Students' Ability to Solve Geometry Problems with Emphasis on Interdisciplinary Relations

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Abstract **– In this paper we focus on the interesting relationship of geometry and music from the perspective of problem-based learning. This teaching has to be designed with the intention of supporting students' self-education. We are looking for a connection between teaching geometry, especially congruent transformations in plane and selected musical works. We also provide specific examples of these works and analysing the occurrence of congruent transformations in them. Subsequently, we created tasks for students from this field. Our interpretations of the findings from student solutions are also presented.**

Keywords **– Interdisciplinary relations, Congruent transformations in plane, Musical works, Worksheet, Problem-based learning.**

1. Introduction

For the interaction of mathematics and music we can help at the beginning with one quote from a master violinist and also the teacher Novak: "Music is only one branch of mathematics". But in fact, mathematics and music are truly independent. We do not even realize how much mathematics can be found in the music works itself. And it can also be found in music notation, in the composing of musical instruments or in various sounds. The aim of our work was to find this connection of mathematics and musical works.

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At first, we did not believe that the relationship between mathematics and music is so deep. We focused only on geometry, specifically for interesting congruent transformations in plane. Another of our aim was to present this theme to students, to put into the teaching process interesting geometric tasks somehow related to music works.

We have noticed that students are accustomed to working in mathematics lessons exclusively with basic geometrical figures (segment, line, triangle, rectangle, square or circle). This fact we have also found in teaching of congruent transformations. Students do not consider at other shapes in which we can look for the elements and properties of these transformations. So we wanted them to point out the occurrence of congruent transformations in musical works as well as in different parts of musical instruments.

Many of researches published in Slovakia were focused on congruent transformations in plane (more see [1], [2], [3]). This part of geometry is interesting, but difficult for students, so it is advisable to teach it using problem-based tasks. As well as official document called "State Educational Program – Mathematics ISCED 3A" specifies the mathematical standards at Slovak secondary schools. Standards also set the objectives, such as to lead students to discover congruent transformations in the world around us or to appreciate the importance of symmetry in everyday life, especially in creating an aesthetic environment [4].

2. General Background

Generally, geometry is one part of mathematics that students do not like. But just, teaching geometry can be made interesting and non-traditional.

Students have to realize that understanding in geometry is more important than memorizing. Generally, it applies that there is more chance for success in building for long term understanding rather than just when students learning only facts. An example of such learning is active learning.

For our needs active learning means that students take increasing responsibility for their learning. Of course there is a teacher, who is mainly activator of learning, rather than lecturer or deliverer of solutions.

In a "traditional" learning, it is common for some students to participate with solution of problems in only form of asking or responding to questions to their teacher. Active learning is generally defined as any instructional method that engages students in the learning process. In short, active learning requires students to do meaningful learning activities and think about what they are doing in the classroom [5].

As author [6] says the teacher has several suitable methods of active learning to present new problem, but also to practice and apply this problem, as well as to reflect on it. We focus on problem-based learning method. We can say that elementary and secondary schools in Slovak republic have adopted problem-based learning.

All good teaching is characterized by proper teaching methods, and priority may be given to improve the capacities and professional competencies of the teachers to apply appropriate teaching methods that may enhance learning of students [7].

Problem-based learning is a model which centered on students, develops active and motivated learning, problem solving skills and broad field knowledge, and based on the deep understanding and problem solving [8]. Students in problem-based learning environments typically have greater opportunity to learn mathematical processes associated with communication, representation, modeling, and reasoning ([9], [10], [11], [12]).

A condition of successful realization of problem instruction is the orientation on the basic structure of the teaching material, setting reasonable, unequivocal and controllable goals in instruction, use of general algorithms and heuristic processes, students' motivation and a differentiated approach to them [13].

3. Methodology and Materials

In this paper, we examine how interesting problems were used to support the problem-based learning goals. Students construct explanations of issues focused on congruent transformations in plane and also reason effectively. In this process students become self-directed learners while maintaining a student-centered learning process.

Therefore, when preparing pupil activities, it was necessary to do:

- focus on activities with the possibility of involving all students,
- focus on achieving objectives,
- prepare problems demanding for higher thinking processes,
- accept the diversity of students in their learning styles.

We used the PAR method in the teaching process. Teaching consists of three phases: Present, Activities, Review (see more in [14]). All three elements are needed. In the first phase the learning goals were explained to students, also knowledge, reasoning or theories are presented to them, too. Theoretical knowledge we added with concrete examples and presented it with active involvement of pupils. Then students got geometrical problems that require them to apply their knowledge or theories of congruent transformations in plane (second phase). These were practical tasks to link mathematical knowledge to music-related tasks. We tried to praise and encourage all students, not just high attainment, but also their effort or progress. Finish phase is especially important at the start or finish of topics. We decided to test as learning strategies for students.

In order to confirm student knowledge of congruent transformations in plane, we have included geometrical tasks that were somehow connected with music or musical instruments (see Figure 1).

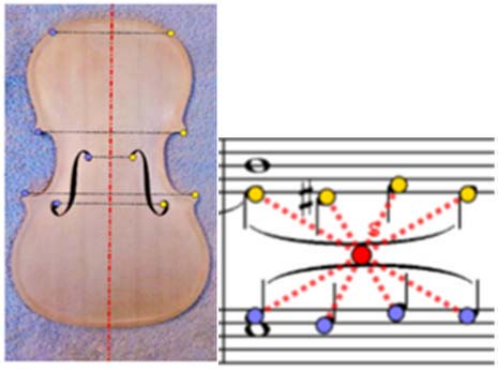

Figure 1. Example of symmetry on a musical instrument and between notes

Subsequently, we prepared a test for them where the described interdisciplinary relations were used. The tasks in this test were focused on translation, rotation and reflection.

4. Results of our Findings

Our tasks were solved by 34 students who are not musicians. That is, they do not know musical notes and they see the notes only as a form, symbolism, something unknown, unreadable. Musical notes can be seen as musical letters, but also a specific geometric figure, so we could try to work with them in the lessons of mathematics.

Text of the test was colorful and should motivate students to a positive attitude to work with it. We will present examples of students' solutions and we used formative assessment.

In the first task, students were to draw notes that are symmetrical about the o_1-axis and the o_2-axis (solution is in the Figure 2).

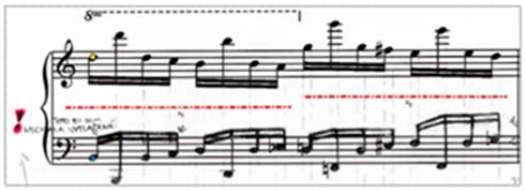

Figure 2. Example of correct student´s solution of the 1st task

Students solved the task very well, half the pupils had a 100% solved it. The rest of the students were largely wrong in not noticing the two symmetry axes, but only the first one, and therefore displayed all the musical notes along the o_1-axis.

In the second task, students had to require to mark musical notes in the picture (Figure 3) that are symmetrical according to the given axes.

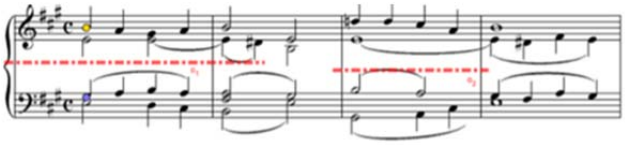

Figure 3. Picture for solution in the 2nd task

Accurate construction and measurement was required in the task; from our findings many pupils did not want to do this. Finally, only six students did not resolve it correctly. We offered students that they can look for another solution to the task, respectively other symmetry. Of all pupils, only one student tried to do so (Figure 4).

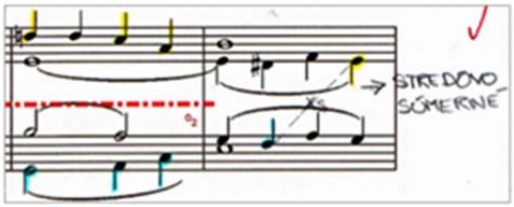

Figure 4. Part of task solution

Musical instrument was also used in the test (3rd task). The task was to finish the second half of the violin bridge as accurately as possible. Two thirds of pupils enjoyed the task very much and did not mean that it is anything exceptionally difficult for them. This is also evidenced by the results of their work, 23 pupils (it is 69 percent) solved this task perfectly. The correct solution of the student is presented in the Figure 5.

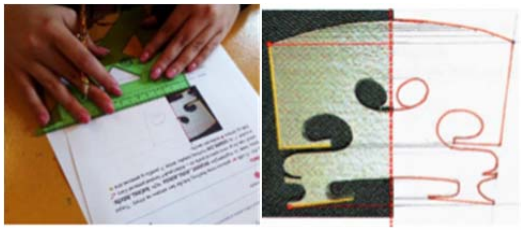

Figure 5. Student´s work

Other task (4th task) was focused of translation. The pupils have been specified translation vector and the musical notes that create congruent transformations. Their task was to correctly identify from a given number of musical notes exactly those for which the given translation. Pupils worked on this task mostly without questions. As many as nineteen pupils considered the task simple and for eight pupils this task seemed appropriate given their knowledge. This was also reflected in the success assessment, where 25 pupils solved the task excellently (see one of the student's solutions in Figure 6).

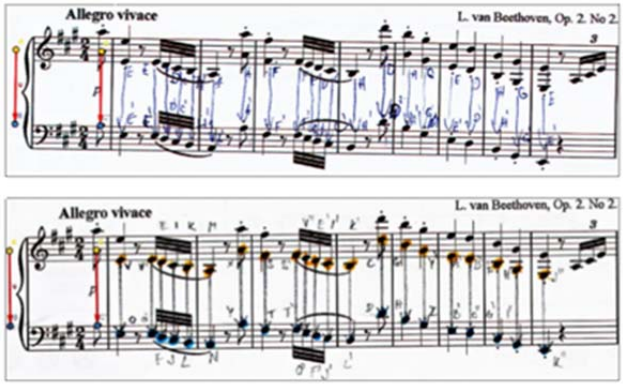

Figure 6. Translation in the 4th task

Translation as one congruent transformation also occurred in another task (5th task). Students should draw a specific translation vector for each musical notes. In the musical notation students were given pre-image of notes and their image.

This task was very difficult for fourteen years old pupils. We think that the mistakes were probably because many pupils were unable to understand the fact that they did not have to draw the transformation vector between particular basic geometrical figures. Now they had "irregular shapes" that were created by combining the selected musical notes. The Figure 7 shows the work of a student who color-coded translation vectors.

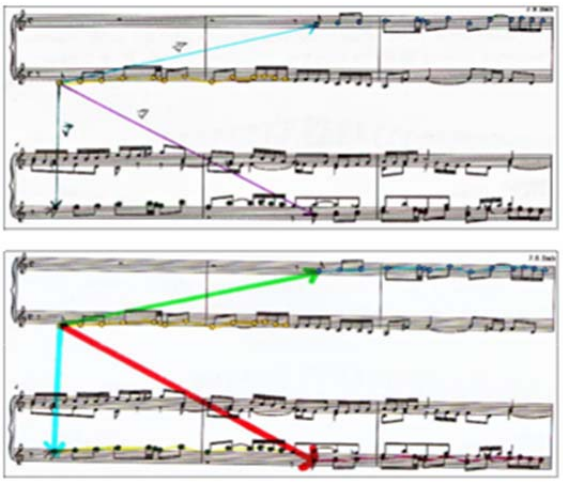

Figure 7. Example of correct student´s solution of the 5th task

The next task (6th task) was also focused on translation. But it was more difficult task for students, which is also confirmed by the results of their solutions. Find the pre-image of the yellow musical notes as well as vector of translation (all the marked musical notes are displayed in the same translation). So the students did not have a given vector of translation, so only the note image was given.

Up to twenty students had a completely wrong solution to this task. Half of the students identified it as a truly difficult task for them. Despite these facts, thirteen students correctly solved the task (see Figure 8).

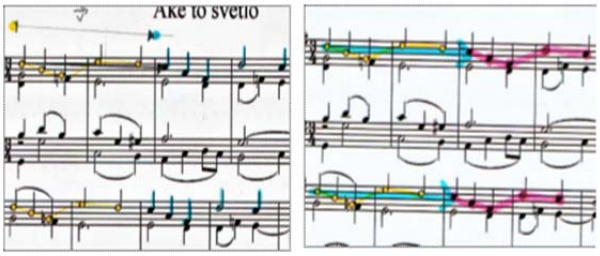

Figure 8. Example of correct student´s solution of the 6th task

In the last task (7th task) the students were to select musical notes that are symmetrical according to the marked center of symmetry S.

In this task up to ten students were not clear assignments of task, the other fourteen pupils therefore called the last task as difficult. However, there were also fifteen students for whom it was easy to solve the 7th task. In the Figure 9 shows the most precise student solutions of the last task.

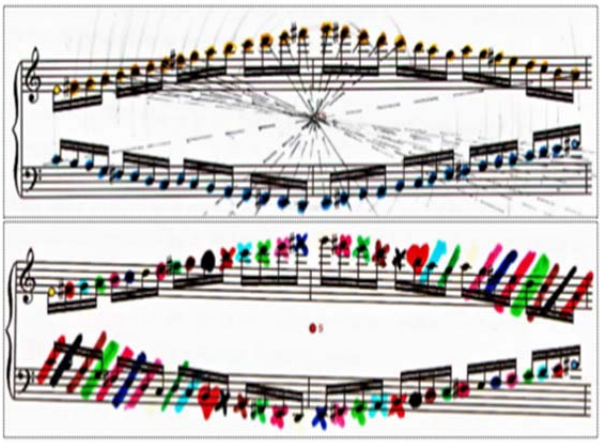

Figure 9. Best student solutions of 7th task

5. Discussion

Tasks were solved by 34 pupils and they could get a maximum of 14 points. Their average percentage success rate was 67% and the average score per pupil was 9.41. The following Table shows the percentage success of students in each task (see Table 1).

Table 1. Overview of students according to their score (Source: authors´ own processing)

Score	Quantity (%)	Number of students with given score
14	23.53	8
13	2.94	1
12	8.82	3
11	8.82	3
10	8.82	3
9	5.88	2
8	11.8	4
7	2.94	1
6	2.94	1
5	8.82	3
4	8.82	3
3	5.88	2
2	0	0
1	0	0
0	0	0

Here are some students' opinions on the tasks:

- "It is great idea to combine music with mathematics. In the first task, however, I would have the whole musical notes displayed, because this can confuse the person. Thank you ☺."
- "Task assignments were creative and unusual. Most of the tasks were manageable, but with some I had some problems. However, it was quite a pleasant educational process."
- "I need to learn more."
- "It's difficult for me, but I tried."
- "Some ta are incomprehensible. When I understood them so super, top."

6. Conclusion

In general, good students can understand mathematics also by passive methods, but this is not enough for weaker students. Both types of students significantly improve their learning when they are obliged to use problem-based learning. As stated [15], it is essential to realize how appropriate given problem is the focus for acquiring knowledge and reasoning strategies for students.

We can say that through our work with students their interest in this topic increase. From the presented results it shows that on average each student correctly solved more than half the tasks.

From the available materials, we understand that there is indeed a link between mathematics and music. Therefore, understanding the nature of this interaction can create interesting problem tasks.

By connecting mathematics and music, we wanted to point out interesting interdisciplinary relations that we can also use in the educational process. It certainly motivates students to work, leads to the popularity of solving tasks from everyday life and learning mathematics concurrently.

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