Digital Algorithms for the Discrete Frequency Selection of Signals that Do Not Use Algorithmic Multiplication Operations

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Abstract – The issues related to the problem of minimizing hardware costs in the digital algorithms' hardware and software implementation for the discrete signals' frequency selection on programmable logic devices (PLD) and special processor microelectronic devices are considered. Possible ways to solve these problems are described based on the computational algorithms for non-recursive digital filtering (NDF) and difference digital filtering with integer coefficients use. A necessary and sufficient condition is given for using a computational algorithm's hardware and software implementation of discrete signals' difference digital filtering for their multi-stage discrete Fourier transform without performing arithmetic multiplication operations.

Keywords – cascade digital filtering, difference digital filtering, frequency selection of signals, multi-band digital filtering, NDF.

1. Introduction

The widespread microelectronic devices are used for digital signal processing and the need to save such devices' computing resource actualizes the search for solutions to pressing problems of reducing and

minimizing hardware costs for digital algorithms' hardware and software implementation for the frequency selection of discrete signals on programmable logic devices and special processor microelectronic devices [1]. These tasks can and should be solved by cascading digital filtering methods with a minimum number of arithmetic multiplication operations and multi-band digital filtering (MDF) methods without performing arithmetic multiplication operations [2], [3], [4].

Computational cascade digital filtering algorithms with a minimum number of arithmetic multiplication operations can be implemented either on the basis of NDF, the amplitude-frequency characteristic (AFC) of which has the symmetry property, on the basis of Walsh NDF, or on the basis of homogeneous and triangular digital filters [5]. Computational MDF algorithms without arithmetic multiplication operations can and should be implemented on the NDF basis with low-bit coefficients or on the basis of difference digital filters (DDF) with low-bit coefficients, or on the DDF basis with integer coefficients [6], [7]. The use of such algorithms for the MDF complex signal $\{x(n \cdot T)\}$ with a sampling period T using a low-pass digital filter (LDF) allows the signal's N -point discrete Fourier transform (DFT) to be performed only by adding and shifting its n -th time samples at $n = 0, 1, 2 \dots N - 1$ [8].

The purpose of the study is a comparative analysis of discrete signals' frequency selection digital methods to build its algorithms without algorithmic multiplication operations and determine the necessary and sufficient conditions for using such methods for multi-stage DFT of discrete signals without performing arithmetic multiplication operations [9], [10].

The study used computational procedures for cascade digital filtering algorithms with a minimum number of algorithmic multiplication operations and MDF without performing algorithmic multiplication operations [11], [12].

The comparative analysis results as well as hardware and software modeling of such algorithms have shown and confirmed the reducing hardware

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costs possibility of discrete frequency selection of signals with their MDFs only by adding and shifting these signals' time samples [13], [14].

2. Theoretical Basis

The theoretical basis for digital algorithms for the frequency selection of discrete signals that do not use the algorithmic multiplication operations can and should be considered the Nyquist-Shannon sampling theorem, which allows to determine the required signal sampling frequency ω_D value, and convolution theorem as applied to digital convolution calculation $\{s(K, n \cdot T)\}$ of all k -th coefficients $\{h(k)\}$ of K -th order and n -th time samples digital filtering $\{x(n \cdot T)\}$ of a digital signal with a sampling period $T = 2 \cdot \pi / \omega_D$ according to the formula (1) when $k = 0, 1, 2 \dots K - 1$ and $n = 0, 1, 2 \dots N - 1$:

$$s(K, n \cdot T) = \sum_{k=0}^{K-1} h(k) \cdot x(n \cdot T - k \cdot T). \quad (1)$$

3. Methodology

During the study, there were used the NDF hardware and software modeling methods with symmetric AFC, Walsh NDF, homogeneous and triangular digital filters, NDF and DDF with low-bit coefficients, DDF with integer coefficients and methods for comparative analysis of these digital filters' software modeling results. The research materials used were digital filters and time sequences of discrete signals' digital samples.

3.1. Methods of Cascade Digital Filtering with a Minimum Number of Arithmetic Multiplication Operations

3.1.1. Cascade digital filtering method based on non-recursive digital Walsh filters

Methods of cascade digital filtering can minimize the number of arithmetic multiplication operations in the frequency selection of signals algorithms up to a complete refusal to perform arithmetic multiplication operations [15], [16]. The problem of minimizing the number of arithmetic multiplication operations by these methods can be solved in various ways by using non-classical digital filters' structures in which arithmetic multiplication operations are excluded altogether [17], [18].

One way of reducing the number of arithmetic multiplication operations in frequency selection algorithms is to use for such cascade digital filtering selection based on NDF, known as Walsh filters [8].

The main idea of using the Walsh filter, which does not fundamentally require arithmetic multiplication operations, is that it carries out

preliminary only "coarse" digital filtering, and a digital filter equalizer cascaded to it along with compensation for distortions in the passband that introduces into the signal the Walsh filter, provides the final digital filtering of the signal. Since the requirements for a digital equalizer filter are weakened in comparison with the requirements for a NDF as a whole, fewer coefficients are used to implement it and, accordingly, fewer arithmetic multiplication operations are required [11], [12].

However, NDF based on Walsh filters, which does not require arithmetic multiplication operations, still requires arithmetic multiplication.

3.1.2. Cascade digital filtering method based on NDF, AFC of which has the symmetry property

A. A. Lanne, D. I. Kaplun, T. V. Merkuicheva, D. V. Minenkov and A. Yu. Supyan developed a cascade digital filtering method that minimizes the number of arithmetic multiplication operations in a digital filter by balancing its amplitude-frequency characteristic (AFC) [18].

The required digital filter is obtained by connecting elementary links with different parameters, selected in such a way to provide this filter's best-specified characteristics. Its impulse characteristic is decomposed into the sum of polynomials, each of which is realized by a serial connection of the digital filter's links, forming the so-called "filter branch". A parallel connection of the "filter branches" gives the desired digital filter's impulse characteristics of the [16].

Obviously, when digitally filtering the n -th complex signal's time samples $\{x(n \cdot T)\}$ with a sampling period $T = 2 \cdot \pi / \omega_D$ at $n = 0, 1, 2 \dots N - 1$ the n -th time samples' values $\{y_1(n \cdot T)\}$ at the first link's output will be calculated by the formula (2):

$$y_1(n \cdot T) = x(n \cdot T) + x(n \cdot T - T). \quad (2)$$

Accordingly, the n -th time samples' values $\{y_2(n \cdot T)\}$ at the second link's output will be calculated by the formula (3):

$$y_2(n \cdot T) = y_1(n \cdot T) + y_1(n \cdot T - T) = x(n \cdot T) + 2 \cdot x(n \cdot T - T) + x(n \cdot T - 2 \cdot T). \quad (3)$$

Then, the n -th time samples' values $\{y_3(n \cdot T)\}$ at the third link's output will be calculated by the formula (4):

$$y_3(n \cdot T) = y_2(n \cdot T) + y_2(n \cdot T - T) = x(n \cdot T) + 3 \cdot x(n \cdot T - T) + 3 \cdot x(n \cdot T - 2 \cdot T) + x(n \cdot T - 3 \cdot T). \quad (4)$$

The coefficients of each subsequent link are determined based on previous links. And k -th coefficients $\{h_{lm}(k)\}$ of the K -order digital filter are

represented by the polynomial coefficients of degree I_m (the digital filter link order), which are calculated by the formula (5) with $k=0,1,2...K$:

$$h_{Im}(k) = \begin{cases} 1, k = 1 \\ ((\prod_{i=0}^{k-2} (I_m - i)) / (k-2)!), k > 1. \end{cases} \quad (5)$$

Reducing the AFC of a K -order digital filter to a symmetrical form allows it to reduce its multipliers' number by reducing the non-zero coefficients number of its impulse characteristics $\{h(k)\}$ for $k=0,1,2...K-1$. A special case of AFC digital filter symmetry is double symmetry [19], [20]. A digital filter whose AFC has a symmetry property has almost half the number of non-zero coefficients compared to a digital filter whose AFC does not have a symmetry property. Given the AFC digital filter's double symmetry, its impulse characteristics can contain up to a quarter of non-zero coefficients.

If the initial requirements for the AFC filter are not symmetric, the balancing by tightening the filter requirements can be performed there:

- by cascade method;
- by direct symmetrizing method;
- by increasing the sampling rate.

However, symmetrizing the K -th order digital filters' AFC leads to toughening the requirements for them and, consequently, to increasing their order K of such a digital filter. But the number of non-zero filtration coefficients of the K -th order $\{h(k)\}$ with such an operation, as a rule, decreases for $k=0,1,2...K-1$.

3.1.3. Cascade digital filtering method based on homogeneous and triangular digital filters

The cascade digital filtering method based on homogeneous and triangular digital filters allows to completely abandon the arithmetic multiplication operations in algorithms for frequency selection of signals through the use of unified special substructures cascade connections organized on the basis of homogeneous and triangular digital filters [13], [14].

Cascade digital filtering by homogeneous digital filters of the K -th order does not require the multipliers use if $K = 2^q - 1$ at $q=1,2,3...Q$, since the arithmetic multiplying operation by 2^q is reduced in fixed-point systems to the multiplied trivial digits' shifts. A triangular filter can be represented as a cascade connection of two identical homogeneous ones.

However, homogeneous and triangular digital filters do not provide high selectivity and small unevenness of the AFC cascade digital filtering in the delay band [8], [9].

3.2. NDF Methods without Performing Arithmetic Multiplication Operations

3.2.1. NDF method with low-bit coefficients

NDF method proposed by L.V. Sabaev, allows to replace the values of the impulse characteristics' k -th samples $\{h(k)\}$ of the NDF's K -th order with the values of their mutual successive increments $\{h(k) - h(k-1)\}$ at $k = 0,1,2...K-1$ and $h(k) = 0$ if $k \leq 0$. These increments $\{h(k) - h(k-1)\}$ are essentially the difference coefficients' values $\{h_p(1, k_p)\}$ DDF of K_p -th order and 1st order of difference according to formulas (6) and (7) for $k_p = 0,1,2...K_p-1$ and $K_p=K+1$:

$$h_p(1, k_p) = h(k_p) - h(k_p - 1) \quad (6)$$

$$h_p(1, k+1) = 0. \quad (7)$$

Therefore, the n -th time samples $\{y(n \cdot T)\}$ at the LDF's K -th order output the at MDF for the N -point DFT of the complex signal $\{x(n \cdot T)\}$ with a sampling period $T = 2 \cdot \pi / \omega_D$ performed according to formula (8) at $n = 0,1,2...N-1$:

$$y(n \cdot T) = \sum_{k=0}^{K+1} \sum_{k_1=0}^k h_p(1, k_1) \cdot x(n \cdot T - k_1 \cdot T). \quad (8)$$

Some decrease in the required bit depth of $MULT_{MDF}(N, K+1)$ multipliers and $CELL_{MDF}(N, K+1)$ memory cells during hardware DFT implementation using the N -band MDF method based on a NDF of L.V. Sabaev is ensured by some increase in the number of $SUMM_{MDF}(N, K+1)$ adders in the K -th order LDF. But the number of $MULT_{MDF}(N, K+1)$ multipliers and the number of $CELL_{MDF}(N, K+1)$ memory cells for MDF-based DFTs by one NDF of L.V. Sabaev remains the same as when using the traditional K -th LDF:
 $CELL_{MDF}(N, K+1) = CELL_{DFT}(N),$
 $MULT_{MDF}(N, K+1) = MULT_{DFT}(N).$

However, with scale-of-two (binary) values $(-2^q, +2^q)$ at $q = 1,2,3...Q$, and (or) trivial values $(-1, 0, +1)$ of k_1 -th difference coefficients $\{h_p(1, k_1)\}$ NDF of L.V. Sabaev does not require arithmetic multiplication operations at $k_1 = 0,1,2...K$.

3.2.2. Method of difference digital filtering with low-bit difference coefficients

The method of difference digital filtering of high-order difference proposed by V.S. Kuzkin and developed by V.S. Kuzkin, Yu. S. Shinakov, Yu. Ya. Burov and A. Yu. Burova can significantly reduce the bit depths of both $MULT_{MDF}(N, K_p)$ multipliers and $CELL_{MDF}(N, K_p)$ memory cells required for DFT's hardware implementation by the MDF method

based on K -order LDF [10]. Such hardware savings are ensured by the special processor implementation of the K -th order LDF ((DDF $K + J$ -th order and the $-$ th difference order). For this, the difference coefficients' values $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K + j - 1$, $(j + 1)$ -th order of the difference, $j = 1, 2, 3, \dots, J$, are replaced by the values of the difference coefficients $\{h_p(j+1, k_j)\}$, $k_j = 0, 1, 2, \dots, K + j$, of the following difference order $(j + 1)$, $j = 1, 2, 3, \dots, J$, provided that $h_p(j, k_j) = 0$, if $k_j \leq 0$ by formula (9):

$$h_p(j + 1, k_j) = h_p(j, k_j) - h_p(j, k_j - 1). \quad (9)$$

However, the DFT's hardware implementation of the complex $\{x(n \cdot T)\}$, $n = 0, 1, 2, \dots, N - 1$, $T = 2 \cdot \pi / \omega_D$, by the N -band MDF method using K -order

LDF (DDF $K + j$ -th order and the $-$ th order of difference for $j = 1, 2, 3, \dots, J$) requires a significantly larger number of $SUMM_{MDF}(N, K+j)$ adders than the DFT's hardware implementation using the same N -band MDF method with K -th order LDF, but based on a NDF of L.V. Sabaev:

$$SUMM_{MDF}(N, K + j) \gg SUMM_{MDF}(N, K + 1) \text{ at } j > 1.$$

This is due to an increase in the number of arithmetic addition operations in the differential $(K+j)$ -th order digital filtering algorithm with an increase in its order of the difference j to calculate the n -th time samples of the complex signal's N -point sample $y(n \cdot T)$ at the DDF output $(K+j)$ -th order and j -th order of difference in accordance with formula (10) [6]:

$$y(n \cdot T) = \sum_{k_0=0}^{K_p-1} \sum_{k_1=0}^{k_0} \dots \sum_{k_{j-1}=0}^{k_{j-2}} \sum_{k_j=0}^{k_{j-1}} h_p(j, k_j) \cdot x(n \cdot T - k_j \cdot T). \quad (10)$$

For scale-of-two (binary) values $(-2^q, +2^q)$, $q = 1, 2, 3, \dots, Q$, and (or) trivial values $(-1, 0, +1)$ of difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K_p - 1$, difference digital filtering of V.S. Kuzkin does not require arithmetic multiplication.

3.2.3. Method of difference digital filtering with integer difference coefficients

The effectiveness of applying the difference digital filtering method with integer difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K_p - 1$, DDF of the difference's k -th order and J -th order was noted by V.S. Kuzkin [20].

The integer difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K_p - 1$, the difference's DDF of the K -th order and J -th order at $|h_p(j, k_j)| = 0, 1, 2, \dots, 2^{Z-1} - 1$ and the digit capacity Z of the real numbers representation makes it possible to significantly reduce the number of $MULT_{MDF}(N, K_p)$ multipliers and the digit capacity $MULT_{MDF}(N, K_p)$ of multipliers and $CELL_{MDF}(N, K_p)$ memory cells required for the DFT hardware implementations by the MDF method using K -th order LDF, $K = K_p - J$.

Such hardware savings can be realized with the special processor implementation of the K -th order LDF based on the K_p -th order differential filtering algorithms and the K -th order J -th difference filtering, $K_p = K + J$, if each arithmetic multiplication operation by an integer difference

$h_p(j, k_j)$, $k_j = 0, 1, 2, \dots, K_p - 1$, by multiple algebraic additions of the n -th time samples $\{x(n \cdot T)\}$, $n = 0, 1, 2, \dots, N - 1$, $T = 2 \cdot \pi / \omega_D$ arriving at the input of such an LDF (DDF K_p -th order and J -th order difference), in accordance with the magnitude of the numerical value modulus of this coefficient $|h_p(j, k_j)| \leq 2^{Z-1} - 1$, for the digit capacity Z of the real numbers representation.

Obviously, for integer values $-2^{Z-1} \leq |h_p(j, k_j)| \leq +2^{Z-1}$ difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K_p - 1$, difference digital filtering of the K -th order and the J -th order of difference does not require arithmetic multiplication to calculate the n -th time samples of the complex signal N -point sample $\{y(n \cdot T)\}$ at the output of DDF K_p -order and J -th order difference.

However, the DFT hardware implementation of the complex signal $\{x(n \cdot T)\}$, $n = 0, 1, 2, \dots, N - 1$, $T = 2 \cdot \pi / \omega_D$, by the N -band MDF method using K -order LDF with integer coefficients (DDF $K + J$ -th order and $-$ th difference order with integer difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K + J - 1$), by the formula (11) requires a significantly larger number of $SUMM_{MDF}(N, K+J)$ adders than the DFT hardware implementation by the same N -band MDF using LDF of the same K -th order, but with low-bit coefficients (DDF $K + J$ -th order and $-$ th difference order with low-bit difference coefficients $\{h_p(j, k_j)\}$, $k_j = 0, 1, 2, \dots, K + J - 1$) [6]:

$$y(n \cdot T) = \sum_{k_0=0}^{K+J-1} \sum_{k_1=0}^{k_0} \dots \sum_{k_{j-1}=0}^{k_{j-2}} \sum_{k_j=0}^{k_{j-1}} \sum_{i=0}^{|h_p(J,k_j)|} \text{Sign}[h_p(J,k_j)] \cdot x(n \cdot T - k_j \cdot T). \quad (11)$$

4. Discussion

The comparative analysis and hardware-software modeling results of well-known cascade digital filtering algorithms with a minimum algorithmic multiplication number and MDF operations without performing algorithmic multiplication operations showed and confirmed the possibility of reducing the digital signal processing hardware cost only by adding and shifting their time samples. The minimal hardware costs for the PLD implementation of the DDF computational algorithm with integer coefficients made it possible to develop deductive methods for the multistage discrete Fourier transform of digital signals based on recurrence algorithms for their difference digital filtering [6].

These coefficients' triviality is a necessary and sufficient condition for using a computational algorithm's hardware and software implementation for difference digital filtering of discrete signals for their multi-stage discrete Fourier transform without performing arithmetic multiplication operations.

5. Conclusion

A comparative analysis of discrete signals' frequency selection digital methods for constructing its algorithms without algorithmic multiplication operations allowed to determine the conditions for reducing the digital processing hardware costs of such signals and to determine the necessary and sufficient condition for the difference digital filtering methods and algorithms effective use for multistage discrete Fourier transform (DFT) of discrete signals that do not require arithmetic multiplication operations. The idea of reducing the number of algorithmic multiplication operations in digital algorithms for the discrete signals' frequency selection is due to the hardware use in order to achieve equal speeds of arithmetic multiplication operations and addition in elementary microelectronics devices. Therefore, digital algorithms for the discrete signals' frequency selection are in demand. Their hardware implementation significant simplification is achieved through the single digital filter for all spectral bands use and provide hardware savings in cases where it is possible to synthesize NDF. They that do not require arithmetic multiplication operations, since in FPLDs (Field programmable logic device) and special processor microelectronics devices the arithmetic equality execution speeds multiplication and addition

operations are achieved mainly due to hardware costs.

The successful application of difference digital filtering methods with integer difference coefficients in the special processor implementation of digital algorithms for the discrete signals' frequency selection is possible. However, that can only be done through the further development methods of transverse difference digital filters synthesis with integer difference coefficients of different orders of difference.

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