The Development of Fast Algorithm for Continuous-Time Subsectional Polynomial E-Pulse Synthesis

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Abstract **– This paper discusses the development in the E-pulse technique, also known as the method of extinction pulse, which is an aspect-independent approach to ultra-wideband radar target discrimination in which each target can be characterized by the set of its natural resonances. It is shown that subsectional polynomial E-pulse can be constructed without composing a linear problem and further solution of the underlying matrix equation set. The key concept of the proposed algorithm consists of several steps, where the first one is building a skeleton E-pulse of an especial waveform, the second step is its extension, and the final step is the series of integration. The polynomial structure of the pulse allows above listed steps to be performed over the coefficients of basic functions rather than the functions themselves. As a result, the proposed solution could perform up to a thousand times faster than one based on direct matrix solution. It also provides the coefficients of the polynomial E-pulse sections without solving a linear problem associated with ill-conditioned sparse matrix in its left-hand side. The E-pulse signals synthesized by means of the fast algorithm are proven to be exactly the same as one synthesized by the direct approach. The numerical example given in the paper exposures the main features of the E-pulse technique.**

The discrimination scheme where two aircraft scaled model targets are involved is simulated.

It was shown that the E-pulse discrimination number provides the effective tool for measuring the energy of the late-time part of the convolution as a measure of the difference of two pole sets belonging to the responses under comparison.

Keywords **– E-pulse, extinction pulse, natural resonances, radar target discrimination, ultra-wideband radar.**

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1. Introduction

The recent advance in ultrawide-band radar system exploiting digital signal processing techniques shows that the problem of developing brand-new and enhancing the existing methods of radar target discrimination. Those methods are expected to be based on the set of aspect invariant features being extracted from the signals obtained as the result of receiving electromagnetic signals measured as electromagnetic fields scattered by the target when their field are incident by short ultrawide-band pulses [1], [2].

According to the Singular Expansion Method (SEM) proposed by C.E. Baum, the signal reflected by the radar target superimposed by electric currents induce by incident waves can be divided in time domain into two adjacent intervals [3]. Supposing that TL denotes the time required to the returned propagation of the wave within the physical length of the radar target, one can define the early-time interval: $t < TL$, when the incident electromagnetic wave is reflecting from point scatterers distributed over the target body. The rest of the time called late-time interval happens after the wave comes over the whole target for $t > TL$. The explanation of the entire process for targets of complex shapes with conductive surface was further developed that is given in [4]. The proposed model states that the early-time part of the target response can be expressed as superposition of many reflections received from the set of effective reflection points. Since its relative strength and position observed from the distant radar station can easily vary, in that case the aspect of the target or wave polarization change, the signal of the early-time part could not be robust for massive target discrimination, although some identification schemes supported by artificial neural networks can be applied to cope with it [5], [6]. In contrast, in the late-time interval, one can observe dumping with time natural oscillations induced by the current whose distribution on the target surface is assumed to be independent from the aspect and other factors.

According to SEM, the late-time part of target response $r(t)$ can be expressed in the compact closed-form consisting of complex dumping sinusoids

$$r(t) = \sum_{n=1}^{2N} R_n e^{s_n t}, \quad t > T_L, \tag{1}$$

or equivalently in real-valued form:

$$r(t) = \sum_{n=1}^{N} A_n e^{\sigma_n t} \cos(\omega_n t + \varphi_n), \quad t > T_L, \tag{2}$$

where $s_n = \sigma_n + j\omega_n$ are the poles which characterize the dominant resonant complex frequencies of natural oscillations. The real-valued signal implies that poles s_n engaged in the response can be naturally grouped into N complex-conjugate pairs. Each pair is in charge of the partial response, which is the dumping sinusoids with some amplitude A_n determined by the resonance strength and some initial phase φ_n. Those taken together can be rewritten as a phasor $R_n = A_n\exp(j\varphi_n)$, which can be also thought as residual in the Laplace transform of the target response.

Both amplitude and phase are usually aspect-dependent while the poles are bound to be independent. In other words, each target can be described by its unique feature that is the set of poles $\{s_n\}$ corresponding to the complex frequencies or resonances of the target body. The one of the most efficient techniques regarding processing the resonance oscillation of the target is the E-pulse methods which are shortly described in the next section of the paper. In the third section, the algorithm which significantly increases the speed of the E-pulse synthesis is presented. The fourth section describes the identification scheme and gives the example of E-pulse synthesis. The paper ends with conclusion where the main benefits are highlighted.

2. The Theory of E-pulse Concept

E-pulse (extinction-pulse) denoted by $e(t)$ is a signal of the finite duration TE which is fitted to the signal $r(t)$ in such a way that the convolution $c(t)$ of the signals $e(t)$ and $r(t)$ is equal to zero for all time starting from the moment TE: $t \geq TE$ [7].

The E-pulse can be represented in the analytical form combining two conditions together. The first one is the finite duration:

$$supp\{e(t)\} = [0; T_E], \tag{3}$$

where supp means the support of the signal which is the interval where the signal is not zero. The second condition is necessary one for the E-pulse to exist in time domain:

$$c(t > T_E) = 0,$$
$$c(t) = e(t) * r(t) = \int_0^{T_e} e(\tau) r(t-\tau) d\tau. \tag{4}$$

The application of the Laplace transform allows one to write the necessary condition in the complex-frequency domain in the following way:

$$\forall s_n : E(s_n) = 0 \tag{5}$$

where $E(s)$ is the image (or the result of the Laplace transform) of the extinction pulse $e(t)$. In other words, it means than image $E(s)$ has to turn to zero when any pole of the set is substituted into its expression.

The authors of the original method made the suggestion, that, among all evident ways to the effective synthesis of the signal $e(t)$ satisfying the conditions (3) and (4), the most preferable approach is the parametric one [8]. According to it, an E-pulse is considered to be a linear combination of the finite number of linearly independent basis functions $f_m(t; \boldsymbol{\theta}_m)$:

$$e(t) = \sum_{m=1}^{M} \alpha_m f_m(t; \boldsymbol{\theta}_m), \tag{6}$$

where $\boldsymbol{\theta}_m$ denotes the vector of parameters describing the m-th basis function $f_m(t; \boldsymbol{\theta}_m)$, α_m is a coefficient which is independent from $\boldsymbol{\theta}_m$ and time t. If the functions $f_m(t; \boldsymbol{\theta}_m)$ do not overlap one another in time, they can be naturally treated as individual sections and the E-pulse, which is made of such set of functions, and it can be called subsectional, and it is consisting of M sections.

For practical purpose, we can consider the synthesis of subsectional polynomial E-pulse that is the signal which consists of the sections of the same length where the partial signal within each section is written in the form of a polynomial [9]. The analytical expression of such sort of E-pulse is a particular case of the general model expression (6) and it can be expressed:

$$e(t) = \sum_{m=1}^{M} \sum_{q=0}^{Q} k_{mq} t^q h_m(t), \tag{7}$$

where Q is a chosen polynomial order, M is the total number of sections, Δ denotes the width of any section. The latter can be simply chosen as the whole E-pulse duration TE divided in equal parts: $\Delta = TE/M$. The set of polynomial coefficients k_{mq} consists of $M \times (Q+1)$ scalars to be estimated. The term $h_m(t)$ is used to strictly limit the existence of the m-th polynomial within the m-th section:

$$h_m(t) = u(t - m\Delta + \Delta) - u(t - m\Delta), \quad (8)$$

where $u(t)$ denotes the Heaviside, or the unit step function.

The essential step in E-pulse synthesis is the solving problem consisting in the calculation of the set formed of all the parameter vectors $\{\boldsymbol{\theta}_m\}$ which have to be satisfying the necessary condition written as (4) in time domain or, equivalently, as (5) in complex-frequency domain. Since the problem has been formulated as linear, it can be solved as a set of linear equations. This can be expressed in matrix form as follows:

$$\boldsymbol{Ax} = \boldsymbol{b}, \quad (9)$$

where **A** and **b** are correspondingly the matrix of the system and right-hand side vector, and **x** is the solution vector made of all elements of all vectors $\boldsymbol{\theta}_m$, bringing out the classical solution to the problem under consideration.

It was shown that the matrix **A** is a block matrix, thus it possesses a complex structure. Actually, it consists of several groups of linear equations which are primary equations introducing the necessary conditions of E-pulse existence, then implying the conditions of continuity and smoothness of the E-pulse waveform at the sections' edges and at the edges of the E-pulse itself. The latter leads to more compact spectrum, which makes the transmission of the signal more practical since it demands the devices with more narrow frequency band. In addition, the matrix **A** can be extended with equations leading to some special cases which are called as forced and natural E-pulses or the E-pulse with zero DC component, that may be useful if the signal is to be radiated.

3. Fast Synthesis Algorithm

The complex structure of the left-hand side part of the matrix equation (9) demands the essential effort during its algorithmic assembling and the further evaluation of its elements can be computationally expensive as it is shown in. Moreover, the increase in the order of polynomial function forming the sections as well as the increase in the number of poles will make the set of linear equations more and more ill-conditioned. The order in the decimal representation of the matrix condition number depends quasi-linearly on the number of pole pairs and the order of the polynomial used for section assembling. In addition to that, the matrix turns to be sparse due to the natural sparsity in the matrix structure as well as the sparsity of its building blocks, which are block submatrices.

The overcoming of all difficulties which are inevitable part of the direct approach of polynomial coefficient estimations, the new algorithm was invented by the author of the current paper for the evaluation of a subsectional polynomial E-pulse. The further development of that idea has led to the algorithm where the E-pulse identical to one synthesized by traditional approach can be found directly without any construction of matrix equation and its further solution for the coefficients.

The bright idea lying behind the developed methods consists in solving the task in two main steps. The first step is the fast synthesis of the skeleton E-pulse. This E-pulse is made of delta-functions only:

$$e_K(t) = \sum_{n=0}^{N} b_n \delta(t - n\Delta), \quad (10)$$

where bn is the weight of the delta-function that occurs at time instant n_Δ. It will be shown below that this can be performed without any matrix manipulation.

The second step is the linear transformation of the skeleton E-pulse turning it into the polynomial one by means of series of similar linear filtrations.

According to the fundamental theorem of algebra, the polynomial in s representing the Laplace transform of the skeleton E-pulse can be expressed in the form of the product:

$$E_K(s) = b_0 \cdot \prod_{n=1}^{N} (1 - a_n e^{-s\Delta}). \quad (11)$$

The deeper investigation of (11) can lead to the fact that the skeleton E-pulse $e_K(t)$ can be synthesized in time domain by means of sequential linear convolution:

$$e_K(t) = C_{n=1}^{N} [\delta(t) - exp(p_n\Delta)\delta(t - \Delta)] \quad (12)$$

where «**C**» denotes the multiple convolution of the several indexed operands in a row, which is similar to the notation of a sum or product.

From a practical point of view, the set of coefficients $\{b_n\}$ is worth being considered as a digital signal, named $b[n]$, which can be obtained as the result of the digital convolution of N signals. Each of those signals consists of two samples only:

$$b[n] = C_{n=1}^{N} \{1, -a_n\}. \quad (13)$$

At the next step, we can apply convolution of the E-pulse with the specially fitted sequence of delta-functions to extent the skeleton E-pulse. The basis function for generating this sequence is a simple sequence made of two delta-functions:

$$\psi(t) = \delta(t) - \delta(t - \Delta). \quad (14)$$

In case in which the convolution is carried out for e_K(t) and $\psi(t)$, the signal consisting of N+2 delta functions and exhibiting zero DC component will be obtained. Its first integration will lead to the signal made of rectangles. In other words, one will get the subsectional E-pulse satisfying the conditions (3) and (4) where each section describes by the constant, or zero-order polynomial.

If one wants to generate E-pulse whose sections are described with polynomials of the order Q, they require to conduct the extension of the skeleton E-pulse by Q+1 convolutions with ψ(t) and then perform Q+1 integrations. The commutative property of the convolution allows writing this signal via expression:

$$e_{EXT}(t) = e_K(t) * C_{q=1}^{Q+1} \psi(t), \quad (15)$$

where $e_{EXT}(t)$ denotes the extended skeleton E-pulse consisting of N+Q+2 delta-functions.

The practical evaluation of the signal $e_{EXT}(t)$ can be interpreted as calculation of the weights of the delta-functions in its representation:

$$e_{EXT}(t) = \sum_{n=0}^{N+Q+1} b_{EXT}[n]\delta(t - n\Delta), \quad (16)$$

where $b_{EXT}[n]$ can be treated as sample of the underlying digital signal.

The signals $e_w(t)$ obtained at each of the Q+1 integration steps leading to the polynomial of the Q-th order:

$$e(t) = \int_0^t \int_0^{\tau_1} \dots \int_0^{\tau_Q} e_{EXT}(\tau_{Q+1}) d\tau_{Q+1} d\tau_Q \dots d\tau_2 d\tau_1, \quad (17)$$

are also subsectional polynomial functions of time, each is E-pulse indeed, whose general expression can be given as:

$$e_w(t) = \sum_{m=1}^{M} \sum_{q=0}^{w} k_w^q[m] \cdot t^q \cdot h_m(t), \quad (18)$$

where k denotes the coefficient at term t^q in the m-th section for intermediate E-pulse of the order w having obtained at appropriate integration step. It is crucial to notice that those coefficients can be evaluated by means of a finite-step iterative procedure. Thus, the coefficients of the polynomials in the sections of the signals $e_w(t)$ being obtained at all steps except the initial one, in other words, where $w \geq 1$, can be evaluated using the following formula:

$$k_w^q[m] = \begin{cases} q^{-1} \cdot k_{w-1}^{q-1}[m], \quad \text{if } 1 \leq q \leq w; & \\ 0, \quad \text{if } (q = 0) & (m = 1); \\ k_w^q[m-1] + \sum_{s=1}^{w} \{(k_w^s[m-1] - k_w^s[m]) \times & \\ \times [(m-1)\Delta]^s\}, \quad \text{if } (q = 0) & (m \geq 2); \\ 0, \quad \text{otherwise.} & \end{cases} \quad (19)$$

The first line in the formula corresponds naturally to the element-wise integration of a polynomial function. The condition in the second line is in charge for the continuity at the origin, while the one in the third line provides the continuity at the intermediate points between sections. It is important to notice that the continuity at the right edge of each $e_w(t)$ is satisfied deliberately since the square under curve of $e_{w-1}(t)$ is zero. The latter is completely guaranteed by the chosen way of the E-pulse extension (15).

The iterative evaluation of the coefficients given by (19) provides one with the sequence of the signals $e_w(t)$. At the final step, the result will be the set of coefficients which corresponds to the E-pulse of order Q exhibiting continuity and smoothness of the order Q–1 at any point of time including all its edges.

The result of the fast algorithm is the E-pulse which identical, might be up to the multiplier Q!, to the E-pulse which can be reconstructed by the coefficients obtained in the direct approach based on linear problem solution. Since the convolution (4) is linear, both E-pulses can be treated as the same waveform. Basically, they can be normalized dividing each by the square root of its energy which results in equal waveform.

4. Results and Discussion

Two scaled model of aircrafts Boeing 707 (B-707) и McDonnel Douglas F-18 which original sizes are 33 and 38.5 meters correspondingly has been chosen as targets to be discriminated for carrying out the numerical simulation of E-pulse discriminating scheme. Each of the models are described in terms of their pole sets derived in [4].

As a bright example in this paper, the response of radar scattered signal of the aircraft model F-18 described by its set of poles was used as a starting point for sub-sectional E-pulse synthesis. There have been synthesized several energy normalized E-pulses made of polynomials of different order Q using the proposed technique described above as the fast algorithm. The E-pulses are depicted in Figure 1. which includes waveforms made of delta-functions: $Q = -1$, rectangles $Q = 0$, linear $Q = 1$, parabolas $Q = 2$. The waveforms reveal that continuity holds for polynomials possessing order $Q \geq 1$ whereas smoothness is valid for $Q \geq 2$.

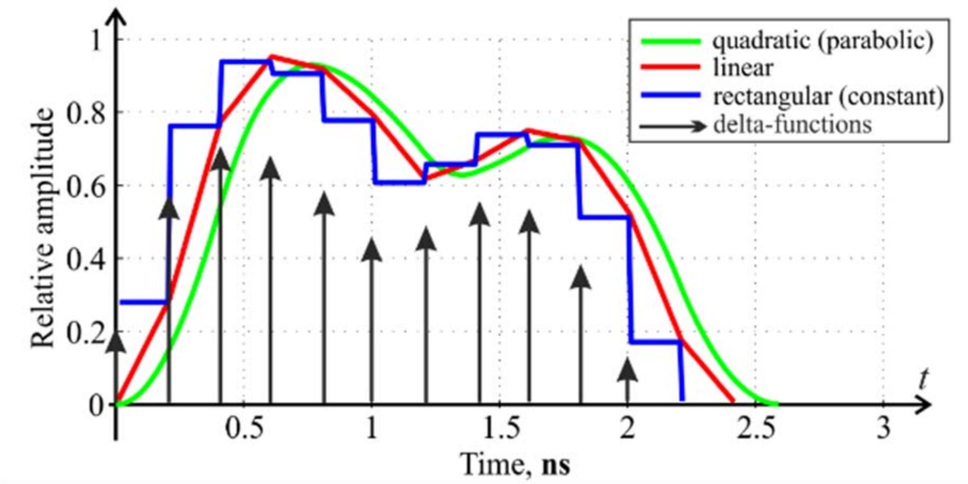

Figure 1. E-pulses fitted to F 18 pole set

The convolutions of the E-pulse with the response whose poles were used for the E-pulses synthesis are shown in Figure 2. There, one can see that the convolutions are the finite-time signals as well as E-pulses themselves. Moreover, the condition (3) satisfies exactly since the duration of the convolution is equal to the duration of E-pulse, which the convolution was evaluated with.

It is important to highlight that the speed of the E-pulses synthesis was significantly quicker by usage of the fast algorithm in comparison with the direct approach based on linear problem solving. Using GNU/Octave as the simulation software running on Core i5 personal computer, the synthesis of the E-pulse of order 7 for the set of 10 poles took 280 μs only while the direct approach required 170 ms.

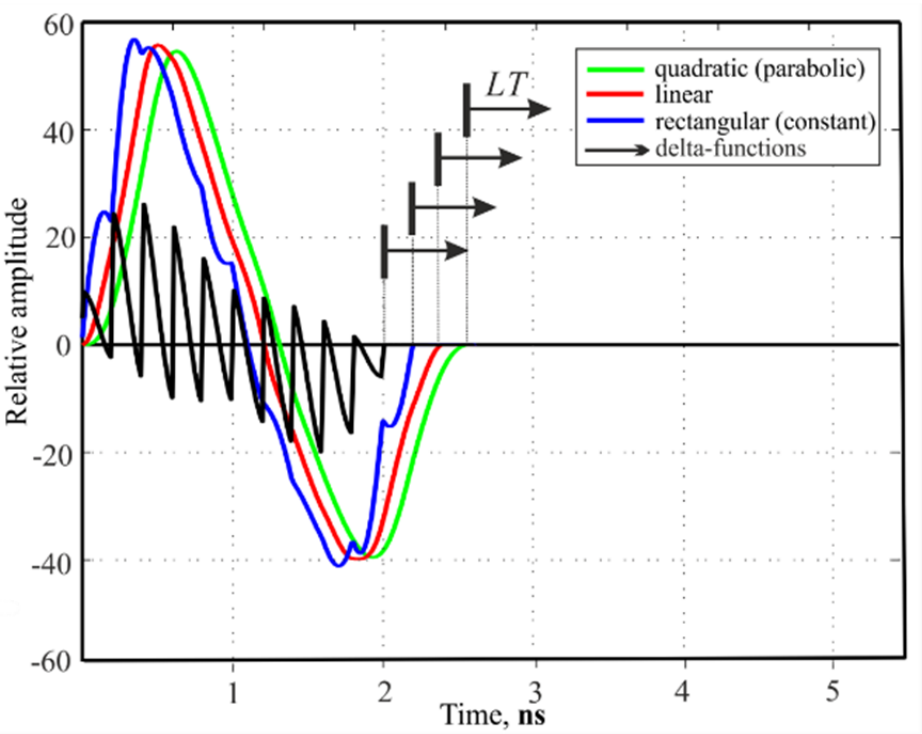

Figure 2. The convolution of F 18 response with the E-pulses fitted to F 18 poles

The basic idea lying behind of the discrimination technique based on E-pulse method consists in the fact that the convolution of E-pulse $e_X(t)$ that is fitted to the pole set describing some radar target X with the response of this target $r_X(t)$ vanishes during the late-time part (LT) of the convolution: $c_{XX}(t > T_{LT}) \equiv 0$. On the other hand, the convolution of the same E-pulse with the signal describing the response of another radar target Y $r_Y(t)$ in its late-time part will considerably distinct from zero: $c_{XY}(t > T_{LT}) \neq 0$. The cross convolution of the radar response of F-18 with the E-pulse fitted to B-707 is shown in Figure 3. The cross convolution of the radar response of F-18 with the E-pulse fitted to B-707 is shown in Figure 4.

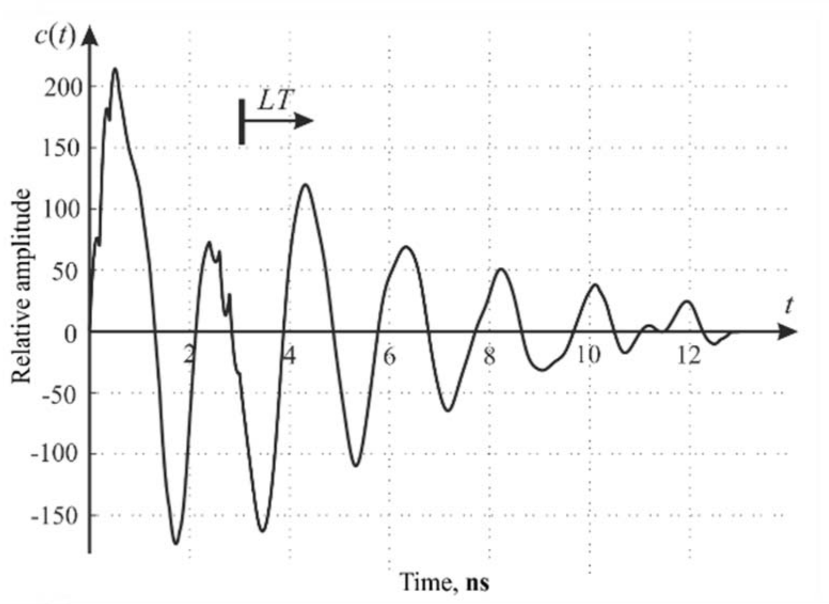

Figure 3. Cross convolution of F-18 response with the E-pulse fitted to B-707 pole set

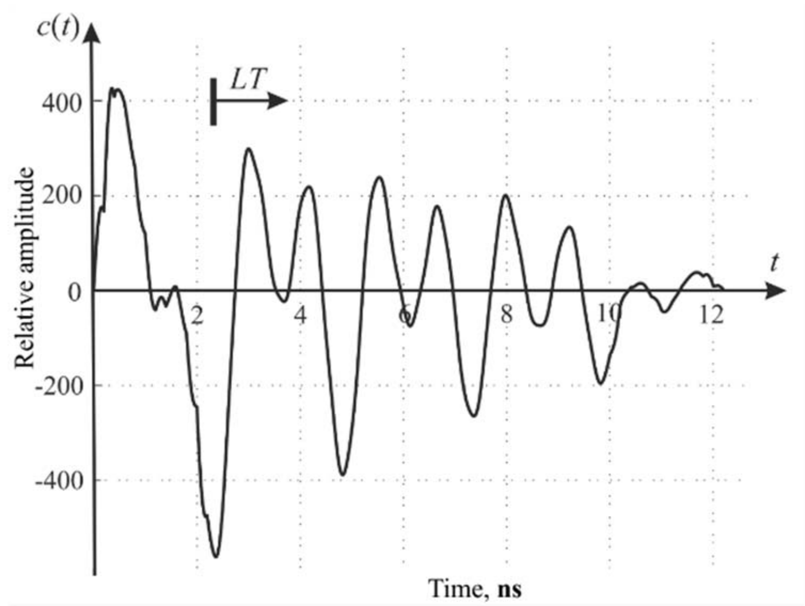

Figure 4. Cross convolution of B-707 response with the E-pulse fitted to F-18 pole set

The quantitive assessing of the identification performance can be carried out using E-pulse discrimination number (EDN) [10]. The EDN is calculated for the convolution of the E-pulse built for the target *X* with the response of the target *Y*:

$$EDN_{XY} = \frac{\int_{T_E}^{\infty} [c_{XY}(t)]^2 dt}{\int_{0}^{T_E} [e_X(t)]^2 dt} = \frac{\int_{T_E}^{\infty} [e_X(t) * r_Y(t)]^2 dt}{\int_{0}^{T_E} [e_X(t)]^2 dt}. \quad (20)$$

The EDN parameter can be evaluated by means of the numerical methods. Its physical meaning is straightforward: it is the ratio of the energy of the convolution $c_{XY}(t)$ measured in the late-time part ($t >$

T_{LT}) to the energy of the E-pulse itself $e_X(t)$. Evidently, if the targets coincide $Y = X$, the discrimination number will be equal to zero $EDN_{XX} = 0$.

The structural scheme of the system designed for target discrimination is shown in Figure 5. It is designed for E-pulse technique where the target natural resonances are involved as it is considered [11]. Some further modifications of these scheme can be found in.

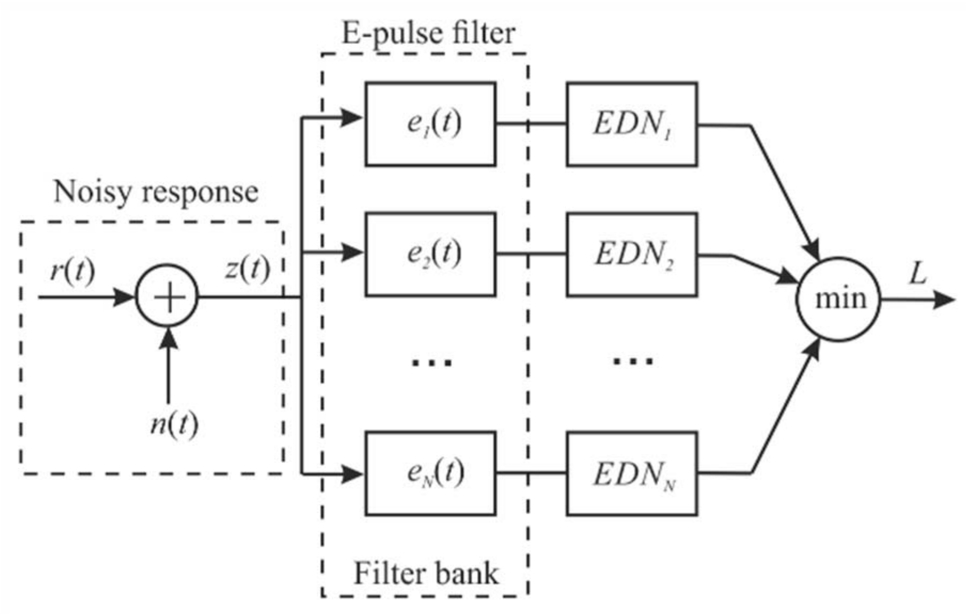

Figure 5. The structural scheme of the target discrimination system

The noise signal $z(t)$ scattered by the radar target is fed to the input of the system. This signal goes through the set of N E-pulse filters which implements the impulse responses that are proportional to the E-pulse waveforms fitted to N radar targets considered as reliable patterns. At the output of each filter, the convolution is obtained $c_i(t)=e_i(t)*z(t)$, $i = 1,\dots,N$. Then, the estimation of EDN is made for each convolution $c_i(t)$ in each channel by means of measuring the energy of late-time part ($t > T_{LT}$) resulting in the set of EDN_i, ($i = 1,\dots,N$). The channel with minimal discrimination number $EDN_{min} = \min\{EDN_i\}$ is chosen among all the channels. Providing that its number is L, the processed response will be related to the target L taken from the pattern library.

5. Conclusion

The E-pulse technique considered in the paper can be successfully applied to solving the problem of ultrawide-band radar target identification based on the invariant set of features contained in time-domain response of the target such as the target impulse response. The well-known direct approach to E-pulse synthesis of subsectional polynomial E-pulse by means of solving the set of linear equations cannot be considered as numerically effective especially in cases where the order of the polynomial or the number of sections increase when the number of poles in the set describing the target natural resonances enlarges.

The fast algorithm described in this paper has been developed to overcome the main drawbacks of the direct algorithm. It can provide the identical E-pulse without solving the large equation set. The main benefit of the proposed fast algorithm consists in avoiding the linear matrix problem usually involving ill-conditioned system matrix in its left-hand side. The conducted numerical simulation has revealed that time elapsed to the E-pulse synthesis using the fast algorithm is about one thousand times smaller than time elapsed by the direct algorithm This has a practical value for multiple E-pulse synthesis where the multiple target discrimination is being carried out using the pattern bank of feature sets which are poles describing target natural resonances.

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