00	0.00
UTI-1	0.00	0.00	0.00	-8546.05	8445.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T1-P	0.00	0.00	18.78	8445.95	-8464.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FDR-A	0.00	0.00	1587.30	0.00	0.00	-1593.02	2.82	2.90	0.00	0.00	0.00	0.00	0.00
T2-S	0.00	0.00	0.00	0.00	0.00	2.82	-2.82	0.00	0.00	0.00	0.00	0.00	0.00
T3-S	0.00	0.00	0.00	0.00	0.00	2.90	0.00	-2.90	0.00	0.00	0.00	0.00	0.00
FDR-B	0.00	0.00	763.36	0.00	0.00	0.00	0.00	0.00	-766.00	2.64	0.00	0.00	0.00
T4-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.64	-2.64	0.00	0.00	0.00
FDR-C	0.00	0.00	1098.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1108.50	2.76	6.84
T5-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.76	-2.76	0.00
T6-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.84	0.00	-6.84

Figure A.2 Susceptance matrix (nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	0.001377	0.001377	0.001031	0.000128	0.000143	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031
AUX-1	0.001377	0.065330	0.001031	0.000128	0.000143	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031	0.001031
MAIN	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.001686	0.001686	0.001686	0.001686	0.001686
UTI-1	0.000128	0.000128	0.000209	0.000420	0.000416	0.000209	0.000209	0.000209	0.000209	0.000209	0.000209	0.000209	0.000209
T1-P	0.000143	0.000143	0.000234	0.000416	0.000468	0.000234	0.000234	0.000234	0.000234	0.000234	0.000234	0.000234	0.000234
FDR-A	0.001031	0.001031	0.001686	0.000209	0.000234	0.002436	0.002436	0.002436	0.001686	0.001686	0.001686	0.001686	0.001686
T2-S	0.001031	0.001031	0.001686	0.000209	0.000234	0.002436	0.061620	0.002436	0.001686	0.001686	0.001686	0.001686	0.001686
T3-S	0.001031	0.001031	0.001686	0.000209	0.000234	0.002436	0.002436	0.045578	0.001686	0.001686	0.001686	0.001686	0.001686
FDR-B	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.003256	0.003256	0.001686	0.001686	0.001686
T4-S	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.003256	0.061542	0.001686	0.001686	0.001686
FDR-C	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.001686	0.001686	0.002776	0.002776	0.002776
T5-S	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.001686	0.001686	0.002776	0.058529	0.002776
T6-S	0.001031	0.001031	0.001686	0.000209	0.000234	0.001686	0.001686	0.001686	0.001686	0.001686	0.002776	0.002776	0.014957

Figure A.3 Resistance matrix (nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	0.034302	0.034302	0.033035	0.005209	0.005271	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035
AUX-1	0.034302	0.412262	0.033035	0.005209	0.005271	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035	0.033035
MAIN	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.034155	0.034155	0.034155	0.034155	0.034155
UTI-1	0.005209	0.005209	0.005386	0.009264	0.009255	0.005386	0.005386	0.005386	0.005386	0.005386	0.005386	0.005386	0.005386
T1-P	0.005271	0.005271	0.005450	0.009255	0.009365	0.005450	0.005450	0.005450	0.005450	0.005450	0.005450	0.005450	0.005450
FDR-A	0.033035	0.033035	0.034155	0.005386	0.005450	0.034785	0.034785	0.034785	0.034155	0.034155	0.034155	0.034155	0.034155
T2-S	0.033035	0.033035	0.034155	0.005386	0.005450	0.034785	0.389889	0.034785	0.034155	0.034155	0.034155	0.034155	0.034155
T3-S	0.033035	0.033035	0.034155	0.005386	0.005450	0.034785	0.034785	0.379927	0.034155	0.034155	0.034155	0.034155	0.034155
FDR-B	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.035465	0.035465	0.034155	0.034155	0.034155
T4-S	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.035465	0.414338	0.034155	0.034155	0.034155
FDR-C	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.034155	0.034155	0.035065	0.035065	0.035065
T5-S	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.034155	0.034155	0.035065	0.397465	0.035065
T6-S	0.033035	0.033035	0.034155	0.005386	0.005450	0.034155	0.034155	0.034155	0.034155	0.034155	0.035065	0.035065	0.181225

Figure A.4 Reactance matrix (nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	1356.46	-16.04	-819.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AUX-1	-16.04	15.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MAIN	-819.67	0.00	4026.66	0.00	-319.28	-1333.33	0.00	0.00	-636.94	0.00	-917.43	0.00	0.00
UTI-1	0.00	0.00	0.00	20207.86	-17985.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T1-P	0.00	0.00	-319.28	-17985.6	18304.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FDR-A	0.00	0.00	-1333.33	0.00	0.00	1376.81	-17.34	-24.40	0.00	0.00	0.00	0.00	0.00
T2-S	0.00	0.00	0.00	0.00	0.00	-17.34	16.90	0.00	0.00	0.00	0.00	0.00	0.00
T3-S	0.00	0.00	0.00	0.00	0.00	-24.40	0.00	23.18	0.00	0.00	0.00	0.00	0.00
FDR-B	0.00	0.00	-636.94	0.00	0.00	0.00	0.00	0.00	655.01	-17.60	0.00	0.00	0.00
T4-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-17.60	17.16	0.00	0.00	0.00
FDR-C	0.00	0.00	-917.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1026.33	-17.94	-86.41
T5-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-17.94	17.94	0.00
T6-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-86.41	0.00	82.09

Figure A.5 Conductance matrix (off-nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	-428.26	2.71	411.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AUX-1	2.71	-2.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MAIN	411.52	0.00	-3879.87	0.00	18.78	1587.30	0.00	0.00	763.36	0.00	1098.90	0.00	0.00
UTI-1	0.00	0.00	0.00	-8546.05	8445.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T1-P	0.00	0.00	18.78	8445.95	-8464.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FDR-A	0.00	0.00	1587.30	0.00	0.00	-1593.48	2.89	3.05	0.00	0.00	0.00	0.00	0.00
T2-S	0.00	0.00	0.00	0.00	0.00	2.89	-2.82	0.00	0.00	0.00	0.00	0.00	0.00
T3-S	0.00	0.00	0.00	0.00	0.00	3.05	0.00	-2.90	0.00	0.00	0.00	0.00	0.00
FDR-B	0.00	0.00	763.36	0.00	0.00	0.00	0.00	0.00	-766.14	2.71	0.00	0.00	0.00
T4-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.71	-2.64	0.00	0.00	0.00
FDR-C	0.00	0.00	1098.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1109.24	2.76	7.20
T5-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.76	-2.76	0.00
T6-S	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.20	0.00	-6.84

Figure A.6 Susceptance matrix (off-nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	0.001377	0.001413	0.001031	0.000128	0.000143	0.001031	0.001058	0.001085	0.001031	0.001058	0.001031	0.001031	0.001085
AUX-1	0.001413	0.065403	0.001058	0.000131	0.000147	0.001058	0.001086	0.001114	0.001058	0.001086	0.001058	0.001058	0.001114
MAIN	0.001031	0.001058	0.001686	0.000209	0.000234	0.001686	0.001730	0.001775	0.001686	0.001730	0.001686	0.001686	0.001775
UTI-1	0.000128	0.000131	0.000209	0.000420	0.000416	0.000209	0.000214	0.000220	0.000209	0.000214	0.000209	0.000209	0.000220
T1-P	0.000143	0.000147	0.000234	0.000416	0.000468	0.000234	0.000240	0.000247	0.000234	0.000240	0.000234	0.000234	0.000247
FDR-A	0.001031	0.001058	0.001686	0.000209	0.000234	0.002436	0.002499	0.002564	0.001686	0.001730	0.001686	0.001686	0.001775
T2-S	0.001058	0.001086	0.001730	0.000214	0.000240	0.002499	0.061748	0.002631	0.001730	0.001775	0.001730	0.001730	0.001821
T3-S	0.001085	0.001114	0.001775	0.000220	0.000247	0.002564	0.002631	0.045841	0.001775	0.001821	0.001775	0.001775	0.001868
FDR-B	0.001031	0.001058	0.001686	0.000209	0.000234	0.001686	0.001730	0.001775	0.003256	0.003341	0.001686	0.001686	0.001775
T4-S	0.001058	0.001086	0.001730	0.000214	0.000240	0.001730	0.001775	0.001821	0.003341	0.061714	0.001730	0.001730	0.001821
FDR-C	0.001031	0.001058	0.001686	0.000209	0.000234	0.001686	0.001730	0.001775	0.001686	0.001730	0.002776	0.002776	0.002922
T5-S	0.001031	0.001058	0.001686	0.000209	0.000234	0.001686	0.001730	0.001775	0.001686	0.001730	0.002776	0.058529	0.002922
T6-S	0.001085	0.001114	0.001775	0.000220	0.000247	0.001775	0.001821	0.001868	0.001775	0.001821	0.002922	0.002922	0.015257

Figure A.7 Resistance matrix (off-nominal turns ratio)

	G-1	AUX-1	MAIN	UTI-1	T1-P	FDR-A	T2-S	T3-S	FDR-B	T4-S	FDR-C	T5-S	T6-S
G-1	0.034302	0.035196	0.033035	0.005209	0.005271	0.033035	0.033896	0.034773	0.033035	0.033896	0.033035	0.033035	0.034773
AUX-1	0.035196	0.414073	0.033896	0.005345	0.005409	0.033896	0.034779	0.035680	0.033896	0.034779	0.033896	0.033896	0.035680
MAIN	0.033035	0.033896	0.034155	0.005386	0.005450	0.034155	0.035045	0.035952	0.034155	0.035045	0.034155	0.034155	0.035952
UTI-1	0.005209	0.005345	0.005386	0.009264	0.009255	0.005386	0.005526	0.005669	0.005386	0.005526	0.005386	0.005386	0.005669
T1-P	0.005271	0.005409	0.005450	0.009255	0.009365	0.005450	0.005592	0.005737	0.005450	0.005592	0.005450	0.005450	0.005737
FDR-A	0.033035	0.033896	0.034155	0.005386	0.005450	0.034785	0.035691	0.036615	0.034155	0.035045	0.034155	0.034155	0.035952
T2-S	0.033896	0.034779	0.035045	0.005526	0.005592	0.035691	0.391725	0.037570	0.035045	0.035958	0.035045	0.035045	0.036889
T3-S	0.034773	0.035680	0.035952	0.005669	0.005737	0.036615	0.037570	0.383685	0.035952	0.036889	0.035952	0.035952	0.037844
FDR-B	0.033035	0.033896	0.034155	0.005386	0.005450	0.034155	0.035045	0.035952	0.035465	0.036389	0.034155	0.034155	0.035952
T4-S	0.033896	0.034779	0.035045	0.005526	0.005592	0.035045	0.035958	0.036889	0.036389	0.416211	0.035045	0.035045	0.036889
FDR-C	0.033035	0.033896	0.034155	0.005386	0.005450	0.034155	0.035045	0.035952	0.034155	0.035045	0.035065	0.035065	0.036910
T5-S	0.033035	0.033896	0.034155	0.005386	0.005450	0.034155	0.035045	0.035952	0.034155	0.035045	0.035065	0.397465	0.036910
T6-S	0.034773	0.035680	0.035952	0.005669	0.005737	0.035952	0.036889	0.037844	0.035952	0.036889	0.036910	0.036910	0.185013

Figure A.8 Reactance matrix (off-nominal turns ratio)