Formation of Investment Behavior Strategy using the Game-theoretic Method

Andrey Zaytsev [1], Nikolay Dmitriev [1], Rinat Fayzullin [2], Ekaterina Mihel [1]

[1] *Peter the Great Saint-Petersburg Polytechnic University, St. Petersburg, Russia*
[2] *Izhevsk State Technical University, Izhevsk, Russia*

***Abstract* – This work is aimed at developing the behavior strategy of an economic entity involved in investment activities using the game-theoretic method. To build a qualitative model, the authors considered the economic aspects of game theory and developed an algorithm for adopting the game-theoretic criteria to investment planning. The authors analyzed the investment portfolios available for business entities on the Russian market to test the model. The outcome of the article is the formation of the authors' approach to rationalizing the behavior of an economic entity in the course of investment activities. A practical example is given in which the suggested method is used when various investment portfolios are available. The approach was tested to prove that it can be practically applied and further used in the construction of investment strategies.**

***Keywords* – investment portfolio, game theory method, investment behavior, game theory, investment strategy.**

1. Introduction

Economics faces the irremovable problem of effective decision-making in the time when the external environment is forever changing.

DOI: 10.18421/TEM102-23
https://doi.org/10.18421/TEM102-23

Corresponding author: Andrey Zaytsev,
Peter the Great Saint-Petersburg Polytechnic University, St. Petersburg, Russia.
Email: andrey_z7@mail.ru

Received: 19 October 2020.
Revised: 24 February 2021.
Accepted: 03 March 2021.
Published: 27 May 2021.

Economic agents create a countless number of situations when the decisions taken by some entities have a direct or indirect impact on the decision-making processes of other entities [1]. A conflict nature of this situation makes it necessary to substantiate the behavior of economic agents, since efficiency directly depends on the decisions they take. Moreover, participants in mutually dependent processes can be statical or dynamic, changing and developing all the time [2]. One of the ways to satisfy diverse interests is to set objectives to take mathematically-based decisions in the time of uncertainty.

Using the achievements of mathematics in economic research is a classical form of obtaining quality results to rationalize processes [3]. The game theory method can be used to model conflict situations, where alternative options of behavior can be chosen. Thus, mathematical formulation of goal-setting in decision-making and substantiation of approaches to practical analysis of these decisions help us to see the situation critically and develop the best behavior strategy to maximize the gain [4], [5].

Today, the excitement around game theory has run its course. It is not because this subject has given round, but because its influence has spread on the entire subject of economics and turned game theory into a standard economic tool [6]. Game theory as an economic tool can be used to model feasible solutions in the time of conflict and competitive struggle between fighting parties. Game theory models can be built to define the best strategies considering the negative conditions of the external environment and reducing risk level [4]. Scientists developed various game theory criteria to search for the best solutions in the time of uncertainty. For example, the criteria of Laplace's, Wald's, Savage's, Hurwitz's, Bayes-Laplace's. Lots of criteria can be used and modified depending on a specific situation [7]. Each of the above criteria makes it possible to design an investment behavior strategy and rationalize business administration processes in an enterprise. However, in order to adopt the best strategy, alternatives to every event have to be taken into account.

Conflict situations are caused by competing interests of the parties, when the goals of economic entities are varied or if the targets of one of the players are not defined clearly. In case one of the participants in conflict does not understand clearly what his goals are and the other one has a strategy of purposeful actions, such game is called “a game with nature” [8], [9], [10]. In the case of a game with nature, only one force demonstrates conscious actions (Player 1). Such a participant can be an economic entity. The second force is expressed with objective reality and acts spontaneously, without any clear goals (Player 2). For instance, the second player can be the stock market, which is relatively uncertain for external players [11]. After building the models of behavior strategies of Player 1, it is possible to evaluate the effectiveness of each available option and analyze if the resources are sufficient for taking any actions. Investment conditions are largely dependent on economic cycles and market conditions, so they can be both favorable and negative. From the perspective of game theory, the actions of Player 2 are traditionally considered as the worst-case scenario of an event for Player 1, the occurrence of which is accompanied with a maximum level of risk [12].

This research considers the possibility for applying game theory to investment analysis in order to formulate behavior strategy with maximum efficiency. The conditions of the research are based on the fact that any economic entity forms an investment portfolio, taking into account its interests, the admissible level of risk, and thus, chooses the relevant types of assets. During investment planning, it is possible to control the investment processes, evaluate their effectiveness and further correct them, which can also be the basis of the game theory approach [3], [8]. Generally speaking, the game theory method helps to optimize the investment portfolio of an economic entity based on the data of the average market return on assets for a concrete period [11].

The object of the research is the investment behavior strategies accessible in the course of capital investment. The subject of the research is the methodological tools of the game theory method applied in the field of investment design.

The study is aimed at adapting the game theory criteria to investment analysis and defines the best investment behavior strategy when there are lots of alternatives. In order to reach this goal the following objectives have to be accomplished: the existing research studies on applying game theory in economics, which substantiate the authors’ proposals, have to be analyzed; a game matrix adapted to the investment conditions has to be formed and the game theory criteria have to be applied to this matrix; an algorithm has to be built to solve an antagonistic game and finally choose the best strategy; the algorithm has to be tested on investment portfolios.

The scientific novelty of the research is in developing the theoretical-methodological approach to investment strategy formulation in the time when there are many alternative solutions, which can be diversified in the most rational proportions. The authors’ approach represents a new view on the interaction between the criteria of game theory and investment analysis, which opens the door to further search for solutions to investment problems in the conditions of uncertainty.

2. Literature Review

As early as in the last century, they started to consider the possibilities of taking the best economic and management decisions in case there are many alternatives to choose from. The use of game theory in economics traces back to the major work «Theory of Games and Economic Behavior», written by J. Von Neumann & O. Morgenstern [13]. Many researchers investigated in detail the game theory method in economics, for instance: A. Dixit & B. Nalebuff [4], W.P. Fox & R. Burks [7], J. Newton [10], H.T. Smit & L. Trigeorgis [12]. These studies contain a sufficient amount of theoretical knowledge and many practical ways concerning the adaptation of the game theory method to making economic decisions. In terms of this study, interest is represented by research papers where individual game theory aspects are considered, for example, works by F. Allen & S. Morris [8], G. Askari, M.E. Gordji & C. Park [14], V. Bieta & P. Smelyanets [15], E. Bonabeau [2], R. Breen [9], M. Breton [11], C. Daskalakis, A. Deckelbaum & A. Kim [16], L. Wei & Q. Hao-peng [5]. Thus, there are many applied works on this topic. However, using game theory in the field of investing is still a problem that has not been investigated thoroughly and has to be researched further.

In order to adapt game theory to investment analysis, it is suggested that research papers on the matters of rationalizing investment activities at different levels of management are to be analyzed. The paper by [17] analyzes the processes of formulating the enterprise strategy, in particular, the significance of optimizing investment activities for long-term development of a business entity, as well as the possibility of using mathematical methods for multicriteria optimization of capital and cost structure. The paper by [18] substantiates the formation of the investment program of an industrial enterprise given the hierarchical priority of objects in the conditions when resources are tight, which causes the need to conduct investment activities aimed at improving the resulting efficiency in the strategic perspective. The study by [19] suggests a process for developing a corporate investment program that can be represented in a form of an optimization task of efficiency improvement, considering the presence of resource constraints. The studies by [20] and [21] reflect a growing need for using more up-to-day methods to evaluate the investment efficiency of enterprises, related to government structures, based on the methods of rationalization and simulation of investment behavior strategy. The works by [22] describe approaches to assessing the role of the investment development mechanism at the macro level, using the existing methods, which can applied to build models for optimizing investment behavior strategies. The significance of rationalizing investment activities at the macro level is also considered in the paper by [21], which study the matters of long-term investment and suggest ways to develop various management strategies. The paper by [23] analyzes some aspects of a company's economic activity, including investment activities in the strategic perspective. Thus, most research studies in the field of investment analysis are aimed at finding models for making activity more efficient through reducing potential risk and here mathematical methods can be used, such as the game theory criteria.

A number of research studies make attempts to adapt mathematical methods, in particular, game theory, to various aspects of economic relations, including investment activities. For example, the paper by [24] substantiates the necessity to build a model for determining the intensity of investment processes in an enterprise based on mathematical methods and decision-making models that consider uncertainty and risk factors, which the game theory criteria can be referred to. The paper by [25] analyzes the theory of option games for investing into intellectual development of an enterprise and innovations, using asymmetric information and cooperative behavior. The application of the mathematical tools of game theory on financial markets is considered in the paper by [15]. The paper by [26] addresses the processes of selecting a behavior strategy by an organization, using game theory, and presents mathematical models for analyzing the interaction between economic agents. These works are a good basis for conducting the authors' research, since they explain individual aspects of rationalizing the investment processes of business entities using the achievements of game theory.

Based on the existing works, the method, proposed by the authors for finding the best investment behavior strategies using the game theory method, can be considered substantiated. However, it has been revealed that the studies have constraints, since the application of game theory is mostly studied from the perspective of choosing the only admissible decision while there are many. In this study, we consider the possibility of maximizing the positive effect taking into account the choice of several decisions that can be implemented in certain proportions.

3. Methodological Aspects of Adapting the Game Theory Criteria to Investment Strategy

The first step in the authors' approach is to build a payment matrix of the game. Investment behavior strategies, whose totality can be unified in investment portfolios, are considered in the rows of the matrix. The columns are the time periods within which specific investment strategies worked. An example of the game matrix is presented in Figure 1. Note: ip_i is the type of investment behavior strategy (investment portfolio); t_j is the time period; iv_{ij} is the efficiency value of investment strategy in a specific period (efficiency should be understood as return on investment).

Player 2

Player 1

	t_1	t_2	t_3	…	t_n
ip_1	iv_{11}	iv_{12}	iv_{13}	…	iv_{1n}
ip_2	iv_{21}	iv_{22}	iv_{23}	…	iv_{2n}
ip_3	iv_{31}	iv_{32}	iv_{33}	…	iv_{3n}
…	…	…	…	…	…
ip_m	iv_{m1}	iv_{m2}	iv_{m3}	…	iv_{mn}

Figure 1. Game matrices

In order to make calculations, it is reasonable to use linear programming methods and a simplex method. An economic entity can have a different number of investment strategies *($i \in 1, 2, 3, \dots m$)*, while investments can be meant for a specific number of time periods *($j \in 1, 2, 3, \dots n$)*. Based on the linear programming methods, the available options of investment behavior with the use of the game theory criteria are calculated with a return-on-investment matrix ($A = (iv_{ij})_{n\times m}$), which can be presented in expanded form as follows:

$$A = \begin{pmatrix} iv_{11} & iv_{12} & iv_{13} & iv_{1n} \\ iv_{21} & iv_{22} & iv_{23} & iv_{2n} \\ iv_{31} & iv_{32} & iv_{33} & iv_{3n} \\ iv_{m1} & iv_{m2} & iv_{m3} & iv_{mn} \end{pmatrix}, (1)$$

The matrix element A (iv_{ij}) shows what level of return can be obtained from choosing a specific investment behavior strategy in various time periods. The most optimal proportions of the implemented investment strategies maximize the guaranteed return independently on time periods. From the perspective of the game theory criteria, this problem is antagonistic (Player 1 is an economic entity; Player 2 is a time period). Thus, it can be concluded that the unstable dynamics of a time period does not let the economic entity to maximize the efficiency of investment activities and can lead to considerable losses [3]. Correspondingly, the actions of Player 2 entail the highest risk for the economic entity (investor). Thus, the authors' model is aimed at obtaining return on investment in case the damage from the time period is at its maximum. When making an investment, the economic entity can choose the investment behavior strategy based on an aggressive or passive portfolio. Using the built game matrix, the highest and the lowest value of the game as well as the saddle point must be determined [16]. The concluding matrix is shown in Table 1.

Table 1. Game matrix with the determined highest and lowest value

	t_1	t_2	t_3	…	t_n	min
ip_1	iv_{11}	iv_{12}	iv_{13}	…	iv_{1n}	$iv_{1\ min}$
ip_2	iv_{21}	iv_{22}	iv_{23}	…	iv_{2n}	$iv_{2\ min}$
ip_3	iv_{31}	iv_{32}	iv_{33}	…	iv_{3n}	$iv_{3\ min}$
…	…	…	…	…	…	…
ip_m	iv_{m1}	iv_{m2}	iv_{m3}	…	iv_{mn}	$iv_{m\ min}$
max	$iv_{max\ 1}$	$iv_{max\ 2}$	$iv_{max\ 3}$	…	$iv_{max\ n}$	–

The lowest value of the game: $\alpha = max_i\ min_j\ \alpha_{ij} = max\ \{iv_{1\ min};\ iv_{2\ min};\ iv_{3\ min};\ \ldots;\ iv_{m\ min}\}$.

The highest value of the game: $\beta = min_j\ max_i\ \beta_{ij} = min\ \{iv_{max\ 1};\ iv_{max\ 2};\ iv_{max\ 3};\ \ldots;\ iv_{max\ n}\}$.

The inequation of the highest and lowest value $(\alpha \neq \beta)$ leads to the lack of the saddle point in the antagonistic game. Consequently, the absence of pure strategy is observed, while the value of the game is within a range: $\alpha \leq v \leq \beta$. In order to find the solution to the game in mixed strategies, an optimization model of two strategies has to be built: Player 1 is strategy *X*; Player 2 is *Y*. The mathematical expectation of return on investment cannot be lower than the value of the game (v).

The maximum efficiency from the investment behavior strategy is reached by the economic entity if the admissible return is achieved with minimum risk. This efficiency is calculated mathematically by multiplying the mixed strategy of Player 1 (X) or the investment behavior strategy by the transposed return-on-investment matrix (A^T). The strategy of Player 1 must tend to maximizing return ($\geq v$): $X * A^T = (x_1;\ x_2;\ x_3;\ \ldots;\ x_n) * (iv_{ij})_{n\times m}\ |\geq v$.

As a result, the following system of inequations can be built to find the expected value of return from the implemented investment behavior strategy. In this model (Formula 2): $x_1 + x_2 + x_3 + x_n = 1,\ x_1 \geq 0;\ x_2 \geq 0;\ x_3 \geq 0;\ x_n \geq 0$. In order to simplify the mathematical calculations, it is suggested that the coefficient p_n, equal to the following dependence be introduced: $p_1 = x_1 / v$; $p_2 = x_2 / v$; $p_3 = x_3 / v$; $p_n = x_n / v$. In the modernized model (Formula 3): $p_1 + p_2 + p_3 + p_n = 1/v,\ p_1 \geq 0;\ p_2 \geq 0;\ p_3 \geq 0;\ p_n \geq 0$. The maximization of the gain of Player 1 in the modernized model is in the minimization of the value $1/v$.

$$\begin{cases} \mathrm{iv}_{11} * x_1 + \mathrm{iv}_{21} * x_2 + \mathrm{iv}_{31} * x_3 + \mathrm{iv}_{m1} * x_n \geq v \\ \mathrm{iv}_{12} * x_1 + \mathrm{iv}_{22} * x_2 + \mathrm{iv}_{32} * x_3 + \mathrm{iv}_{m2} * x_n \geq v \\ \mathrm{iv}_{13} * x_1 + \mathrm{iv}_{23} * x_2 + \mathrm{iv}_{33} * x_3 + \mathrm{iv}_{m3} * x_n \geq v \\ \mathrm{iv}_{1n} * x_1 + \mathrm{iv}_{2n} * x_2 + \mathrm{iv}_{3n} * x_3 + \mathrm{iv}_{mn} * x_n \geq v \end{cases}, (2)$$

$$\begin{cases} \mathrm{iv}_{11} * p_1 + \mathrm{iv}_{21} * p_2 + \mathrm{iv}_{31} * p_3 + \mathrm{iv}_{m1} * p_n \geq 1 \\ \mathrm{iv}_{12} * p_1 + \mathrm{iv}_{22} * p_2 + \mathrm{iv}_{32} * p_3 + \mathrm{iv}_{m2} * p_n \geq 1 \\ \mathrm{iv}_{13} * p_1 + \mathrm{iv}_{23} * p_2 + \mathrm{iv}_{33} * p_3 + \mathrm{iv}_{m3} * p_n \geq 1 \\ \mathrm{iv}_{1n} * p_1 + \mathrm{iv}_{2n} * p_2 + \mathrm{iv}_{3n} * p_3 + \mathrm{iv}_{mn} * p_n \geq 1 \end{cases}, (3)$$

In this case, the strategy of Player 2 (Y) is the opposite of the actions of the economic entity and considers the maximum risk of market uncertainty, while the expected value cannot exceed the value of the game ($\leq v$): $Y * A^T = (y_1;\ y_2;\ y_3;\ \ldots;\ y_n) * (iv_{ij})_{n\times m}\ | \leq v$. In the model of the system of inequations for elemental forces (time period) (Formula 4): $y_1 + y_2 + y_3 + y_n = 1,\ y_1 \geq 0;\ y_2 \geq 0;\ y_3 \geq 0;\ y_n \geq 0$. In order to simplify the mathematical calculations, it is suggested that the coefficient q_n, equal to the following dependence be introduced: $q_1 = y_1 / v$; $q_2 = y_2 / v$; $q_3 = y_3 / v$; $q_n = y_n / v$. In the modernized model (Formula 5): $q_1 + q_2 + q_3 + q_n = 1/v,\ q_1 \geq 0;\ q_2 \geq 0;\ q_3 \geq 0;\ q_n \geq 0$. Player 2 goes after maximizing $1/v$, i.e. reducing the value of the game.

$$\begin{cases} \mathrm{iv}_{11} * y_1 + \mathrm{iv}_{21} * y_2 + \mathrm{iv}_{33} * y_3 + \mathrm{iv}_{m1} * y_n \leq v \\ \mathrm{iv}_{12} * y_1 + \mathrm{iv}_{22} * y_2 + \mathrm{iv}_{32} * y_3 + \mathrm{iv}_{m2} * y_n \leq v \\ \mathrm{iv}_{13} * y_1 + \mathrm{iv}_{23} * y_2 + \mathrm{iv}_{33} * y_3 + \mathrm{iv}_{m3} * y_n \leq v \\ \mathrm{iv}_{1n} * y_1 + \mathrm{iv}_{2n} * y_2 + \mathrm{iv}_{3n} * y_3 + \mathrm{iv}_{mn} * y_n \leq v \end{cases}, (4)$$

$$\begin{cases} \mathrm{iv}_{11} * q_1 + \mathrm{iv}_{21} * q_2 + \mathrm{iv}_{33} * q_3 + \mathrm{iv}_{m1} * q_n \leq 1 \\ \mathrm{iv}_{12} * q_1 + \mathrm{iv}_{22} * q_2 + \mathrm{iv}_{32} * q_3 + \mathrm{iv}_{m2} * q_n \leq 1 \\ \mathrm{iv}_{13} * q_1 + \mathrm{iv}_{23} * q_2 + \mathrm{iv}_{33} * q_3 + \mathrm{iv}_{m3} * q_n \leq 1 \\ \mathrm{iv}_{1n} * q_1 + \mathrm{iv}_{2n} * q_2 + \mathrm{iv}_{3n} * q_3 + \mathrm{iv}_{mn} * q_n \leq 1 \end{cases}, (5)$$

Thus, the best strategies look as follows: Player 1: $P = p_1 + p_2 + p_3 + p_n \rightarrow min$; Player 2: $Q = q_1 + q_2 + q_3 + q_n \rightarrow max$. In order to solve both strategies, it is necessary to use a simplex method and build vectors: $P = (p_1;\ p_2;\ p_3;\ p_n)$; $Q = (q_1;\ q_2;\ q_3;\ q_n)$. Since $P_{min} = Q_{max} = 1 / v$, then $v = 1 / Q_{max} = 1 / P_{min}$. Hence the players' strategy vectors are equal: $X = (x_1;\ x_2;\ x_3;\ x_n)$; $Y = (y_1;\ y_2;\ y_3;\ y_n)$. The result of the suggested method is a model for obtaining the guaranteed return by the economic entity, equal to the value of the game (v), even in case unfavorable events take place. The best strategy of the economic entity is to pursue the investment behavior strategy in the given proportions: $ip_1 = x_1$; $ip_2 = x_2$; $ip_3 = x_3$.

Thus, the authors have suggested an algorithm of action in order to find the best investment behavior strategy, which is presented in Table 2.

Table 2. An algorithm for formulating the investment behavior strategy

1	To form investment portfolios *(ip_i)*, available for implementation. $i \in 1, 2, 3, \ldots m$
2	To define the time periods for investing *(t_j)*. $j \in 1, 2, 3, \ldots n$
3	To find the lowest (α) and the highest (β) value of the game and determine the presence of the saddle point. $\alpha = max_i\ min_j\ \alpha_{ij} = max\ \{iv_{1\ min};\ iv_{2\ min};\ iv_{3\ min};\ \ldots;\ iv_{m\ min}\}$ $\beta = min_j\ max_i\ \beta_{ij} = min\ \{iv_{max\ 1};\ iv_{max\ 2};\ iv_{max\ 3};\ \ldots;\ iv_{max\ n}\}$ If $(\alpha = \beta)$, there is only one solution; If $(\alpha \neq \beta)$, the solution is within a range between α and β, i.e. $\alpha \le v \le \beta$.
4	To build the vectors of the mixed strategy of the economic entity (investor) (X) and elemental forces (Y).
5	To design the best strategies and find the value of the game (v). Investor: $P = p_1 + p_2 + p_3 + p_n \rightarrow min$ Elemental forces: $Q = q_1 + q_2 + q_3 + q_n \rightarrow max$ $P_{min} = Q_{max} = 1 / v$ $v = 1 / Q_{max} = 1 / P_{min}$
6	To obtain the vectors of the investment behavior strategy. $X = (x_1;\ x_2;\ x_3;\ x_n)$
7	To make investment in the most effective proportions. $ip_1 = x_1$; $ip_2 = x_2$; $ip_3 = x_3$

4. Testing the Proposed Algorithm by a Practical Example

It is proposed to consider the testing of this method in the example of investment portfolios available for implementation for a concrete industrial enterprise on the Russian market. Six heterogeneous investment portfolios *(ip)* have been selected, which is sufficient to confirm that the authors' methodology works. The following vectors correspond to the selected investment portfolios:

$ip_1 = (0.0248;\ 0.0539;\ 0.061;\ 0.1286);$
$ip_2 = (0.0611;\ 0.0344;\ 0.3036;\ 0.1556);$
$ip_3 = (0.1539;\ 0.0812;\ 0.1036;\ 0.2945);$
$ip_4 = (0.069;\ 0.0663;\ 0.0814;\ 0.039);$
$ip_5 = (0.174;\ 0.151;\ 0.1084;\ 0.1306);$
$ip_6 = (0.025;\ 0.0599;\ 0.1158;\ 0.1001).$

The average increment as of early 2020 is considered for the following time periods (t): 3 months, 6 months, 1 year, and 3 years. The game matrix is shown in Table 3. The return-on-investment matrix is presented by Formula 6.

Table 3. Game matrix

	3 months	6 months	1 year	3 years
ip_1	0.0248	0.0539	0.061	0.1286
ip_2	0.0611	0.0344	0.3036	0.1556
ip_3	0.1539	0.0812	0.1036	0.2945
ip_4	0.069	0.0663	0.0814	0.039
ip_5	0.174	0.151	0.1084	0.1306
ip_6	0.025	0.0599	0.1158	0.1001

$$A = \begin{pmatrix} 0{,}0248 & 0{,}0539 & 0{,}061 & 0{,}1286 \\ 0{,}0611 & 0{,}0344 & 0{,}3036 & 0{,}1556 \\ 0{,}1539 & 0{,}0812 & 0{,}1036 & 0{,}2945 \\ 0{,}069 & 0{,}0663 & 0{,}0814 & 0{,}039 \\ 0{,}174 & 0{,}151 & 0{,}1084 & 0{,}1306 \\ 0{,}025 & 0{,}0599 & 0{,}1158 & 0{,}1001 \end{pmatrix}, (6)$$

In order to determine the presence of the saddle point, the game matrix is built in Table 4 and the lowest and highest value is defined. Figure 2 presents the graphical illustration of the return of the considered investment portfolios.

Table 4. Game matrix with the determined lowest and highest value

	1 month	3 months	6 months	1 year	min
ip_1	0.0248	0.0539	0.061	0.1286	0.0539
ip_2	0.0611	0.0344	0.3036	0.1556	0.0344
ip_3	0.1539	0.0812	0.1036	0.2945	0.0812
ip_4	0.069	0.0663	0.0814	0.039	0.039
ip_5	0.174	0.151	0.1084	0.1306	0.1084
ip_6	0.025	0.0599	0.1158	0.1001	0.025
max	0.174	0.151	0.3036	0.2945	–

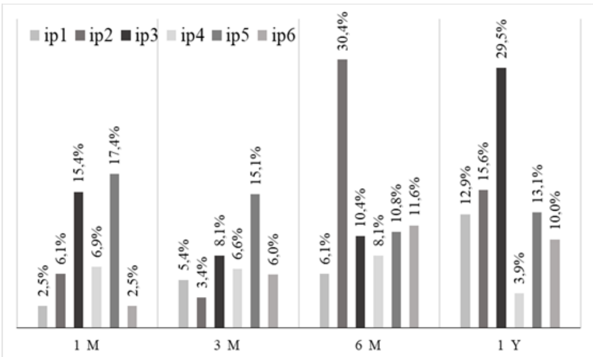

Figure 2. Return of the considered investment portfolios

The lowest value: $\alpha = max_i\ min_j\ \alpha_{ij} = max\ \{0.0539;\ 0.0344;\ 0.0812;\ 0.039;\ 0.1084;\ 0.025\} = 0.1084$.

The highest value: $\beta = min_j\ max_i\ \beta_{ij} = min\ \{0.174;\ 0.151;\ 0.3036;\ 0.2945\} = 0.151$.

Since $\alpha = 0.1084$ and $\beta = 0.151$ ($\alpha \neq \beta$), then there is no saddle point. Hence, the value of the game is within a range of $0.1084 \le v \le 0.151$. Based on the obtained values the return on investment of the business entity will be no less than 10.84% and can even reach 15.1%, if a stable growth of the assets in the investment portfolio is observed.

1. The system of inequations for Player 1 (Formula 7).
2. The simplified system of inequations for Player 1 (Formula 8).
3. The system of inequations for Player 2 (elemental forces) (Formula 9).
4. The simplified system of inequations for Player 2 (Formula 10).

$$\begin{cases} 0{,}0248 * x_1 + 0{,}0611 * x_2 + 0{,}1539 * x_3 + 0{,}069 * x_4 + 0{,}174 * x_5 + 0{,}025 * x_6 \geq v \\ 0{,}0539 * x_1 + 0{,}0344 * x_2 + 0{,}0812 * x_3 + 0{,}0663 * x_4 + 0{,}151 * x_5 + 0{,}0599 * x_6 \geq v \\ 0{,}061 * x_1 + 0{,}3036 * x_2 + 0{,}1036 * x_3 + 0{,}0814 * x_4 + 0{,}1084 * x_5 + 0{,}1158 * x_6 \geq v \\ 0{,}1286 * x_1 + 0{,}1556 * x_2 + 0{,}2945 * x_3 + 0{,}039 * x_4 + 0{,}1306 * x_5 + 0{,}1001 * x_6 \geq v \end{cases}, \quad (7)$$

$$\begin{cases} 0{,}0248 * p_1 + 0{,}0611 * p_2 + 0{,}1539 * p_3 + 0{,}069 * p_4 + 0{,}174 * p_5 + 0{,}025 * p_6 \geq 1 \\ 0{,}0539 * p_1 + 0{,}0344 * p_2 + 0{,}0812 * p_3 + 0{,}0663 * p_4 + 0{,}151 * p_5 + 0{,}0599 * p_6 \geq 1 \\ 0{,}061 * p_1 + 0{,}3036 * p_2 + 0{,}1036 * p_3 + 0{,}0814 * p_4 + 0{,}1084 * p_5 + 0{,}1158 * p_6 \geq 1 \\ 0{,}1286 * p_1 + 0{,}1556 * p_2 + 0{,}2945 * p_3 + 0{,}039 * p_4 + 0{,}1306 * p_5 + 0{,}1001 * p_6 \geq 1 \end{cases}, \quad (8)$$

$$\begin{cases} 0{,}0248 * y_1 + 0{,}0611 * y_2 + 0{,}1539 * y_3 + 0{,}069 * y_4 + 0{,}174 * y_5 + 0{,}025 * y_6 \leq v \\ 0{,}0539 * y_1 + 0{,}0344 * y_2 + 0{,}0812 * y_3 + 0{,}0663 * y_4 + 0{,}151 * y_5 + 0{,}0599 * y_6 \leq v \\ 0{,}061 * y_1 + 0{,}3036 * y_2 + 0{,}1036 * y_3 + 0{,}0814 * y_4 + 0{,}1084 * y_5 + 0{,}1158 * y_6 \leq v \\ 0{,}1286 * y_1 + 0{,}1556 * y_2 + 0{,}2945 * y_3 + 0{,}039 * y_4 + 0{,}1306 * y_5 + 0{,}1001 * y_6 \leq v \end{cases}, \quad (9)$$

$$\begin{cases} 0{,}0248 * q_1 + 0{,}0611 * q_2 + 0{,}1539 * q_3 + 0{,}069 * q_4 + 0{,}174 * q_5 + 0{,}025 * q_6 \leq 1 \\ 0{,}0539 * q_1 + 0{,}0344 * q_2 + 0{,}0812 * q_3 + 0{,}0663 * q_4 + 0{,}151 * q_5 + 0{,}0599 * q_6 \leq 1 \\ 0{,}061 * q_1 + 0{,}3036 * q_2 + 0{,}1036 * q_3 + 0{,}0814 * q_4 + 0{,}1084 * q_5 + 0{,}1158 * q_6 \leq 1 \\ 0{,}1286 * q_1 + 0{,}1556 * q_2 + 0{,}2945 * q_3 + 0{,}039 * q_4 + 0{,}1306 * q_5 + 0{,}1001 * q_6 \leq 1 \end{cases}, \quad (10)$$

The best investment behavior strategy is in minimizing the values *(p)*, i.e. $P = p_1 + p_2 + p_3 + p_4 + p_5 + p_6 \rightarrow$ min. The strategy of elemental forces is in maximizing the values *(q)*, i.e. $Q = q_1 + q_2 + q_3 + q_4 + q_5 + q_6 \rightarrow max$. Using a simplex method, the values *p* and *q* that satisfy the best strategies are discovered. Both specialized and standardized programs can be used for calculculations, for example the add-in "Solver" in Microsoft Excel.

The following values are obtained in this example:

P = (0; 1.005307; 0.038276; 0; 6.372911; 0);

Q = (0.799379; 1.566597; 0.790362; 0.11567; 0.042683; 4.101803).

Since $P_{min} = Q_{max} = 7.416493$, then $v = 0.135$

Hence, the vectors of the players' strategies are equal:

X = (0; 0.1356; 0.0052; 0; 0.8593; 0);

Y = (0.1078; 0.2112; 0.1066; 0.0156; 0.0057; 0.5531).

Thus, the best strategy of the enterprise *(X)* is graphically illustrated in Figure 3.

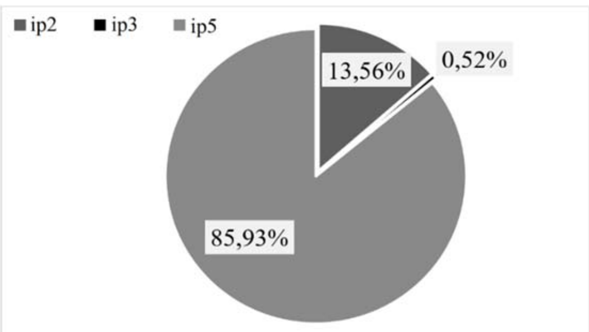

Figure 3. The best strategy of the business entity

It can be argued that as a result of the proposed investment behavior strategy, the enterprise will obtain the guaranteed level of return in the amount of 13.5%. In order to achieve this value, using the following proportions of the investment portfolios is the most rational: 13.56% – (ip_2); 0.52% – (ip_3); 85.93% – (ip_5). Other available portfolios are recommended not to be considered.

5. Discussion of the Results

In order to substantiate the effectiveness of the data obtained, it is suggested that the Sharpe ratio be used. The average return of the selected investment portfolios in the calculated proportion is equal to 14.08%, while risk is reduced to a minimum and equals 4.11%. The average level of return between the maximum interest on bank deposits and federal loan bonds, which equals to the value of 7.2%, is taken as a risk-free asset. Thus, the Sharpe ratio for the suggested investment behavior strategy equals: *S = (14.08% – 4.11%) / 7.2 = 1.67*. The value obtained demonstrates the high effectiveness of the created investment portfolio based on the return and risk ratio, which proves the significance of the authors' approach to the problem of formulating the investment behavior strategy.

Then, it is proposed to compare the data obtained using the authors' approach to the classical criteria of game theory:

1. Laplace criterion.

$ip_1 = (0.0248 + 0.0539 + 0.061 + 0.1286) * (\frac{1}{4}) = 0.067075$

$ip_2 = (0.0611 + 0.0344 + 0.3036 + 0.1556) * (\frac{1}{4}) = 0.138675$

$ip_3 = (0.1539 + 0.0812 + 0.1036 + 0.2945) * (\frac{1}{4}) = 0.1583$

$ip_4 = (0.069 + 0.0663 + 0.0814 + 0.039) * (\frac{1}{4}) = 0.063925$

$ip_5 = (0.174 + 0.151 + 0.1084 + 0.1306) * (\frac{1}{4}) = 0.141$

$ip_6 = (0.025 + 0.0599 + 0.1158 + 0.1001) * (\frac{1}{4}) = 0.0752$

According to this criterion, the maximum value should be selected, i.e. the portfolio ip_3. With the equal possibility of occurrence of all events, this portfolio will bring the biggest return. However, with the lack of diversification, the risk level that can emerge during market fluctuations and affect the return is not considered. The following most important portfolios are ip_5 and ip_2. They suggest lower return, but are less risky and more significant according to the authors' approach.

2. Wald's maximin model:

In case of the worst-case scenario unfolds on the market and the economic entity has no uncertainty about the time of realization of the investment portfolio, risks must be minimized and the maximum level of return on the assets be ensured at the same time. Thus, the portfolio that corresponds to the lowest value of the game is selected: $max_i \ min_j \ \alpha_{ij} = max \ \{0.0539; 0.0344; 0.0812; 0.039; 0.1084; 0.025\} = 0.1084$. In this example, the investment portfolio ip_5 is the best, which is also consistent with the authors' approach.

3. Savage criterion:

Maximum values: *max (1 month) = 0.174; max (3 months) = 0.151 max (6 months) = 0.3036; max (1 year) = 0.2945*. Table 5 considers the risk matrix.

Table 5. Risk matrix

	1 month	3 months	6 months	1 year
R ip_1	0.1492	0.0971	0.2426	0.1659
R ip_2	0.1129	0.1166	0	0.1389
R ip_3	0.0201	0.0698	0.2	0
R ip_4	0.105	0.0847	0.2222	0.2555
R ip_5	0	0	0.1952	0.1639
R ip_6	0.149	0.0911	0.1878	0.1944

Finally, we obtain the following values: $min \ R = \{0.1492; 0.1166; 0.2426; 0.2555\} = 0.1166$. Thus, the investment portfolio ip_2 is admissible from the perspective of risk, but its return is not the best. In the authors' approach this portfolio takes the second place in terms of significance and it is suggested that 13.56% of the available funds are be given to it. The data obtained demonstrate that this portfolio is significant for reducing the aggregate risk of the investment.

Implementing other game theory criteria, including the Hurwitz criterion and the Laplace–Bayes criterion, calls for a more detailed analysis of probabilities, which is less reasonable from the perspective of their practical application in investment activities in the time of uncertainty. However, the data obtained bear evidence that the portfolio ip_5 is maximum in terms of return; the portfolio ip_3 is the portfolio with the minimum risk and lowest return; the portfolio ip_2 is the best from the perspective of risk and return ratio. Consequently, the diversified investment behavior strategy proposed by the authors combines the totals of the classical examples.

In practical work it is common to be faced with a situation when there is lack of consent on the matter and there are many alternatives that can be implemented. An inability to choose the best solution in the time when data are scarce is caused by a majority of balances generated by a specific game. Applying the game theory criteria to investment modelling allows us to considerably reduce potential risks of lower return and direct the actions of the economic entity towards maximizing return.

The theory of rational choice relies on the idea that people are rational in pursuing their goals according to their personal interests. A rational choice in investment activities can be understood as analysis of potential profits or losses and consequent search for actions, taking into account potential risks and possibility of occurrence of some events. In this research [14], some significant aspects of choosing can be added to consider the conditions of the external environment, the type of behavioral interaction in this environment and the system used to evaluate the impact of the environment on the decisions of the entity. Many personality traits of investors and decision-makers can also be assessed, which has been analyzed in the paper by [27]. Game theory can be supplemented by the discussion of the behavioral factor, such as reciprocation, internal motivation and status.

As it is said in the work by L. Samuelson [6], economists are distinctive among social scientists due to their faith in methodological individualism, the essence of which is in explaining social phenomena through studying individual behavior that has certain consistency and stability in preferences. Thus, using game theory allows us to select the best alternative among a set of possible ones, which, considering the given preferences, has a higher grade in comparison with the other ones. Modelling investment strategies of both economic entities and individual investors is accompanied with strong uncertainty. Mathematical methods are the most significant for reducing risk in unstable conditions. If investment strategy is devised based on the game theory method, return on investment can be maximized, taking into account the high volatility and variability of the market [28].

6. Conclusion

The approach proposed in the authors' paper will help to reduce the level of risk and maintain the optimal return on investment. The specific feature of the method is that it can be adapted to various situations on the Russian market. Nevertheless, it cannot be called comprehensive, the same as a variety of other methods for rationalizing investment activities described in scientific literature. The constraint of the approach does not reduce its significance for those investors whose strategic thinking is focused on searching for new ways to increase return. Any sensible investor should choose to apply new methods of evaluating investment portfolios and revise his strategy as frequently as possible, as the market is not still but, instead, constantly changing [4].

In case when there is no saddle point in the game matrix, no single investment portfolio can be formed. However, portfolios can be diversified in certain proportions to rationalize investment behavior strategy. Such tactics result in maximizing return for a given period with simultaneous maintenance of the admissible level of risk. It should be noted that in order to obtain more reliable data, the gained return of every asset in the investment portfolio has to be analyzed and its aggregate level within the investment portfolio has to be defined [3].

In the example considered, the enterprise should not devise its investment behavior strategy based on the implementation of one investment portfolio. After making some calculations using the authors' model, it is suggested that the resources are to be diversified between three investment portfolios in the following proportion: 13.56% of the capital should be put into the second investment portfolio, 0.52% into the third, and 85.93% into the fifth. Such distribution of the capital will help to maximize return and reduce potential risks, considering the previous levels of return on assets of each portfolio. This approach can also be applied to other investment portfolios, which will allow us to define the best investment behavior strategy in order to improve the efficiency of the financial resources due to combining various proportions of investment portfolios, and to reduce investment risks.

The testing of this method proves its effectiveness. Nevertheless, to make the analysis more comprehensive, it is suggested that further research should be directed towards expanding the scope of application of the approach considered. It is expected that the research will be adapted to selecting the most profitable assets on developing markets, to the investment processes taking place when the level of the company's intellectualization is changing, as well as to implementing investment projects at the level of territorial units. For example, now there is no doubt that the investment attractiveness of a region has a direct impact on the influx of investment resources [29]. Applying the game theory criteria makes it possible to select the projects that are not only the most profitable but also have the biggest significance for the development of the infrastructure of the territorial unit. Referring to the promotions on the developing markets, their increased volatility and specific features, such as development of the raw material and industrial sector, have to be considered [30]. Thus, it is suggested that error factors are to be introduced to supplement the game theory method.

The game theory method can be modernized to devise investment behavior strategy using correlation and regression analysis, thanks to which it will be possible to reveal the factors seriously affecting the variables of the game matrix, and suggest alternative strategies for a given time period.

Acknowledgements

This research work was supported by the Academic Excellence Project 5-100 proposed by Peter the Great St. Petersburg Polytechnic University.

References

[1]. Mohammaditabar, D., Ghodsypour, S. H., & Hafezalkotob, A. (2016). A game theoretic analysis in capacity-constrained supplier-selection and cooperation by considering the total supply chain inventory costs. *International Journal of Production Economics*, *181*, 87-97. https://doi.org/10.1016/j.ijpe.2015.11.016

[2]. Bonabeau, E. (2002). Agent-based modeling: Methods and techniques for simulating human systems. *Proceedings of the national academy of sciences*, *99*(suppl 3), 7280-7287. https://doi.org/10.1073/pnas.082080899

[3]. Zaitsev, A., Kichigin, O., & Korotkova, A. (2019, October). Standard dynamic financial analysis and control tools of an enterprise in the time of digital economy. In *Proceedings of the 2019 International SPBPU Scientific Conference on Innovations in Digital Economy* (pp. 1-7). https://doi.org/10.1145/3372177.3373305

[4]. Dixit, A. K., & Nalebuff, B. J. (2010). The Art of Strategy: A Game Theorist's Guide to Success in Business and Life.

[5]. LU, W., & QIN, H. P. (2018). Research on Optimization of Webcast Income Distribution Based on Game Theory. *DEStech Transactions on Engineering and Technology Research*, (pmsms). https://doi.org/10.12783/dtetr/pmsms2018/24943

[6]. Samuelson, L. (2017). Game theory in economics and beyond. *Voprosy Ekonomiki*, (5), 89-115. https://doi.org/10.32609/0042-8736-2017-5-89-115

[7]. Fox W.P., Burks R. (2019) Game Theory. In: Applications of Operations Research and Management Science for Military Decision Making. International Series in Operations Research & Management Science, vol 283. Springer, Cham. https://doi.org/10.1007/978-3-030-20569-0_6

[8]. Allen, F., & Morris, S. E. (2014). Game theory models in finance. In *International Series in Operations Research and Management Science* (pp. 17-41). Springer New York LLC. https://doi.org/10.1007/978-1-4614-7095-3_2

[9]. Breen, R. (2017). Game theory. In The Oxford Handbook of Analytical Sociology, Oxford press, USA. https://doi.org/10.1093/oxfordhb/9780199215362.013.26

[10]. Newton, J. (2018). Evolutionary game theory: A renaissance. *Games*, *9*(2), 31. https://doi.org/10.3390/g9020031

[11]. Breton M. (2018) Dynamic Games in Finance. In: Başar T., Zaccour G. (eds) Handbook of Dynamic Game Theory. Springer, Cham. https://doi.org/10.1007/978-3-319-44374-4_23

[12]. Smit, H. T., & Trigeorgis, L. (2012). *Strategic investment: Real options and games*. Princeton University Press. https://doi.org/10.2307/j.ctvcm4hg0

[13]. Morgenstern, O., & Von Neumann, J. (1953). *Theory of games and economic behavior*. Princeton university press.

[14]. Askari, G., Gordji, M. E., & Park, C. (2019). The behavioral model and game theory. *Palgrave Communications*, *5*(1), 1-8. https://doi.org/10.1057/s41599-019-0265-2

[15]. Bieta, V., & Smelyanets, P. (2007). Game Theory and Financial Markets. *Voprosy Ekonomiki*, (10), 114-124. https://doi.org/10.32609/0042-8736-2007-10-114-124

[16]. Daskalakis, C., Deckelbaum, A., & Kim, A. (2015). Near-optimal no-regret algorithms for zero-sum games. *Games and Economic Behavior*, *92*(C), 327-348. https://doi.org/10.1016/j.geb.2014.01.003

[17]. Mednikov, M. D., Sokolitsyn, A. S., Ivanov, M. V., Sokolitsyna, N. A., & Yuryev, V. N. (2018). Capital structuration as enterprise management strategy elaboration basis. In *Proceedings of the 32nd International Business Information Management Association Conference, IBIMA 2018-Vision 2020: Sustainable Economic Development and Application of Innovation Management from Regional expansion to Global Growth* (pp. 6600-6613).

[18]. Nikolova, L. V., Abramchikova, N. V., & Velikova, M. D. (2019). The investment program of industrial enterprises under conditions of limited resources. In *Proceedings of the 33rd International Business Information Management Association Conference, IBIMA 2019: Education Excellence and Innovation Management through Vision 2020* (pp. 5433-5445).

[19]. Demidenko, D. S., & Dubolazova, Y. A. (2019). Drawing up an optimal investment program for innovative development of an enterprise. In *Proceedings of the European Conference on Innovation and Entrepreneurship, ECIE* (pp. 254-259). https://doi.org/10.34190/ECIE.19.220

[20]. Kuporov, J. J., Kudryavtseva, T. J., & Gorovoy, A. A. (2018). Algorithm for formation of the investment project portfolio of a public utility company. In *Innovation Management and Education Excellence through Vision 2020* (pp. 941-954).

[21]. Dmitriev, N., Zaytsev, A., & Dubanevich, L. (2020). Determining the Strategic Prospects of an Enterprise by Assessing the Dynamics of its Intellectual Rent. *Montenegrin Journal of Economics*, *16*(4), 187-197. https://doi.org /10.14254/1800-5845/2020.16-4.15

[22]. Krasyuk, I. A., Kobeleva, A. A., Mikhailushkin, P. V., Terskaya, G. A., & Chuvakhina, L. G. (2018). Economic interests focusing as a basis of the formation of investment policy. *Espacios*, *39*(28), 518-531.

[23]. Adamenko, A. A., Zolotukhina, E. B., Ulanov, V. A., Samoylova, E. S., Chizhankova, I. V., & Mamatelashvilli, O. V. (2017). Investment management activities of commercial enterprises. *International Journal of Applied Business and Economic Research*, *15*(12), 11.

[24]. Lukashevich, N., Garanin, D., & Konnikov, E. (2019). Modeling of the parameters of the investment project based on the information-statistical approach. In *Proceedings of the 33rd International Business Information Management Association Conference, IBIMA 2019: Education Excellence and Innovation Management through Vision 2020* (pp. 8743-8752).

[25]. Liao, Y. L., & Hong, Q. L. (2011, May). Survey of option game theory of R&D investment. In *2011 International Conference on E-Business and E-Government (ICEE)* (pp. 1-6). IEEE. https://doi.org/10.1109/ICEBEG.2011.5881618

[26]. Shirokova, S. V., & Naidenysheva, E. G. (2017). Application of game theory in the project management of interaction between business structures and public-private partnerships. In *Proceedings of the 29th International Business Information Management Association Conference-Education Excellence and Innovation Management through Vision 2020: From Regional Development Sustainability to Global Economic Growth* (pp. 1150-1156).

[27]. Leider, S. (2019). Behavioral analysis of strategic interactions: Game theory, bargaining, and agency. In The Handbook of Behavioral Operations, 7, John Wiley & Sons, USA. https://doi.org/10.1002/9781119138341.ch7

[28]. Alexander, J. (2016). Evolutionary game theory. In W. Batchelder, H. Colonius, E. Dzhafarov, & J. Myung (Eds.), *New Handbook of Mathematical Psychology* (Cambridge Handbooks in Psychology, pp. 322-373). Cambridge: Cambridge University Press. doi:10.1017/9781139245913.007

[29]. Polyakova, A. G., Polyakova, A. G., Akhmetshin, E. M., Pavlyuk, A. V., & Meshkova, G. V. (2019). Investment appeal of a region and its impact on investment inflows. *Entrepreneurship and Sustainability Issues*, *7*(2), 1089-1097. https://doi.org/10.9770/jesi.2019.7.2(21)

[30]. Akhmetshin, E. M., Ilyasov, R. H., Sverdlikova, E. A., Tagibova, A. A., Tolmachev, A. V., & Yumashev, A. V. (2018). Promotion in emerging markets. *European Research Studies*, *21*, 652-665.