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Dynamics of Flexible Wind Power Generator with Unbalanced Rotor

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***Abstract* – The paper deals with dynamic analysis of a wind power generator as a large flexible structure with high speed rotating machines and considerable masses. The dynamic model is considered as a multibody system of rigid and flexible bodies. Nonstationary and transitional processes caused because of eccentricity of the high speed rotating machines, as well as, of the propeller vibrations are simulated and analyzed. Analytical method is applied for dynamic simulation. The results are verified by numerical procedures. Example of wind power generator with three propellers is presented.**

***Keywords* – rotor dynamics, non-stationary vibrations, wind power generator.**

1. Introduction

Rotating machines play a paramount role in many modern industrial applications. Most of them can be considered as on-board machines affected mainly by mass unbalance and support excitations. Automotive turbocharger, ship and aircraft turbines as well as locomotive electrical generators are examples of rotors on moving base. The undesirable mass unbalance is due to the eccentricity of the center of mass along the rotor axis and can be caused by material non-homogeneities, manufacturing defaults, assembly and service conditions. The rotor balancing aims at minimizing the mass unbalance but generally does not lead to a complete cancellation, see, for example. A rotor can also be excited by the movement of its base which can increase the lateral vibration of the rotor and create a dynamic instability phenomenon. These are wind power generators, powerful turbo-generators subject to inertial and kinematic (seismic) excitations, and etc. Similar structures possess significant vertical stiffness and several times lower stiffness in horizontal direction because of specific equipment placed under them (columns, support, and etc.) [1], [2]. As a result of these particularities a large deflections and unstable areas of oscillations of the wind generators during the process of the rotational acceleration could be observed.

To prove the reliability of the investigations analytical and numerical methods are applied. The analytical method consists in application of asymptotical methods [3–5]. The numerical approach uses novel generalized Newton – Euler dynamic equations for rigid and flexible bodies [6, 7].

The paper presents an approach to dynamics simulation of rigid and flexible multibody systems including high speed rotating machines with considerable inertia mass properties. The subject of the investigation is a wind power generator with three propellers which is considered a system compiled of an elastic column with large height and a rotor with eccentricity. The stiffness and damping of the column, as well as, of the rotor bearings and the shaft are taken into account. The external disturbances are modeled as earthquake vibration excitations. Non-stationary vibrations and transitional processes are analyzed.

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2. Analytical model of a high speed rotor on a sizeable flexible foundation

In Fig. 1. the design scheme of a turbomachine mounted on a large flexible pillar is depicted. The pillar (a beam of steel tube) is placed on a basement. On the top of the beam the stator and the rotor of the turbo generator are mounted. The rotation is along an axis perpendicular to the plane *XY*, while the deflections of the system are in the same plane.

In Fig. 1. (b) the kinematic analog (scheme) is presented and in Fig. 1 (c) the corresponding dynamic model of the system including the reduced mass and stiffness properties. Further down the analytical approach for deriving of the dynamics equations is presented.

The first natural frequency and mode of vibration are obtained taking into account that the pillar is a cantilever beam for which the first and second natural frequencies are computed as follows [4]:[9].

$$\omega_{col_1} = 1.875\sqrt{\frac{E.I}{m_0.H^4}}\,; \qquad \omega_{col_2} = 4.694\sqrt{\frac{E.I}{m_0.H^4}}\,, \tag{1}$$

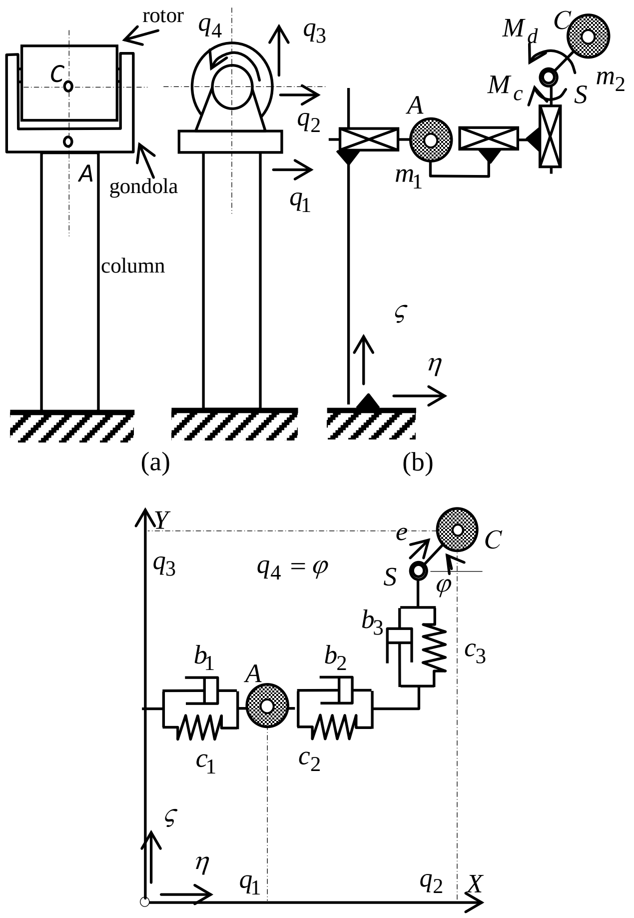


Figure 1. Test model of high speed rotating machine mounted on an elastic column: (a) virtual design scheme; (b) kinematic analogue; (c) dynamic model

where E is Jung modulus of the elasticity [N/m^2]; I is the geometrical inertia moment [m^4];

$m_i = m_0 = \dfrac{m_{col}}{H}$ [kg/m] is point mass distributed along the beam with mass m_{col}; H is the length of the beam. The mass reduced to the beam tip is to be derived equalizing the kinetic energy of the reduced mass and of all distributed masses, i.e.:

$$m_r = \sum_1^n m_0.u_{i1}^2 \Big/ u_{11}^2\,, \tag{2}$$

where m_0 is the mass of the distributed points and u_{i1} is the value of the first modal vector of the corresponding point (Fig. 2.) . The mass pointed out with m_1 (point A) includes the reduced mass of the pillar m_r and the mass of the stator. m_2 of point C is the mass of the rotor.

Taking into account the first natural frequency and the mass reduced to the pillar tip one could estimate

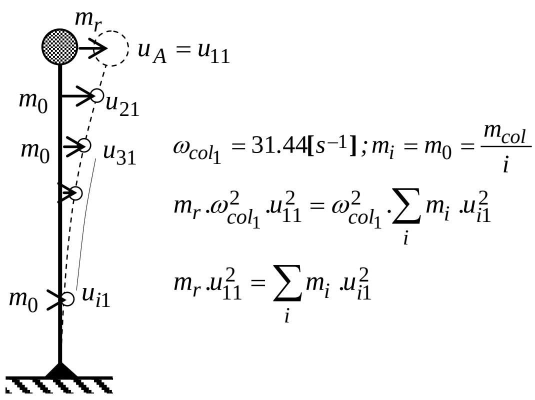


Figure 2. Algorithm for mass reduction of the column

the stiffness coefficient c_1 of bending [N/m]. Using the Lagrange dynamic equation one obtains the following dynamic equations for the system of Fig. 1. (c):

$$\begin{aligned}
&m_1.\ddot{q}_1 + (c_1 + c_2).q_1 + b_1.\dot{q}_1 - c_2.q_2 = \\
&= -c_2.e.cos(q_4) + C_0.\xi + B_0.\dot{\xi}\,; \\
&m_2.\ddot{q}_2 - c_2.q_1 + c_2.q_2 + b_2.\dot{q}_2 = \\
&= c_2.e.cos(q_4) + c_2.\xi + b_2.\dot{\xi}\,; \\
&m_2.\ddot{q}_3 + c_2.q_3 + b_3.\dot{q}_3 = c_2 e.sin(q_4) + c_2.\eta + b_2.\dot{\eta}\,; \\
&J_C.\ddot{q}_4 = M_d(\dot{q}_4) - M_c(\dot{q}_4) - \\
&- c_2.e.(q_2 - q_1)sin(q_4) + c_2.e.q_3.cos(q_4)\,,
\end{aligned} \tag{3}$$

where $B_0 = b_1 + b_2$; $C_0 = c_1 + c_2$; ξ and η are kinematic (seismic) excitations along axes X and Y, respectively; M_d - driving torque; M_c - resistant torque; c_1, c_2, $c_0 = c_1 + c_2$ are the stiffness coefficients [N/m]; b_1, b_2, $b_0 = b_1 + b_2$ are damping coefficients [$N.s/m$]; $q_i, i = (1,2,3,4)$ are generalized coordinates; J_C is mass inertia moment of the rotor [$kg.m^2$] ; e is the rotor eccentricity [m].

With $\xi(t) = 0$, $\dot{\xi}(t) = 0$, $\eta(t) = 0$, $\dot{\eta}(t) = 0$ the frequency spectrum of the conservative system of differential equations that correspond to the system Eq. (3) is obtained using the relations:

$$\left.\begin{matrix}\omega_1^2\\ \omega_2^2\end{matrix}\right\} \to \begin{vmatrix} C_0 - m_1.\omega^2 & -c_2 \\ -c_2 & c_2 - m_2.\omega^2 \end{vmatrix} = 0\,;\ \omega_3^2 = \frac{c_2}{m_2} \tag{4}$$

It should be pointed out that ω_3, from physical point of view, is the critical angular velocity of the shaft for motionless mass m_1. The modal vectors of the vibrations are as follows:

$$\{u\}_i = \begin{Bmatrix} u_{1i} \\ u_{2i} \end{Bmatrix} = u_{1i} \cdot \begin{Bmatrix} 1 \\ \dfrac{u_{2i}}{u_{1i}} \end{Bmatrix} = u_{1i} \cdot \begin{Bmatrix} 1 \\ \dfrac{-c_2}{c_2 - m_2.\omega_i^2} \end{Bmatrix}, (i = 1{,}2) \tag{5}$$

Using quasi-normal coordinates and the method of averaging [8] one obtains the amplitudes of the vibrations in the vicinity of each of the resonances $\omega_i \equiv \Omega$ i.e.:

$$a_i(z) = \frac{f_i}{2.\omega_i \cdot \sqrt{(1-z)^2 + \nu_i^2}}\,;\ z = \frac{\Omega}{\omega_i}; \nu_i = \frac{n_i}{\omega_i}, i = 1{,}2{,}3. \tag{6}$$

The additional resistance torque in case of small disturbances $(\Omega - \omega_i)$, respectively $(1-z)$, is:

$$\Delta L_i(z_i.\omega_i) = \frac{f_i \cdot f_4.\nu_i}{4.\omega_i^2\left[(1-z)^2 + \nu_i^2\right]}, i = 1{,}2{,}3 \tag{7}$$

and the common resistant torque is:

$$\begin{aligned} L_i(z,\omega_i) &= M_c(z,\omega_i) + \Delta L_i(z,\omega_i) = \\ &= k.\omega_i^2.z_i^2 + \frac{f_i \cdot f_4}{4.\omega_i^2} \cdot \frac{\nu}{\left[(1-z^2) + \nu_i^2\right]} \end{aligned} \tag{8}$$

where $f_i = \dfrac{c_2.e.\sum_{k=1}^{2} u_{ik}}{M_i}$, $M_i = m_1.u_{1i}^2 + m_2.u_{2i}^2$, $i = 1{,}2$, $M_3 = m_2$ The analytical procedures for deriving Eqs. (6 - 8) are presented in details in [8].

3. Example based on the analytical investigations

Based on the theoretical results and conclusions and for the geometrical and physical parameters of a wind turbo generator with power about 350 [kW] the frequency spectrum of the pillar, as well as, the amplitudes, modal vectors, stability of the vibrations and others in quasi-stationary transitional modes are calculated. The following input data are used: the pillar – a beam of steel tube with height L = 20 [m]; density ρ = 8.1 $\times 10^3$[kg/m^3]; outer and inner diameters D_0 = 2 [m], D_1 = 1.96 [m], respectively; mass of the gondola (stator) $m^* = 2.3 \times 10^3$ [kg]; distributed beam mass $m_0 = 1.\times 10^3$ [kg]; beam rigid body mass reduced to the beam tip - $m_r = 9.7 \times 10^3$, in accordance with Fig. 1., 2. and Eq. 2; stiff and damping coefficients of the beam c_1 = 9.6 $\times$ 10^6 [N/m], b_1 = (10÷50) $\times$ 10^3 [N.s/m], respectively; rotor mass (mass in point C) m_C = 1.6 $\times$ 10^3 [kg]; mass inertia moment of the rotor J_C = 100 [m^4]; rotor eccentricity e = 0.002 [m]; driving M_d, respectively, resistant M_c torques are of the type

$$\begin{aligned} &M_d(\dot{q}_4) \approx M_d(\Omega) = M_0 - 12.\Omega\,; \\ &M_c(\dot{q}_4) \approx M_c(\Omega) = 0.5.\Omega^2\,;\ f_4 = c_2.e\,, \end{aligned} \tag{9}$$

where Ω is the angular velocity of the rotor, torque for zero angular velocity $M_0 = 300.\tau$ (1.7.τ ≤ 17) . The results are obtained for two cases. The first one discusses extremely elastic shaft with stiff coefficient $c_2 = 0.4 \times 10^4$ [N/m] and damping $b_2 = 11 \times 10^3$ [N.s/m]. The second one is several times stiffer beam, i.e.: $c_2 = 5 \times 10^4$ [N/m]. In Fig. 3. the results for $a_1(z)$, $M_d(z)$, $L(z)$ are depicted in case for transition through the first resonance $\Omega = \omega_1$, i.e. z = 1, $\omega_1 = 15.3$ [s^{-1}] for two different values of ν_1, i.e.: $\nu_1 = 0.22$ [-]; $\nu_1 = 0.16$ [-], where $\nu_1 = \dfrac{n}{\omega_1} = \dfrac{b_1}{2.M_1}$, M_1 is the equivalent mass according to the first natural frequency. Varying the driving torque for zero velocity, M_{d0}, i.e. changing τ, and for quasi-stationary increasing, respectively, decreasing of the rotor velocity, it could be observed violence of the condition for vibration stability. As a result of that it in the frequency range, from point A to point B and for increasing of Ω, respectively z, as well as, from point D to point C (Fig. 3.) for decreasing of Ω, collapse of the vibrations could be observed.

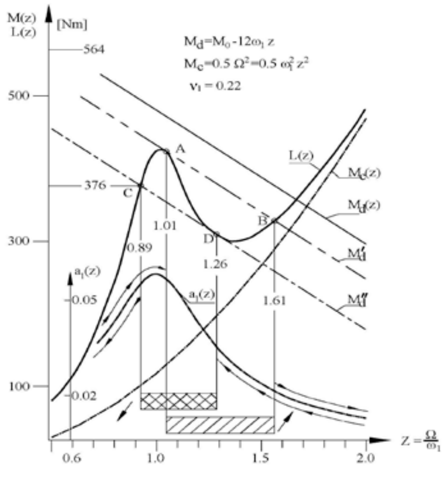


Figure 3. Zones of instability for the rotor rotation

4. Numerical analysis of the rotor for wind power generator

The rotor of Section 3 is analyzed using numerical methods for verification and analysis in case of earthquake excitation. The numerical approach for dynamics simulation of rigid and flexible multibody systems enables many nonlinear effects as contact, friction, external excitations (seismic disturbances) to be taken into account. In Fig. 4. the system of the previous chapter is regarded as a wind power generator with three propellers. In the figure, consequently, the structure of the wind generator, the adjustment of the electric generator to the gondola and flexible deflections of the pillar, as well as, one of the propeller and the flexible degrees of freedom are depicted. With the degree of freedom that presents the ground shaking is denoted.

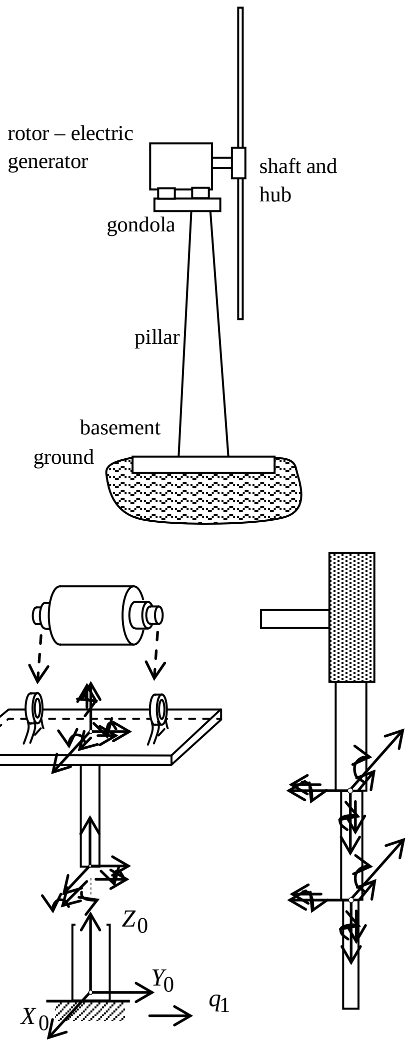


Figure 4. The design scheme of the wind power generator and its units

Using the principle of the virtual work all external forces, including the inertia, stiffness and damping forces are reduced to the system coordinates to derive the dynamic equation that are expressed by a system of **n×n** ODE with respect to the generalized coordinates $\mathbf{q}$, i.e.:

$$\mathbf{M}\cdot\ddot{\mathbf{q}}+\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{S} \tag{10}$$

where **M** is ***n*×*n*** mass matrix**, S** is $n\times 1$ matrix-vector of the generalized forces, $\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})$ is velocity depend term. Numerical analysis of the structure in Section 3, solution of Eq. (10), is conducted. In Fig. 5 the behavior and transitional processes of the turbo generator are displayed that fully coincides with the results of the analytical analysis.

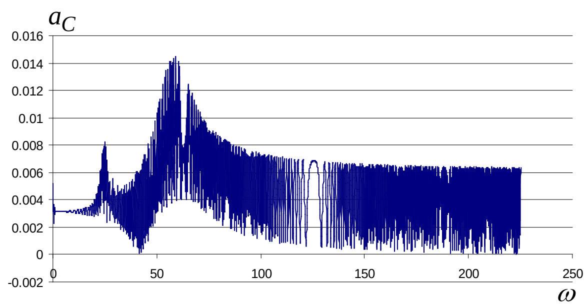


Fig. 5. Transitional behavior of the turbogenerator of Section 3

Earthquake shaking affects forced motion of the structure basement. The ground motion is registered by so called strong-motion accelerating graphs and normally consists in three orthogonal components of the ground acceleration. The velocity and displacement of the ground are obtained integrating the data of the accelerating gram. They also can be analyzed to obtain direct estimation of peak ground motion, duration shaking and frequency. For a specific region using statistical data the accelerating grams for the three orthogonal space displacements are as follows:

$$\ddot{q}_i=\ddot{q}_i(t), i=1,2,3 \qquad (11$$

These are actually reonomic kinematic constraints that depend on time. For a multibody system with degree of freedom n which motion is described by ODE, Eq. (10), subject to m reonomic constraints (Eq. 11) the dynamic equations are presented as follows:

$$\begin{aligned}&\mathbf{M}\cdot[\ddot{q}_1\ \cdots\ \ddot{q}_{k+1}\ \cdots\ \ddot{q}_{k+m}\ \cdots\ \ddot{q}_n]^{\backslash}+\\&+\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})=[S_1\ \cdots\ S_{k+1}\ \cdots\ S_{k+m}\ \cdots\ S_n]^{\backslash}\end{aligned} \tag{12}$$

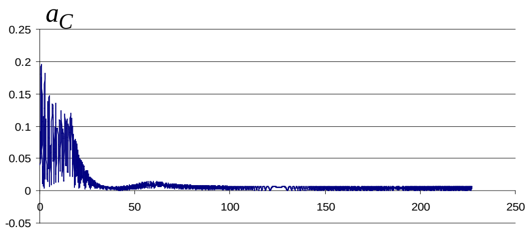
In Eq. (12) the coordinates $\ddot{q}_i=\ddot{q}_i(t)$, $i=k+1,k+2,..,k+m$, included as parameters of the reonomic constraints, are known, while the generalized forces $S_i=S_i(t)$, $i=k+1,$

$k+2...,k+m$ are unknown. In other words, the solution of equations system, Eq. (12), results in solution of mixed direct and inverse dynamic problem to find the values of the coordinates $\ddot{q}_i = \ddot{q}_i(t), i = 1,...,k, k+m+1,...,n$ and the generalized forces $S_i = S_i(t), i = k+1, k+2...,k+m$. The dynamic equations, subject to reonomic constraints, Eq. (12), are transformed with respect to the unknown parameters as follows:

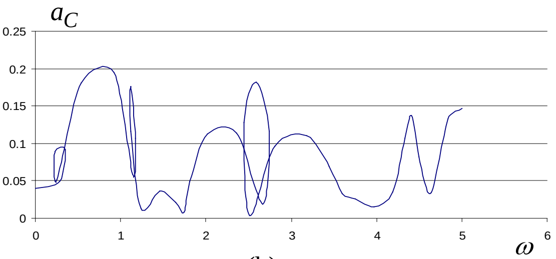
$$\begin{aligned}&\underline{\mathbf{M}} \cdot \left[\ddot{q}_1 \ \cdots \ S_{k+1} \ \cdots \ S_{k+m} \ \cdots \ \ddot{q}_n\right]^{\backslash} + \\ &\mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) = -\underline{\underline{\mathbf{M}}} \cdot \left[\ddot{q}_{k+1} \ \cdots \ \ddot{q}_{k+m}\right]^{\backslash} + \underline{\mathbf{S}}\end{aligned} \tag{13}$$

where the matrices $\underline{\mathbf{S}}$, $\underline{\underline{\mathbf{M}}}$, $\underline{\mathbf{M}}$ are compiled according to the indices of the coordinates subject to the reonomic constraints.

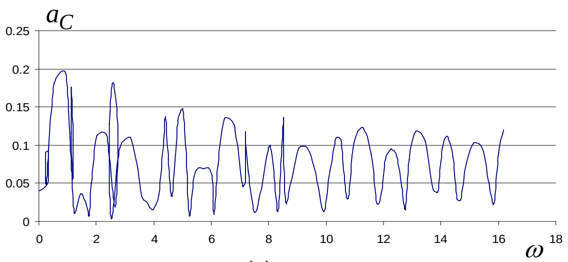
The structure of Fig. 4. is imposed on a earthquake for 5 seconds with acceleration of the excitation $\ddot{q}_1 = 0.2.\mathbf{G}.\pi.cos(\pi.t)$. In Fig. 6. a, b, c the behavior of the system (amplitude of the rotor displacement with respect to the basement as a function of the rotor angular velocity) is displayed. It could be seen that the earthquake causes extremely high system displacement and, even more, in some instants diminish rotor angular velocity.



(a)



(b)



(c)

Fig. 6. Time histories of the rotor displacements function to the rotor angular velocity and the earthquake excitation
(a) for the full time of the computational procedure;
(b) for the first 1.7 second of the earthquake excitation;

5. Conclusions

Analytical approach to dynamics simulation of large structures of rigid and flexible bodies is proposed including high speed rotating machines taking into account design inaccuracy as eccentricity of the rotor and external disturbances.

The results are verified using numerical method for dynamic analysis of multibody systems. Example of wind power generator with three propellers is proposed.

An approach for dynamic analysis of structures subject to seismic excitations is applied for simulation of the wind power generator behavior imposed on an earthquake.

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