Examination of Spatial Ability with Emphasis on Solving Cube Tasks

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Abstract – **In this paper, we focused on verifying the basic geometry knowledge regarding the future primary education teachers with the aim of predicting the success on solving individual test tasks using the statistical method of logistic regression.**

Keywords **– spatial ability, stereometry, primary education.**

1. Introduction

Spatial ability is one of the important abilities of a person, which is of great importance for his or her daily life as well as for further education and professional career. Taking into account the educational process, spatial ability is one of the key competences of the pupil, especially in relation to teaching geometry.

The systematic development of spatial ability places great demands on a pupil. The role of mathematics teacher is therefore irreplaceable, and it is important that he/she should lead the pupil correctly in the early years of his/her studies.

To do this, the teacher himself/herself has a good command of the teaching material and is able to communicate it appropriately to the pupils. In this paper we focused on a non-standard approach to the measurement of spatial students' ability.

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We verified the basic knowledge of geometry for future teachers in primary education with an emphasis on predicting the success of solving individual test tasks using the statistical method of logistic regression.

2. Theoretical Background

The phenomenon of spatial ability is a question of different research approaches and at present there is no unifying concept. [1]

The historical overview of spatial ability research brings [2], within studies oriented on psychometric research, developmental research, differential research, and information processing research are analysed. While psychometric research was focused on defining spatial ability and its factors, developmental research studies analysed the development and levels of spatial ability in relation to developmental opportunities due to the participant's age and localization of capability in brain centres. Differential research was focused on the research of spatial ability under the influence of gender differences, while pursuing several aspects (biological, environmental, psychological). Information processing research was focused on research of spatial ability from the perspective of cognitive system (its goal was to understand the process involved in cognition).

Next, in [2] the author observes that research concerning spatial ability is a "living" area of research in which even more questions need to be addressed. Several research papers [1], [3], [4], [5], [6] present the topicality of the issue.

In this paper, we particularly incline to the phenomenon of spatial ability based on a pedagogical-psychological approach.

In the initial works [7], [8] the author defines three basic factors of spatial ability – spatial relations (based on mental rotation with figures), spatial orientation (ability about imagining figures in different position) and spatial visualization (based on solution of the challenging tasks). In the theory of teaching mathematics, spatial ability is defined by these psychological foundations. [9]

However, it is the ability to visualize the properties and shape of three-dimensional geometric objects, including their position, size and location. Spatial ability is divided into three levels - intuitive visualization, geometric visualization and spatial thinking.

Intuitive visualization is defined as the ability to remember previously seen spatial objects and to visualize them in the mind based on their planar image. Geometric visualization is the ability to abstract geometric properties of objects based on observations of specific models. It is characterized by creating a stock of ideas of geometric figures and building appropriate terminology. Spatial thinking as the highest level of spatial ability is the ability thanks to which a person can imagine various relationships, phenomena and dependencies expressed by a geometric scheme. At the same time, the person can use graphical methods to solve practical tasks.

Thus, introduced phenomenon of spatial ability is related to didactic approach in teaching geometry at Slovak schools. It is based on the intuitive perception of three-dimensional space by children and pupils who are educated in the school environment, and systematically progress from basic thought constructions of spatial images to axiomatic foundations of geometry.

Spatial ability develops throughout the life of an individual, but especially through purposeful, controlled teaching of mathematics at school. [10].

In [11] it is stated that the study of geometric curriculum has to be started at pre-school age. The basic manipulation of solid models, creation of geometric compositions, constructions made of cubes and creation of their own models facilitate the learning among children, and at the same time they prepare for the later development of geometric thinking and the argumentation base in adulthood. [12], [13]

3. Methodology

Research Questions

We have determined the following research questions.

Question 1. What is the relation of selected sample of participants to teaching mathematics? Does the relation to the subject of mathematics at secondary school have a fundamental influence on the final year-end evaluation of the student?

The student is familiar with the basics of visualization comprising selected figures in free parallel projection FPP (cube) and is able to visualize the cube in different views. If he can complete a correct image of cube in right-hand view (standard view) from given elements in a non-standard assignment, we can assume a higher success in the solution of task 2, because he/she has sufficient level of spatial vision to realize positional-metric relations in space.

Question 2. How does success in T1 affect the correct solution of T2?

The student has sufficient knowledge of "cube geometry" and spatial ability at a certain level. The iconic visualization of the cube body is misleading. The student knows that the selected cube walls are in fact squares, and therefore interprets them in an iconic visualization. He does not perceive the difference between the real form of the body and the visual representation in the plane in the form of rhomboids.

Question 3. Do correct solutions of T1 and T2 influence the perception of the planar iconic visualization of the cube buildings?

Participants

The sample consisted of 193 university students, future mathematics teachers, in the first year of teaching in school education at the Slovak schools. It is a sample of participants who have completed full secondary education at two types of Slovak secondary schools - grammar schools and secondary vocational schools. All respondents completed their secondary school leaving exam. The data collection also monitored other characteristics of the sample, such as the last math grade at secondary school certificate (the student did not have to graduate from the subject of mathematics), the declared popularity of the subject of mathematics by the students (disliked subject, subject like any other, favourite subject). There are 58 (30%) secondary grammar school graduates in the sample. The rest 135 (70%) were graduates from secondary vocational schools with school-leaving exams. For the sake of completeness, we added that all participants were women.

Table 1. Characteristics of the sample. GS = grammar school, SVS – secondary vocational school

	GS (N=58)	**SVS (N=135)**	**Total (N=193)**
Grade			
1 excellent	10 (17%)	36 (27%)	46 (24%)
2 very good	31 (53%)	70 (52%)	101 (52%)
3 good	17 (29%)	29 (21%)	46 (24%)
Popularity of mathematics			
disliked	46 (79%)	23 (17%)	69 (36%)
like any other	7 (12%)	97 (72%)	104 (54%)
favourite	5 (9%)	15 (11%)	20 (10%)

Test Tasks and Their Evaluation

In the course of the academic year 2018/2019, the participants were assigned a test containing three basic tasks, designated T1, T2, T3, while the T3 task contained two subtasks T3a, T3b.

The tasks were:

T1: Task 1. *Sketch the image of the ABCDEFGH cube with the given model.*

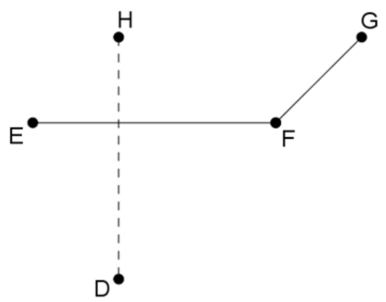

Figure 1. Task T1

T2: Task2. *Draw the centre of the ABCDEFGH cube.*

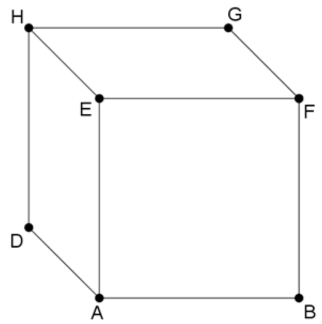

Figure 2. Task T2

T3: Task 3. *The figure shows a cube body. a) How many cubes is it built of? b) How many squares are in the picture? Colour the squares!*

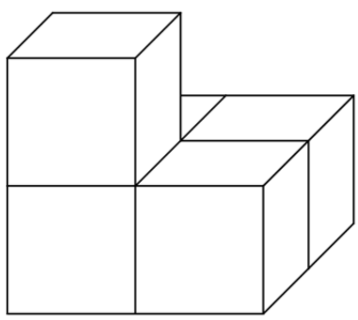

Figure 3. Assignment of T3a,b

Table 2 specifies the criteria for awarding points (score) to assess the success of the solution.

Table 2. Scoring of test tasks

Task	Points	Points awarding criteria
T1	**0 points** incorrect solution	The solution is logically incorrect and the rules for visualizing bodies in FPP are violated.
	1 point partially correct solution	The solution is logically consistent, but the principles of visualization in FPP (visibility of individual edges) are not respected.
	2 points correct solution	The solution is correct and the principles of body visualization in FPP are respected.
T2	**0 points** incorrect solution	The solution is not drawn correctly and contains a logical error.
	1 point correct solution	The solution is drawn correctly.
T3a	**0 points** incorrect solution	The participant's solution is incorrect.
	1 point correct solution	The participant's solutions is correct.
T3b	**0 points** Incorrect solution	Incorrect verbal response supplemented with incorrect graphical designation of required figures (squares).
	1 point partially correct solution	Correct verbal response, but supplemented with incorrect graphical designation of required figures (squares).
	2 points correct solution	Correct verbal response correctly supplemented with correct graphical designation of required figures (squares).

Selected Examples of Student Solutions

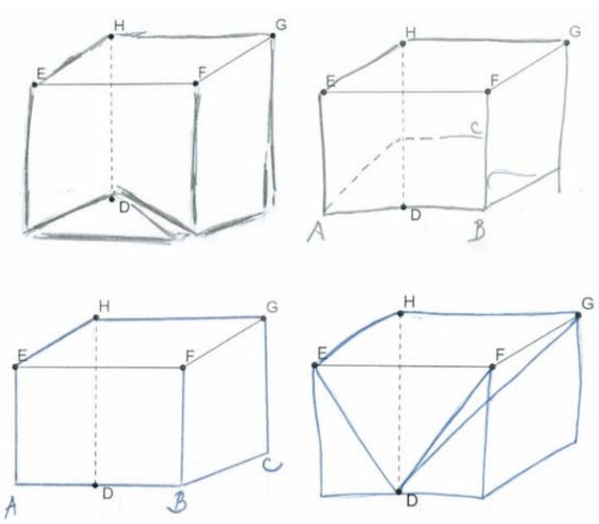

Figure 4. Examples of selected faulty solutions of T1

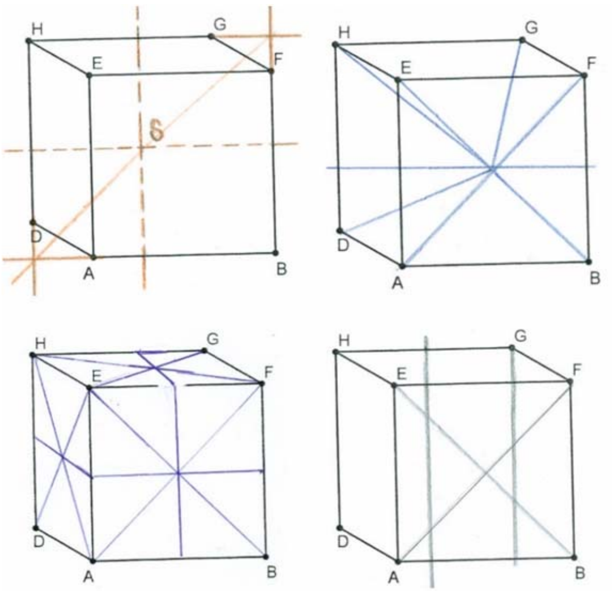

Figure 5. Examples of selected faulty solutions of T2

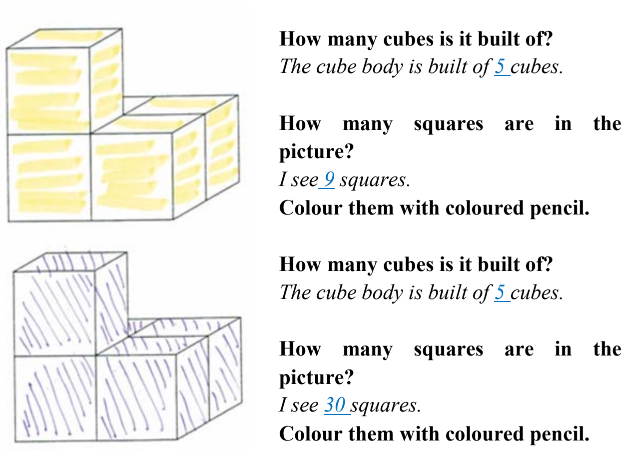

Figure 6. Examples of selected faulty solutions of T3a,b

4. Results

In the test probe only 22 (11%) passed the test at 100% from the total number of the respondents. It means that students correctly solved the task T1 (sketch of the cube image), correctly constructed the center of the cube in the task T2; finally determined the number of cubes in the cube building (task T3a) and identified in connection to graphical output the squares in the figure (task T3b).

Evaluation on the 1st Research Question

The relationship between a pair of ordinal variables - the relation to the subject of mathematics and final year evaluation could not be modelled by the ordinal regression model. There was not the case that the student mentioned mathematics as a favorite subject and achieved the grade „good (3)".

To determine the degree of association, we used an asymmetric version of Somers' D in which the dependent variable is a mark of mathematics and the independent variable represents popularity regarding the subject of mathematics.
The relationship between the variables is shown to have a low linear dependency [14], as Somers' D has reached -0.21 (95% CI $[-0.33; -0.08]$).

Evaluation on the 2nd Rsearch Question

We have obtained these results in solving the research question, which is whether the success in the solution of the task T1 affects the success of the solution of the task T2.
The possible relationship between the binary dependent variable T2 and the categorical independent variable T1 was modelled by the binary regression model. We have found that the model is statistically significant by a likelihood ratio test. ($\chi^2(1) = 21.44;\ p < 0.001$).
The Nagelkerke-pseudo R^2 has a value equal to 0.14. The Kendall order correlation coefficient Tau-α between the estimated probabilities and the observed value of the dependent variable is equal to 0.17.
The chance for correctly solving the task T2 relative to the correct solution of the task T1 increases 2.47 - times (95% CI $[0.51; 1.32]$).

Evaluation on the 3rd Research Question

The relationship between the ordinal variable T3b, which takes three levels (0, 1, 2) and the pair of categorical variables T1 and T2, was modelled by the ordinal regression model.
The model is based on the assumption that there exists a continuous latent variable and a manifest (directly measurable) ordinal variable. The ordinal variable is generated from the discretization of this latent continuum into M ordered classes.
Values on this continuum that define the categories are estimated using the $M - 1$ thresholds $\theta_1, \dots, \theta_{M-1}$. Each threshold $\theta_m, m = 1, \dots, M - 1$ depends only on which of the categories is estimated. Simultaneously the category is independent of the values within the independent variables, and the regression coefficients are the same for all m.
The assumption of the model that the regression coefficients are the same for all m was verified by the parallelism test. As a result, this assumption is fulfilled ($\chi^2(2) = 5.88;\ p = 0.053$).
However, the model cannot be considered sufficiently adequate, because we do not reject the hypothesis that all regression coefficients are equal to zero ($\chi^2(2) = 0.45;\ p = 0.800$).
The Nagelkerke-pseudo R^2 has a very low value equal to 0.002. The Kendall order correlation

coefficient Tau-α between the estimated probabilities and the observed variable value is very low 0.021. Although none of the independent variables is statistically significant in the model, we can interpret the values of the regression coefficients as follows:

a) unit increase in the value of the independent variable T1 results in an 1.10-fold (95% CI [0.76; 1.59]) increase in the chance of correctly solving the task T3b relative to an incorrect (0) or partially correct (1) solution.
b) With unit increase in the value of the random variable T2, the chance of a correct solution in task T3b will be increased by 1.07 times compared to a wrong or partially correct solution. (95% CI [0.59; 1.95]).

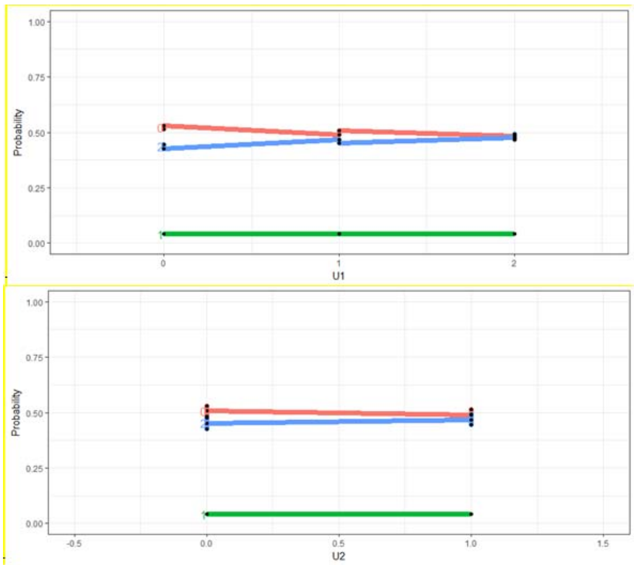

Figure 7. Predicted probabilities for ordinal regression model

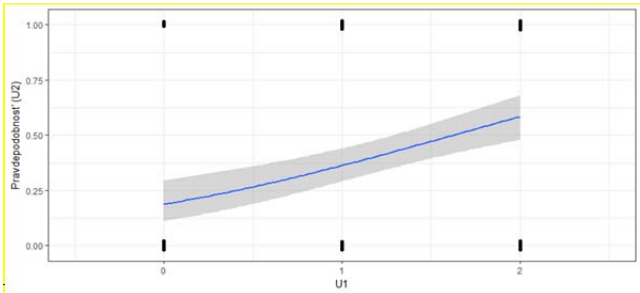

Figure 8. Probability of correct result (blue curve) and its standard estimate (gray band) in the task T2 from the binary regression model

5. Conclusion

Based on the results we state that a more positive relationship to the subject of mathematics affects the grade of mathematics. However, the model shows a low linear dependence, which can be partly explained by a sample of participants.

Study programme for teachers is not one-sidedly focused solely on teaching mathematics at primary school. At the same time, up to 70% of the participants were secondary vocational school graduates. Their secondary education was primarily oriented towards the performance of the future profession. The potential continuation of further university studies was not provisionally considered, which probably influenced their attitude towards academic subjects.

Successful solution of more demanding task T2 is conditioned by correct solution of task T1. The chance that the student will solve the T2 task correctly will increase the correct solution of the T1 task by 2.47 times.

The didactic interpretation is obvious. If a student knows the basics of body visualization in free parallel projection, this will make it much easier for him to solve the T2 task.

It appears that a unit increase in value of the independent variable T1 results in a 10% increase in the probability that in the solution of the T3b task, a score of 1 will be achieved compared to the possibility of gaining 0 points for the T3b task, or a score of 2 compared to a score of 0 or 1 for task T3b.

If the student is aware of the relation between parallelism and cube edge metrics, they have a better chance for answering the question in the T3b task successfully. The interdependence of the tasks is based on the terminological, content conception between the square, and the iconic visualization of the cube wall.

Analogically, with a unit increase in the value of the random variable T2, the chance of solving the task T3b correctly is increased by 7% compared to the incorrect or partially correct solution. However, none of the independent variables is statistically significant in the model.

Based on personal experience in teaching geometry in the above mentioned study programme, we believe that the T2 task is more challenging than T1. It turns out that the intuitive assumption of the right solution of a more demanding task will increase the chances of success of the T3b task is not justified.

The lower chance compared to the previous result can be didactically interpreted as the fact that determining the centre of the cube is a different positional task. Its solution requires a certain level of spatial ability. The student has to be aware that the centre of the cube is the intersection of body diagonals. This is a point whose location is not on the body surface, and it is not related to the concept of a square representing the wall of the cube.

To conclude, we would like to remark that the sample of participants consisted exclusively of women. This is influenced by the fact that profession of being a teacher at primary education has long been perceived in Slovak society as a traditional profession for women.

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