

# Modeling of Some Chaotic Systems with AnyLogic Software

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**Abstract** – The chaotic systems are already known in the theory of chaos. In our paper will be analyzed the following chaotic systems: Rossler, Chua and Chen systems. All of them are systems of ordinary differential equations. By mathematical software Mathematica and MatLab, their graphical representation as continuous dynamical systems is already known. By computer simulations, via examples, the systems will be analyzed using AnyLogic software. We would like to present the way how ordinary differential equations are modeling with AnyLogic software, as one of the simplest software for use.

**Keywords** – Rossler system, Chua system, Chen system, AnyLogic software, ordinary differential equation.

## 1. Introduction

In the era of the rapid rise of computers, many unresolved problems in different scientific areas have been successfully solved. The different program packages offer successful visualization and applications as in [1], [2] and [3].

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Modeling of the ordinary differential equations (ODE) with mathematical software like Mathematica and MatLab is already known as in [4], [5], [6] and [7]. But in this paper, we will test the modeling of ODE with *AnyLogic software*. The Rossler, Chua and Chen systems are reviewed as examples for modeling in the *AnyLogic software*.

The Rossler, Chua and Chen systems are systems of ordinary differential equations defined as continuous dynamical systems. These systems are examples of chaotic systems which are already known in the theory of chaos. In practice, the systems have more forms for use. We will use their simplest representations.

The Rossler system is a nonlinear system of three ordinary differential equations originally defined by Otto Rossler. The Rossler attractor was designed by Otto Rossler in 1976 as originally theoretical equations for continuous-time chaos. But later, the Rossler system has found its application in modeling equilibrium in chemical reactions. The application of the Rossler system as chaotic dynamics system has been used for the design of the cryptographic key in the field of cryptography (as examples [8] and [9]).

The Chua system or Chua circuit is a simple electronic circuit that exhibits specific behavior - chaos. It was constructed by Leon O. Chua in 1983 as a "no periodic oscillator" which produces an oscillating waveform. These oscillating waveforms are not repeats as in an ordinary electronic oscillator [10] and [11].

The Chen system is a simple three-dimensional autonomous system, as the double of the Lorenz system, in a sense defined by Vanecsek and Celikovsky [1996]. Chen found chaotic attractor in the 1999 year, in addition to the already known chaotic attractors of the Lorenz system, [12], [13] and [14].

*AnyLogic software* is a tool for a multi method simulation modeling. The *AnyLogic software* supports agent-based modeling, a discrete event simulation and system dynamics. The *AnyLogic*

software is developed by The AnyLogic Company. Since 2003, the AnyLogic software is focused on business simulation in the domains: market and competition, healthcare, manufacturing, supply chain, logistics, retail, business process, social and ecosystem dynamics, defense, project and asset management, IT Infrastructure, pedestrian dynamics and traffic simulation and aerospace and photovoltaics. One examples of using AnyLogic software are given in [15].

We are going to use AnyLogic software for educational goals, because we have an opinion that it is the great tool for learning mathematical models, [16].

## 2. Modeling of the Rossler system

The Rossler system is given by

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned}$$

where a, b and c are real parameters. The behavior of the Rossler system depends on the values of the parameters a, b and c. Varying of each parameters gave the system possibility to converge toward a periodic orbit, fixed point, or escape towards infinity.

From the system, we can see that the nonlinear character in Rossler system appears in the third equation with quadratic term which generates a chaotic attractor with a single lobe, in contrast to the Lorenz attractor which has two lobes.

The Rossler system has two equilibrium points and they are given by:

$$\left( \frac{c + \sqrt{c^2 - 4ab}}{2}, \frac{-c - \sqrt{c^2 - 4ab}}{2a}, \frac{c + \sqrt{c^2 - 4ab}}{2a} \right)$$

$$\left( \frac{c - \sqrt{c^2 - 4ab}}{2}, \frac{-c + \sqrt{c^2 - 4ab}}{2a}, \frac{c - \sqrt{c^2 - 4ab}}{2a} \right)$$

One of these fixed points is in the center of the attractor.

The behavior of the Rossler system with the AnyLogic software will be given for the values of the parameters a=0.2, b=0.2 and c=5.7. It is a Rossler chaotic attractor.

The model structure in AnyLogic software is simple, since the mathematical expressions can be easily put into the ODEs. In our case, the structure of the Rossler system in AnyLogic software is following:

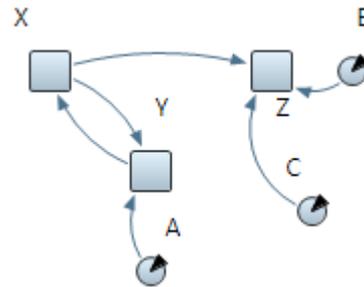


Figure 1. Rossler model in Anylogic software

In AnyLogic software, variables are represented with a element Stock. In System Dynamics, Stocks (also known as levels, accumulations, or state variables) are used to represent the real-world processes (e.g. stocks of material, knowledge, people, money). Stocks define static part of the system. For dependency between variables (stocks) is used element Link. The constants a, b, c are shown with element Parameter (Figure 1.).

In Figure 2. is shown dependency on the variables x, y and z from, the time t, for the initial values x(0)=0, y(0)=0 and z(0)=0.

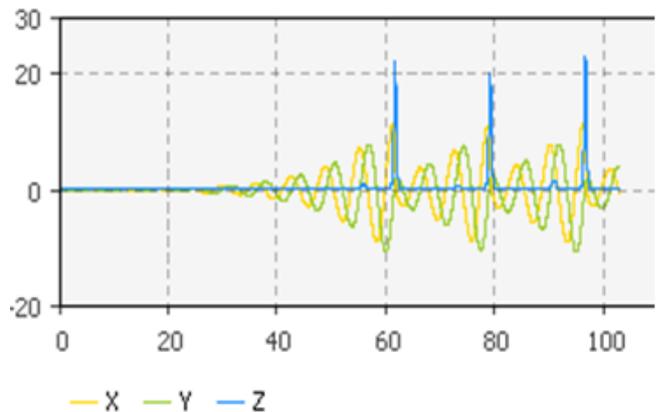


Figure 2. Time dependency of the variables x(t), y(t), z(t) in the Rossler system

In Figure 3. are shown the projecions of a trajectory of the Rossler attractor in x-y, y-z and x-z planes for the same initial values x(0)=0, y(0)=0 and z(0)=0 when the time is changed from t= 0 to t= 250.

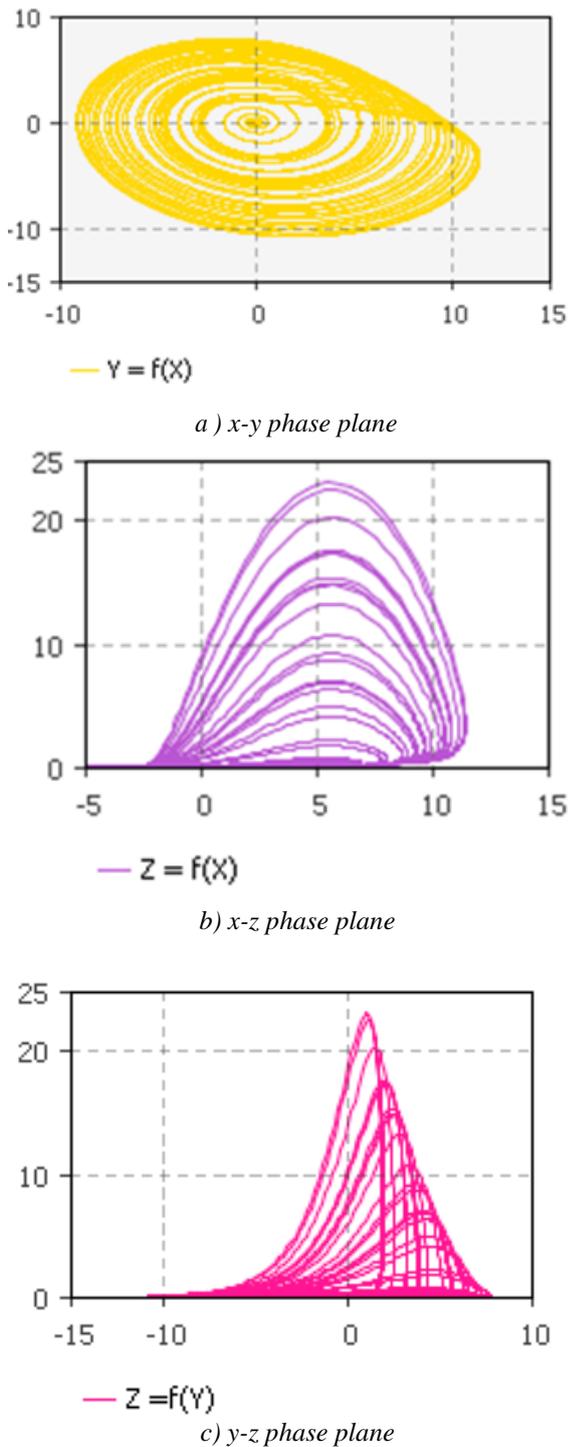


Figure 3. Phase planes of the Rossler attractor in the AnyLogic software

### 3. Modeling of the Chua system

An electronic circuit made from resistors, capacitors, and inductors before it can display chaotic behavior, it must satisfy the following three criteria:

1. one or more nonlinear elements;
2. one or more locally active resistors;
3. three or more energy-storage elements.

These criteria are met from the Chua's circuit.

There are more forms of Chua circuit, but our analyses will be in direction of the nonlinear system of differential equations which is given by

$$\begin{aligned} \dot{x} &= a(y - f(x)) \\ \dot{y} &= x - y + z \\ \dot{z} &= -by \end{aligned}$$

where  $a$  and  $b$  are parameters and the function  $f(x)$  is

$$f(x) = x^5 - \frac{1}{16}x^3 - \frac{1}{6}x$$

From the Chua system, we see that the nonlinear character in the system appears in the first equation.

The Chua system has three equilibrium points and they are:  $(0,0,0)$ ,  $(i,0,i)$ ,  $(-i,0-i)$ ,  $(-\frac{\sqrt{17}}{4}, -\frac{\sqrt{17}}{4}, 0)$ ,  $(\frac{\sqrt{17}}{4}, \frac{\sqrt{17}}{4}, 0)$ .

The behavior of the Chua system with the AnyLogic software will be given for the values of the parameters  $a=10$  and  $b=14$ .

The model structure in AnyLogic software is the following:

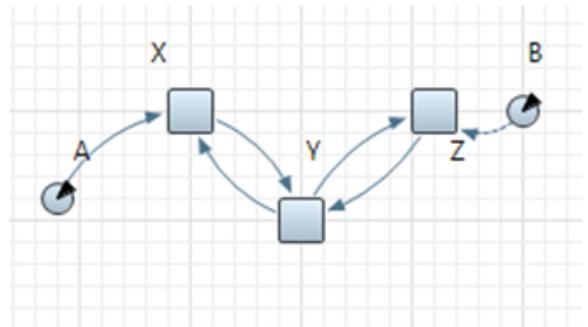


Figure 4. Chua model in Anylogic software

In Figure 5. is shown dependency on the variables  $x$ ,  $y$  and  $z$  from time  $t$ , for the initial values  $x(0)=1$ ,  $y(0)=2$  and  $z(0)=-1$ .

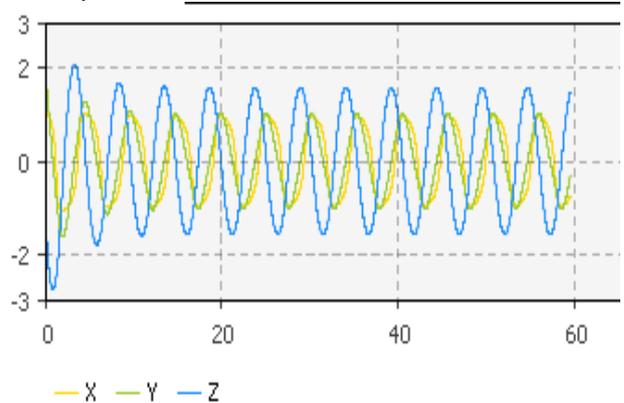


Figure 5. Time dependency of the variables  $x(t)$ ,  $y(t)$ ,  $z(t)$  in the Chua system

In Figure 6. are represented the projections of a trajectory of the Chua attractor in  $x$ - $y$ ,  $y$ - $z$  and  $x$ - $z$  planes for the same values of the initial values  $x(0) = 0$ ,  $y(0) = 0$  and  $z(0) = 0$  when the time is changed from  $t=0$  to  $t=250$ .

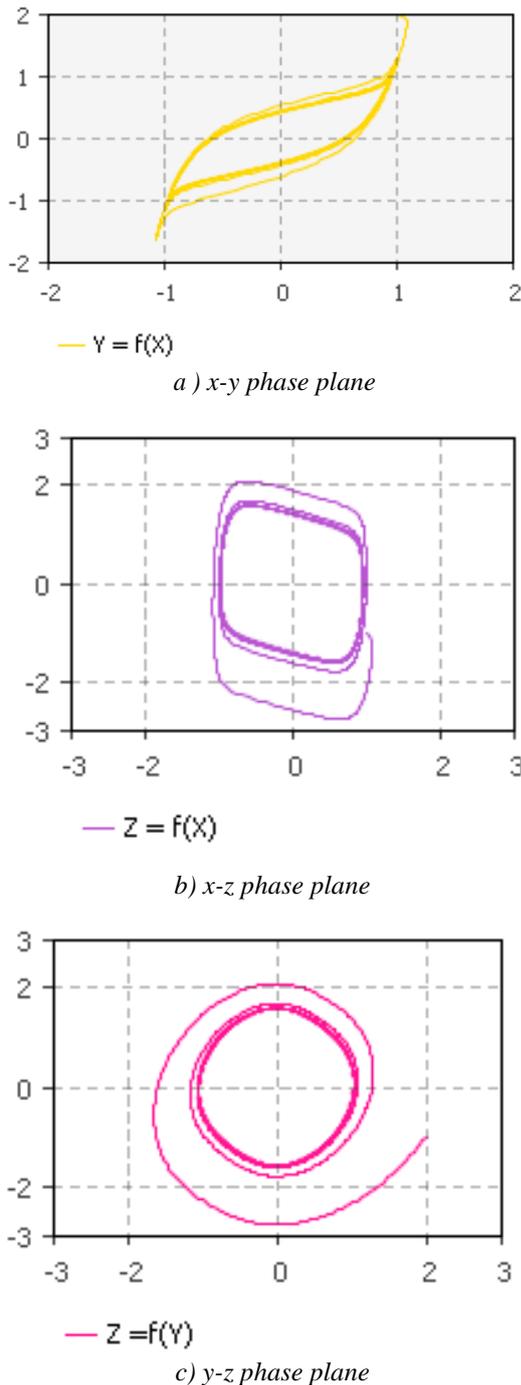


Figure 6. Phase planes of the Chua circuit in the AnyLogic software

#### 4. Modeling of the Chen system

The Chen system as special case from Lorenz system is given by the following system:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x + cy - xz \\ \dot{z} &= -bz + xy \end{aligned}$$

where  $a$ ,  $b$  and  $c$  are real parameters. It is constructed as an example for obtaining new chaotic attractors beside the Lorenz chaotic attractor.

The Chen system has two equilibrium points and they are  $(\pm\sqrt{b(2c - a)}, \pm\sqrt{b(2c - a)}, 2c - a)$ .

The behavior of the Chen system with the AnyLogic software will be given for the values of the parameters  $a = 35$ ,  $b = 3$  and  $c = 28$ , because the system for these values of the parameters has the chaotic solutions.

The model structure in the AnyLogic software is the following:

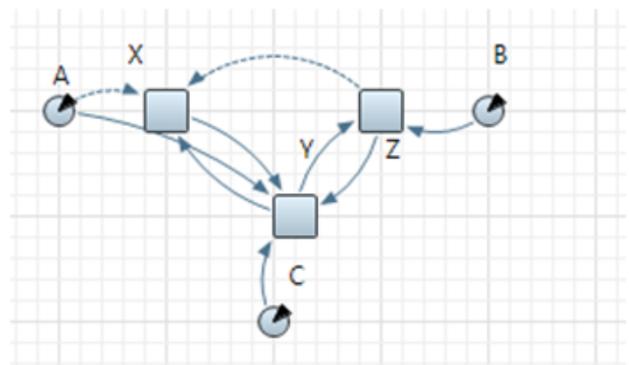


Figure 7. Chen model in AnyLogic software

In Figure 8. is shown the dependency on the variables  $x$ ,  $y$  and  $z$  from, the time  $t$ , for the initial values  $x(0)=0$ ,  $y(0)=10$  and  $z(0) = 20$ .

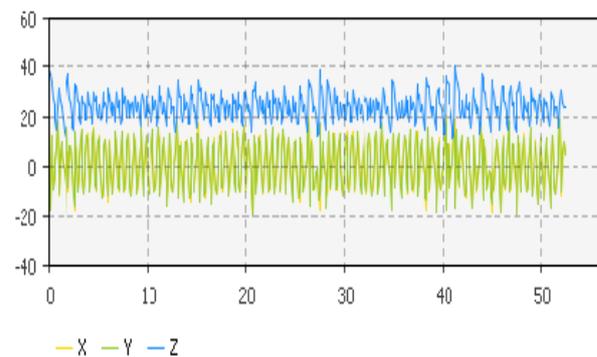


Figure 8. Time dependency of the variables  $x(t)$ ,  $y(t)$ ,  $z(t)$  in the Chen system

In Figure 9. are represented the projections of a trajectory of the Chen attractor in  $x$ - $y$ ,  $y$ - $z$  and  $x$ - $z$  planes for the same values of the initial values  $x(0)=0$ ,  $y(0) = 10$  and  $z(0) = 20$  when the time is changed from  $t=0$  to  $t=100$ .

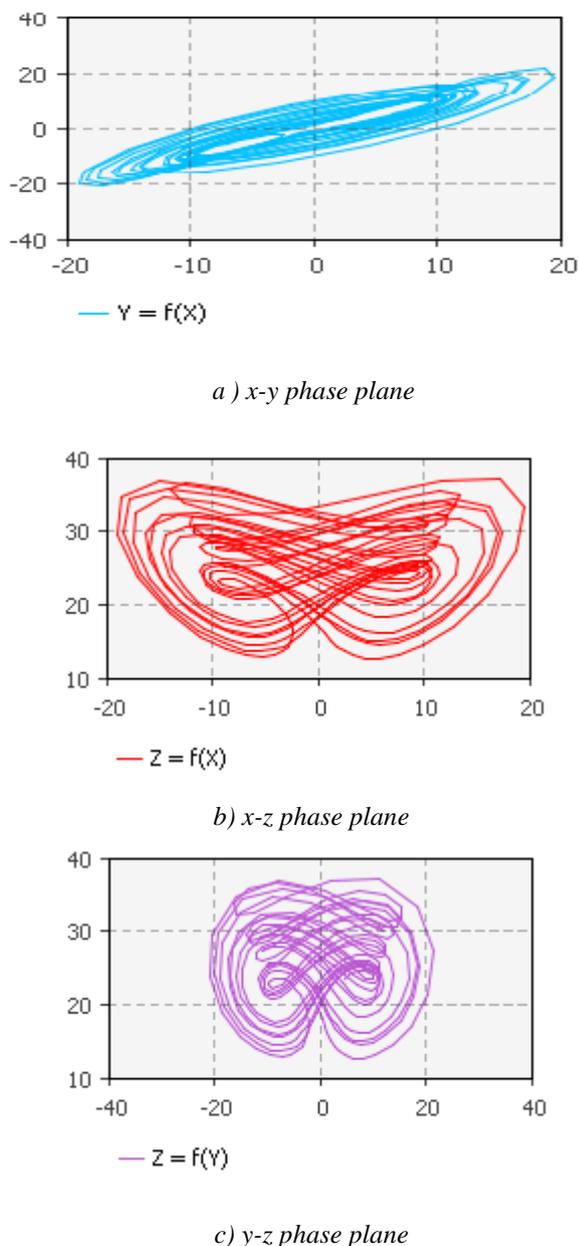


Figure 9. Phase planes of the Chen circuit in the AnyLogic software

## 5. Conclusion

The *AnyLogic* software as a tool for multi-method simulation modeling describes models of ODE in which in explicit way are shown the connections between the variables and also the connections between the variables and the parameters (see Figures 1., 4. and 7.). Thus graphically, these connections could not be seen in another

mathematical software such as Mathematica and MatLab.

For time dependency of the variables  $x(t)$ ,  $y(t)$ ,  $z(t)$  in the systems described in Figures 2., 5. and 8. the *AnyLogic* software gave the same pictures as Mathematica and MatLab software. One advantage in *AnyLogic* software is that we can follow how the whole simulation is drawn, which is not the case in Mathematica and MatLab. But, on the other hand, with Mathematica and MatLab we can draw 3D graphics easily and in *Anylogic* we have to include Jzy3d Java library for drawing 3D graphics. That is some kind of disadvantage of *Anylogic*.

Analogously on the previous paragraph, the *AnyLogic* software for presentation of the phase planes of the systems described in Figures 3., 6. and 9. gave the same images as Mathematica and MatLab software. Again, in *AnyLogic* software, we can follow how the whole simulation is drawn, which is not the case in Mathematica and MatLab.

For this reason, according to our research, *AnyLogic* software is a better tool for dynamical simulations of ODE than the other mathematical software. It is so because *Anylogic* gives the chaotic behavior in the process of ODE presentation, draws out the phase planes as a simulation and shows the connections between the variables and also the connections between the variables and the parameters of the system.

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