Using Chess for Identifying and Correcting "Problem Areas" in the School Math Course

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***Abstract* – The conceptual position of the research is that the integration of mathematical and chess game activities ensures the effective development of cognitive and intellectual abilities of students, motivation to study mathematics, and productive development of its "problem areas". A technology for identifying and correcting the "problem zone" based on a chess game has been developed. All stages of the technology are presented on the example of mastering complex educational construct-combinatorial schemes. The effective influence of the chess game on the development of cognitive and mathematical abilities of students is theoretically and experimentally proved.**

***Keywords* – Teaching mathematics, Mind games, Chess, Integration, Combinatorial problems.**

1. Introduction

Solving a mathematical problem develops students' ability to operate in new conditions, to deal with uncertain situations and changing goals, finding and processing information necessary to master new branches of mathematics and the related knowledge [31], [32]. The proposed approach to selecting math problems and finding new methods for solving them allows expanding and deepening students' experience, forming and developing their intellectual operations and abilities based on the visual modeling that involves representation and correction of functional, operational, and instrumental competencies when mastering mathematics. The integration of math and chess is an effective didactic tool for achieving this goal [33].

The history of human development clearly demonstrates the effectiveness of the formation and development of intellectual and mathematical abilities in the process of playing chess. For example, in his research, the mathematician A. Poincare considered gaming as a type of mathematical creativity. When discussing mathematical thinking, he often referred to chess as a convenient model for studying individual thinking patterns. Poincare believed that by understanding the psychology of players and opponents, one can determine the steps in the game, as well as develop logical thinking and ideas, children's memory, which will have a positive effect on the learning process and understanding of mathematics during school [12]. Another famous mathematician, G. Hardy, gave a similar assessment of chess [15]. He believed that chess allows us to create a "mathematical melody", when understanding the algorithm and strategy of mathematical moves, it allows us to get an advantage in the game as quickly as possible. John von Neumann [21], who is the founder of game theory, considered chess as a visual model. Subsequent research in the field of gamification in education proves the continuity of the formation of intellectual operations of thinking (or universal educational actions) in the process of game practice as an element of non-formal education.

In recent years, the focus has shifted from theoretical to practical aspects of this integration. The Chess in School Committee has been operating in the International Chess Federation for more than twenty years. This body explores the potential of chess in increasing the efficiency of comprehensive school education. Many schools have introduced chess lessons, either in the form of extracurricular activities aimed at general intellectual development of children or elective courses. Currently, the impact of chess on the intellectual development of children, their cognitive abilities, and especially mathematical abilities, the relationship between playing chess and

DOI: 10.18421/TEM101-57
https://doi.org/10.18421/TEM101-57

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Received: 22 October 2020.
Revised: 07 February 2021.
Accepted: 12 February 2021.
Published: 27 February 2021.

understanding math are highly relevant and promising issues in the theory and practice of school math education.

However, the integration of mathematical and chess activities, despite its seemingly increasing popularity, is insufficiently studied, both in theoretical and practical aspects. The following questions remain unexplored:

- The influence of chess on the intellectual development and growth of motivational and emotional-volitional constructs of students in different age categories;
- Establishment of methodological foundations principles and guidelines for integrated teaching of mathematics and chess;
- Selection of tools, methods and forms for integrative mathematics teaching based on a chessboard;
- Identification of "problem areas" of mathematics, the development of which is most productive with the use of intellectual chess.

The above mentioned allowed us to formulate the research problem: What are the technological and didactic mechanisms for identifying and correcting the "problem zone" of mathematical education based on chess that ensure the effective development of cognitive abilities and the formation of subject competencies of students?

2. Literature Review

The integration of math and chess covers psychological, didactic, and social aspects. In this regard, the main idea is the focus of the entire teaching process on the intellectual and cognitive development of a student – the psychological aspect [3], [4], [13]. There are some Russian [3], [8], [13], [17], [27] and many international studies [4], [14], [20], [24], [25] that establish the relationship between students' cognitive abilities and their ability to play chess. Researchers G. Sala, J. P. Foley, F. Gobet [24], as well as A. M. Burjan [5], Zs. Duro [7] claim that teaching chess improves mathematical abilities of students in primary and secondary schools. For instance, they empirically proved that chess has a significant influence on the formation of a wide range of cognitive abilities, such as attention, concentration, planning, memory, logical thinking, and spatial imagination. The authors consider teaching chess an effective educational tool that has a positive cognitive effect on mastering math by children, both in the short and long term. However, a new study by the authors [16] seriously challenges the hypothesis of the "chess effect" in teaching mathematics. Also, S. Hong [16] and W. Bart [1], investigating students of age 8-12, having weak abilities (close to failing), concluded that chess education does not play a role in cognitive development.

According to other international researchers [7], chess is a complex and creative intellectual activity. The scientists obtained an unexpected result: the positive correlation between cognitive abilities and the ability to play chess in children and adults is more pronounced at the low level of chess skills, rather than at higher levels. Therefore, according to Russian and international psychologists and teachers, the introduction of chess into school math course enhances the development of cognitive abilities, increases educational motivation and the quality of mastering math knowledge and skills, and provides synergistic effects when gaining complex mathematical knowledge along with self-development and creative autonomy of the individual.

In the study [19], the interrelation between the determination of the expediency of the choice of chess moves with typical manifestations of cognitive dissonance and consonance, which naturally arise in the process of argumentation. The psychological phenomenon of cognitive dissonance and consonance in the field of argumentation during the game of chess is transformed into the appropriate orientation situations. The results of emotional-logical comparison with logical and internal conflicts are considered. This is evidenced by the subsequent choice, adequately or inadequately expressed in the emotional and behavioral manifestations of the players.

The social role of the integration of chess and math activities implies promoting chess, introducing this game into school course by establishing special organizations that provide additional education and contribute to the balanced development of school children [34]. Many schools now offer chess lessons, either as extracurricular activities aimed at general intellectual development or elective courses. In recent years, experts have devised many programs aimed at teaching chess in comprehensive schools. For instance, Lipetsk region has been running the program "General Chess Education (Chess in School)" since 2016, with over 44 schools of the region participating in it. This project is part of the system of compulsory school education, and playing chess is taught not by professional players, but by teachers. Since 2017, the "Education through Chess in School" project has been implemented in Poland. The project grew out of the need to increase primary mathematical skills. Playing chess improves logical and analytical thinking, problem solving skills, development of spatial orientation, concentration and many others. The project MATE (Multidimensional Analytical Training in Education), based on the chess game and aimed at developing cognitive skills, started in 2016 in schools in Poland, Spain, and Lithuania. Within the framework of this project, results were achieved in the field of chess training, neurobiology, formation and measurement of chess and cognitive skills. The National Core Curriculum [9] brought a revolutionary change with making chess an optional subject in schools. One argument for making chess a subject in schools is that the thinking of a chess player is complex, so knowing it can be beneficial for a large variety of problems or tasks.

Therefore, the educational aspect of the integration implies improving the content, forms, and methods of education aimed at developing the intellectual and cognitive abilities of students. The mathematical problem solving skill improves by learning many problem-solving strategies, which is met by relevant mathematical examples [8]. The chessboard and pieces are used to illustrate various math concepts and operations, for example, geometric shapes as well as odd parity, symmetry, coordinate system, and the rule of the square or triangle [10], [28], [29], [30]. The study of modern mathematical methods, for example, methods and directions of artificial intelligence algorithms (data analysis, methods of optimization of search, truncation of the search tree), is possible based on a chess game [2], [6]. Chess terms can be found in textbooks on combinatorics, graph theory, number theory, or theory of probability. Some educational aspects devoted to solving mathematical problems on a chessboard were investigated in the studies of G. Steinhaus [26], L. Ya. Okunev [22], M. Gardner [11], V. Poloudin [23] and E. Gik [12]. However, these methodological developments focus on the system of additional education.

At the same time, the basic math course has a set of specific problems or math “problem areas” representing the integrative complexes of relevant information, which, in our opinion, can be effectively mastered with a chessboard or chess pieces.

Having performed a comparative analysis of international and Russian publications on this topic, we identified:

- The research goal: to develop a method for identifying and correcting “problem areas” of teaching mathematics in the basic school course through playing chess;
- The research hypothesis: the method for identifying and correcting “problem areas” in teaching mathematics through chess ensures the effective development of cognitive processes when teaching mathematics and provides synergetic effects when mastering complex mathematical knowledge.

3. Methodology

The research of the problem was carried out in 4 stages (2018-2020 academic years):

- at the first stage (ascertaining stage), a theoretical analysis of synergetic approach and founding methodology, dissertation works on the problem, as well as theory and methodology of pedagogical research; the problem, purpose, and research methods are set out. Therefore, the experimental research plan is worked out;
- At the second stage (development of research methodology), a project was developed for a four-stage technology for research and correction of the "problem zone" in teaching mathematics, which consists in step-by-step disclosure of the complex essence of the generalized construct of school mathematics and its integration with the chess game; an integral methodological complex was developed, including diagnostics of cognitive processes (memory, thinking (logical component), attention properties).
- At the third stage of the experiment (the formative stage), experimental work was carried out on the introduction of integrative technology into the practice of training, and the conclusions obtained during the experimental work were analyzed, verified and clarified.
- At the final stage (control and diagnostic stage), the experimental work has been completed, theoretical and practical conclusions have been refined, statistical processing of the data has been carried out, obtained results have been summarized and systematized.

The comparative analysis of the pilot experiment involved students aged from 11 to 12. The experimental sample (n1=25) consisted of students who were taught math (“Combinatorics” section) using chess. The control group (n2=23) mastered math with traditional teaching methods. For the 2018 academic year, 48 children were admitted to the study, for the entire period - 135 people.

During the formation of the control and experimental groups, the requirements of uniformity, non-repetition, and representativeness were met. In particular, selecting control and experimental groups, the principle of creating representative groups was dominant, which is important for statistical processing of the research results. In particular:

1. For each age or class, an equal number of boys and girls were selected with an average level of achievement in mathematics and chess discipline;
2. Considered the same motivational figure, they expressed an interest in mathematics and chess;

3. The choice of teachers in these disciplines was carried out on the principle of the same rating (pedagogical knowledge and skills);
4. The factor of emotional stability and extroversion of the student was taken into account (according to G. Eysenck).

Participants in the control group were observed in natural conditions of the math learning process. For the subjects of the experimental group, special psychological and pedagogical conditions were created, among which we should highlight: training in purposeful visual modeling of complex mathematical problems on a chessboard; organization of training sessions that motivate students to freely search for information and the possibility of revealing their creative potential; setting of problematic interdisciplinary problems, the search for solutions to which is possible in the game intellectual space, etc.

Quantitative assessment on the effect of the game activity among students in the conditions of integration of chess training into mathematical education of school children, which was carried out on the basis of a developed holistic methodological complex, including diagnostics of memory, thinking and attention properties based on valid diagnostic methods that allow fixing this quality (Luria's “Operative memory” method, Lipman's “Logical regularities” method, Munsterberg's method). Assessment of the development of complex mathematical knowledge-combinatorial schemes was performed using quantitative methods of descriptive statistics used to identify differences in the level of formation of subject competencies (mathematical knowledge, skills). As a criterion, the average score of academic performance was selected based on the results of the intermediate certification on the topic “Combinatorics”.

The development of personality is impossible to imagine without the presence of situations of intellectual tension in the conditions of uncertainty and lack of future activities prediction, opportunities to overcome the problem areas that arise in integration of mathematical education and chess playing. The presence of such problem areas and situations of overcoming is directly related to the development of complex knowledge and is an important attribute of qualitative changes in the development of personality [35].

To determine the effectiveness of the developed technology, we used a multidimensional qualimetric tool that includes diagnostics of memory, thinking, and attention properties. Statistical data processing was performed using Pearson's $\chi 2$-test for each diagnosed component separately. Changes in dynamics were calculated based on the average level and integral level indicators. The same criterion was used to assess differences in the level of formation of subject results in mathematics in the control and experimental groups.

4. Results

“The problem area of math education is a set of content, procedural, and personality-adaptive components of teaching mathematics that imply the detection of contradictions and cognitive difficulties in a specific area as well as finding and researching the essence of its complex educational elements” [26]. Let us give an example of studying the “problem area” of a school math course – using chess to solve combinatorial problems. This methodology was tested in a secondary school in Lipetsk region.

When studying the elements of combinatorics in the basic school course, the main objective is to teach students an idea of variability, how to count and enumerate various options and their number that can arise in many practical and everyday situations. Later, combinatorial methods can be applied in high school and pre-vocational training. For example, combinatorial methods are actively used in probability theory, discrete mathematics, and in a wide range of applied problems. Mastery of this material contributes to the formation of students' functional literacy (the ability to perceive and critically analyse information, to understand the ambiguous nature of many real dependencies, and to do combinatorial calculations), acquiring the skills of deductive reasoning, and developing critical and probabilistic styles of thinking.

Let us consider a four-stage technology of studying and correcting the “problem area” in teaching mathematics that includes a stage-by-stage disclosure of the complex essence of the generalized construct of school mathematics and its integration with the chess game.

The main objectives of the first stage of the proposed method, which refers to preparation and organization are as follows: to identify students' difficulties in learning math; to discover the characteristics and preferences of students in cognitive activity (thinking, memory, attention, etc.); to form strong motives for searching and mastering new things when studying math.

The result of this stage was identifying and addressing the “problem area” of school mathematics. Next, it was necessary to establish and study the generalized construct of educational content and to adapt it to the students' knowledge of math. An example of this may be the problem of studying and mastering the basic combinatorial designs as it is difficult to describe the nature and content of the basic concepts of combinatorics (occupancy, permutation, and combination), to visualize and present abstract nature of calculations, or to understand the logical notation of combinatorial formulas. The essence of the generalized construct of "combinatorial designs" determines different approaches to their study: relying on intuitive ideas of students about the connection of elements, sets of different nature, combination, order of objects, or within the set-theoretic approach.

The motivational field: intuitive and visual modeling (video clips, presentations, role plays) of practical situations from real life by classifying objects according to characteristics corresponding to the main combination types (a legend about cutting a board with diamonds; a traveling salesman problem; a 36 officers problem; a 15 schoolchildren problem; a married couples problem, and a meetings problem).

Forms and means include: a research lesson; the analysis of particular situations; work in small groups; practical work with a chessboard; chess computer programs (Kvetka, Arcade Chess 3D, and Absolut Chess).

Chess problems to address the "problem area":

- Problem 1

How many ways are there to arrange eight rooks on the chessboard so that none of them can get the other?

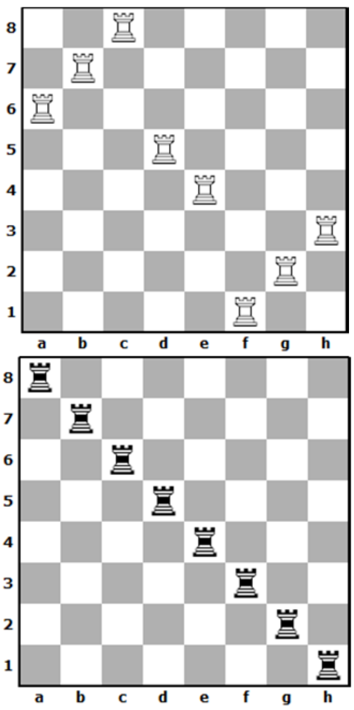


Figure 1. Problem 1. How many ways are there to arrange eight rooks on the chessboard so that none of them can get the other?
Source: Compiled by the authors.

• Solution: The rook can be placed in the first horizontal row in eight ways. After the rook is placed in the first horizontal row, there are only seven squares available to us in the second horizontal row (you cannot put two rooks in the same vertical row). The third horizontal row has only six squares, the fourth – five squares, etc. As a result, we get $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$, i.e. 40,320 possible ways.

Figure 1 present two possible arrangements on a chessboard with eight rooks so that neither of them can threaten the other.

- Problem 2

Determine the maximum number of bishops on the chessboard so that no two of them threaten each other.

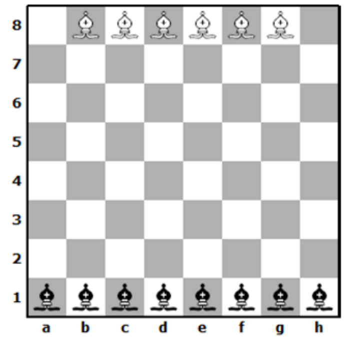


Figure 2. Problem 2. Determine the maximum number of bishops on the chessboard so that no two of them threaten each other
Source: Compiled by the authors.

• Solution: Let us consider the general solution to this problem. We will assume that there is a standard n×n chessboard. The number of diagonals in one direction is (2n-1), of which two diagonals are very short as each contains one (corner) square. These diagonals that have only one square cannot be simultaneously occupied by bishops, because otherwise the bishop scan attack each other along the main diagonal connecting the squares they occupy. Thus, the maximum number of bishops that can be placed on the chessboard so that they do not attack each other is (2n-2). Figure 2 offers one of the solutions to this problem for an 8×8 chessboard. The maximum number of bishops that can be placed on the chessboard so that no two of them threaten each other is 14.

- Problem 3

Determine the maximum ways of placing same-type pieces (queens, bishops, knights, or kings) on the chessboard so that they control all free squares of the board.

At the content and methodological stage, we examined how the generalized construct of the "problem zone" of school math can be adapted to the level of students' math knowledge and learning skills. In this case, one can express the generalized construct of the complex knowledge of the "problem area" (combinatorial designs) through mathematical modeling (by translating a real situation into the language of math). Students should be able to choose the most effective methods (geometric or algebraic) to solve the problem, to calculate and use simple combinatorial algorithms, and to do intelligent computing tasks.

The motivational field: mathematical modeling of real situations with combinatorial notation (the notation of combinatorial formulas). Forms and means: a research lesson, the analysis of particular situations; work in small groups; practical work with a chessboard; chess computer programs (Kvetka, Arcade Chess 3D, and Absolut Chess).

The chessboard problems for studying and correcting the “problem area”:

- Problem 1

Determine the number of ways to arrange eight rooks on the chessboard.

Solution: Using combinatorial formulas, we obtain:

$$C_n^k = \frac{n!}{k!(n-k)!},$$

where n=64, k=8;

$$C_{64}^8 = \frac{64!}{8!(64-8)!} = \frac{64!}{8!56!} = \frac{57\cdot 58\cdot 59\cdot 60\cdot 61\cdot 62\cdot 63\cdot 64}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8} = 4426165368.$$

- Problem 2

How many ways are there in which three rooks can be placed on the chessboard so that they do not beat each other?

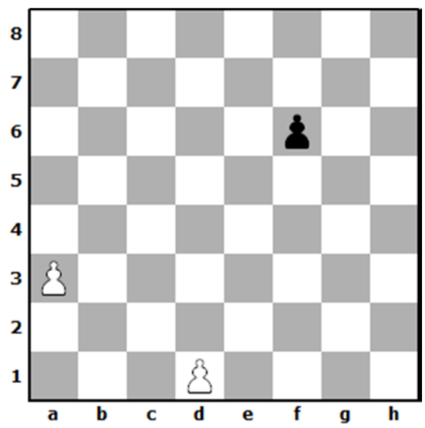
Solution: When solving a problem with k rooks that do not threaten each other on the mxn board, let us use the formula

$$C_m^k C_n^k = \frac{n!m!}{k!(n-k)!(m-k)!}$$

Under the conditions of the problem k=3, n=8, m=8; therefore,

$$C_8^3 C_8^3 = \frac{8!8!}{3!(8-3)!(8-3)!} = \frac{8!8!}{3!5!5!} = \frac{6\cdot 7\cdot 8\cdot 6\cdot 7\cdot 8}{1\cdot 2\cdot 3} = 18816$$

In Figure 3, there are only two ways for arranging the three chess pieces.



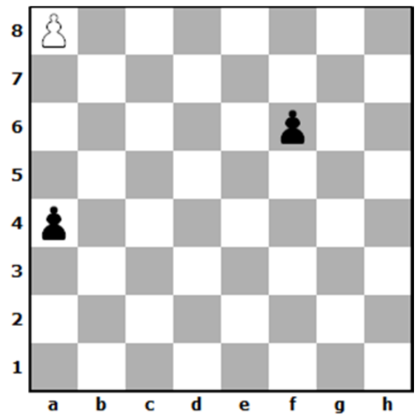


Figure 3. Problem 3. The two ways for arranging the three chess pieces
Source: Compiled by the authors.

- Problem 3

What is the largest number of pawns that can be placed on the chessboard if no two pawns can take the squares symmetric to e4?

- Problem 4

How many ways are there to place white and black kings on the chessboard so that this position does not contradict the rules of the game? Consider all possible positions of the white king on the chessboard.

- Problem 5 (Agakhanov’s problem)

There is a knight on an infinite chessboard. Having made four moves, it returned to the starting square, but has not occupied any square twice. How many ways could it do that?

- Problem 6

What is the largest number of squares that can be marked on the chessboard so that from any of them a knight could reach any other marked square in two moves?

The next stage includes assessment and correction of the proposed method of studying and correcting the “problem area” when teaching mathematics is concerned. The stage involves the continuous monitoring of the educational and cognitive outcomes of school children, the identification of positive and negative dynamics of the parameters and indicators of their learning and cognitive activity, as well as changes in the experience and personal qualities of a student. At this stage, we can recommend problems with variable conditions and data, with the assessment of the efficiency of the chosen solution, or problems with incomplete data. For example, if one introduces additional conditions into a classical combinatorial placement model, then using a chessboard one can demonstrate various cascades of bifurcation solutions.

- Problem 1

In how many ways can you place n rooks on the chessboard nxn that they do not threaten each other? The variations of the problem:

- In how many ways can you place n rooks on the chessboard nxn so that they do not threaten each other and none of them is on the main diagonal?
- In how many ways can you place n peaceful rooks on the chessboard nгn if k of them is white and n–k — black?
- In how many ways can you place n rooks on the chessboard nxn so that they threaten all squares of the chessboard?
- What is the largest number of rooks that can be placed on the chessboard nгn so that each of them is threatened by only one of the others?

- Problem 2

In how many ways can one place n peaceful rooks on then ×n chessboard, if k of them is white and (n − k) are black?
Problem variations:

1.1. In how many ways can you place four peaceful rooks on the 8×8 chessboard, if two of them are white and two are black?
1.2. In how many ways can you place eight rooks that cannot beat each other on an 8×8 chessboard?
1.3. In how many ways can you place four rooks on a standard chessboard?

The final stage of generalization and transformation of studying and correcting the "problem area" in teaching mathematics implies the transfer of the constructed combinatorial models to various areas of knowledge. Exploring a real practical problem, a student formulates various questions and problems, then "translates" them into the language of mathematics in order to solve it using combinatorial methods, and then interprets the solution in line with the problem posed. Solving real problems with combinatorial methods hones students' skills of creative research, increases their learning and cognitive motivation, intellectual self-development, and social adaptation. For example, students are given research tasks on modern scientific problems: architectural combinatorics (the study of constructing architectural shapes based on various combinations), combinatorics in programming (the study of various combinatorial algorithms for computers), or combinatorics of orbits (obtaining new solutions of combinatorial problems by transforming and isolating equivalence classes). Let us consider the latter in detail. When dealing with a combinatorial problem about the possible ways of placing eight rooks on a chessboard (Problem 1, stage 1), one can solve it in several ways by the following transformations: by rotating the chessboard by 90, 180, or 270 degrees, respectively; the axial symmetry regarding the main diagonals; and horizontal and vertical axes of symmetry of the chessboard. As a result, we obtain equivalence classes of possible arrangements transforming into each other, that is, combinatorial orbits.

To assess cognitive processes, we used a holistic methodological complex that included the diagnostics of memory, thinking, and attention properties [18]. We conducted a pilot experiment to test the developed technology. For easier comparative analysis, we split all diagnostic data into levels – low, medium, and high. We singled out three levels depending on the percentage distribution of the subjects, and the average level indicator (ALI) of each property was estimated by with the formula: $ALI = \frac{a+2b+3c}{100}$ where a, b, and c are the percentage of the number of subjects with a low (a), medium (b), and high (c) levels of a property, according to the diagnostic methods used. Table 1 presents the data of the comparative diagnostics for all structural components and their characteristics, including the average level indicator, as well as the integral level indicator (IFI) in the control and experimental groups.

Table 1. Diagnostic results of the cognitive sphere in the control and experimental groups

Diagnostics of cognitive processes		Stages of cognitive processes development, %						ALL	
		Low		Medium		High			
Components of the cognitive sphere	Diagnostic tool	Control groups	Experimental groups	Control groups	Experimental groups	Control groups	Experimental groups	Control groups	Experimental groups
Memory	Luria's method	16-18		19-21		22-24		1.57	2.16
		43.48	24	43.48	36	8.70	40		
Thinking (logical aspect)	Lippmann's method	0-6		7-13		14-20		1.65	2.16
		43.48	24	47.83	36	8.70	40		
Attention (selectivity, concentration)	Münsterberg's method	< 15		16-20		> 20		1.70	2.24
		43.48	20	43.48	36	13.04	44		
Integral Level Indicator								1.64	2.19

The statistical check with Pearson's chi-squared test demonstrated that the levels of cognitive processes development varied greatly for all components in the control and experimental groups (for the absolute frequencies of the property). The main testable hypothesis, which implied that there were no differences in the development of the cognitive sphere by individual components between the control and experimental groups, was rejected ($\chi^2_{emp} = 7.12 \geq \chi^2_{cr}(0.05;2) = 5.99$ for the first component memory; $\chi^2_{emp} = 6.79 \geq \chi^2_{cr}(0.05;2) = 5.99$ for the second component thinking; $\chi^2_{emp} = 6.55 \geq \chi^2_{cr}(0.05;2) = 5.99$ for the third component attention).

We identified a positive dynamic in the all of the students, which describes the level of cognitive processes development for the selected components, as well as significant changes in the integral indicator. All this allowed us to draw a valid conclusion that the introduction of chess into teaching math has a positive effect on the development of all examined indicators.

To assess the subject results of learning in mathematics, we divided the children of the control and experimental groups into three groups according to their academic achievements, including high achievements (NA), middle achievements (MA), and low achievements (LA). Next, we compared the group differences based on the empirical data obtained. The results of statistical testing using the Pearson's22-test showed that differences in the level of subject knowledge of students in the control and experimental groups are statistically significant ($\chi^2_{emp} = 8.51 \geq \chi^2_{cr}(0.05;2) = 5.99$). The number of students with high and medium level of subject competence formation in the experimental group is significantly higher than in the control group, and vice versa: the level of students with low academic achievement is significantly lower than in the control group.

Additionally, we would like to note that the introduction of chess into the learning process allowed us to reveal the motivational and social effect of learning mathematics (Figure 4).

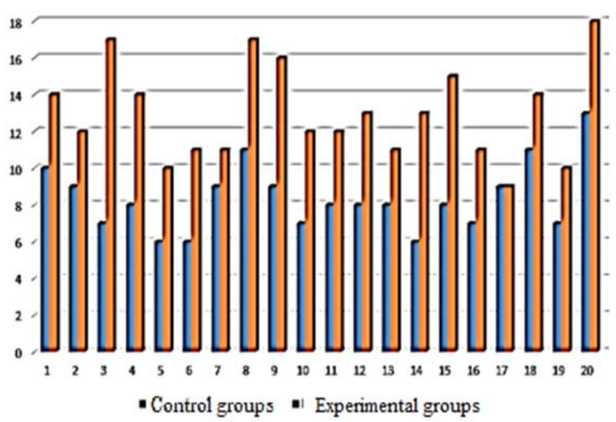


Figure 4. Motivation to study mathematics
Source: Compiled by the authors.

5. Discussion and Conclusion

The problem of integration of mathematical and chess game activities has been solved by researchers before. However, its solution was based on the development of intellectual operations, reflexive abilities, the formation of tolerance to innovation and the ability to predict the results of mathematical activities. Didactic mechanisms and technologies were developed that lead to creative thinking and reflection of one's own decisions. It is established that the integration of logical and intuitive-figurative, characteristic of the chess game, the harmonious construction of this synergy is an effective mechanism not only for the intellectual, social and personal, but also for the creative development of the student. Reasoned solutions of mathematical problems on a chessboard stimulate not only creative independence, but also the activity of structural and functional components of synergy. In particular, based on internal cognitive consonance, non-standard original ideas are identified; by overcoming emotional instability (cognitive dissonance), the logical component of argumentation is strengthened.

This study not only offers a new didactic mechanism for actualizing creativity, cognitive and intellectual abilities of students, motivation to study mathematics, but also involves identifying and correcting the "problem zone" of mathematical education through the introduction of funding complexes of mathematical problems on the chessboard. Mastering the "problem areas" of mathematics was carried out by developing the ability to argue, reflect on their own judgments in the process of solving multi-stage mathematical problems on the chessboard, which will expand the possibilities of personal and creative potential development, self-organization and self-regulation in the process of integrating chess training into mathematical education. In the process of identifying the structural components of cognitive processes, psychodiagnostic tools for diagnosing memory, thinking, and attention properties were developed, and comparative diagnostics was performed for all structural components.

Our research is consistent with the conclusions of philosophers, educators and psychologists (I. Prigozhin, S. P. Kurdyumov, G. Haken, K. Mainzer, A. N. Podyakov, V. S. Stepin, I.S. Utrobin and others), claiming that effective personal development occurs when mastering complex knowledge (different levels of its complexity depending on the personal development of students), creating situations of overcoming difficulties in the process of mastering knowledge and a unified picture of the world are based on a high degree of deployment of educational motivation of students in a single network of interactions, independence and coherence. On the other hand, the history of human development clearly demonstrates the effectiveness of the formation and development of human functional capabilities in the process of gaming. The established relationship between the development of personality in the process of integrative learning of mathematics and game intellectual activity opens up broad prospects for further understanding of the concept of personal development, studying issues related to the development of system thinking.

Having conducted the research, we obtained the following results:

1. We theoretically substantiated the possible influence of chess on the development of students' general and math cognitive abilities and explored the relationship between chess and math skills. We focused on the "problem areas" of school mathematics (for example, combinatorial designs, symmetry, and coordinate system) and demonstrated that they can be successfully mastered with a chessboard or chess pieces.
2. We devised a complex method for identifying and correcting "problem areas" in teaching mathematics based on chess. According to this method, cognitive processes are actively developed in a consistently organized educational process, which includes such stages as preparation and organization, content and methodology, assessment and correction, and generalization and transformation.
3. In order to test the effectiveness of the proposed methodology, we developed a multidimensional qualimetric apparatus aimed at the diagnostics of the main components of the cognitive sphere (memory, thinking, and attention properties). The obtained statistical results proved the effectiveness of the devised method.

The introduction of combinatorial tasks on the chessboard into the mathematics course activates the cognitive processes of students and brings about synergistic effects (motivational and social), which increases the developmental and educational potential of secondary school.

4. Further research can be done in the following areas:

- Improving the methodological educational materials for other topics of school math course with chess;
- Conducting further long-term experimental studies in order to confirm (or possibly refute) the hypothesis of the study.

Further research is planned in the field of methods of teaching in schools and universities to identify and correct other complex topics and sections of mathematics, as well as for the system of inclusive mathematical education based on the chess game. The development of chains of research tasks in mathematics on a chessboard with the actualization of synergy attributes and the manifestation of a creative effect will be implemented with the content of the author's hybrid intellectual learning system in the model of support and support of research activities of schoolchildren.

6. Recommendations

Based on the study, recommendations were developed for the formation of complex mathematical knowledge in the context of modeling a game developmental space based on a chess game:

- To introduce aspects of independent practical and research activities of children through playing activities;
- To conduct further long-term observations of the process of integrating a chess game into the process of teaching mathematics, which implements aspects of visual modeling, identification and correction of complex mathematical knowledge;
- To expand the range of intellectual games (Zhipto, Checkers, Go, Renju) with their subsequent introduction into the practice of teaching mathematics. Reveal the effects of the introduction of other intellectual games, including the deepening of subject competencies in complex areas of mathematics (Algebra, Geometry, Discrete mathematics, Probability theory), the development of research skills, nonlinear thinking, motivation, and creativity.

Acknowledgements

The authors express their gratitude to the administration of the Federal State Budgetary Educational Institution of Higher Education "I.A. Bunin Yelets State University" and personally to Rector E.N. Gerasimova, Doctor of Pedagogical Sciences, Professor, for her assistance in conducting this research and publishing the article.

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